ENGINEERING MATHEMATICS

2ND SEMESTER

- PREPARED BY

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Limit And Continuity

Important Notes And Terms

Geometrically

* let 'f' be a function define in same neighbourhood of expect possible at 'a' and 'L' be number we say that limit of $f(x)$ as 'x' approach to 'a' is 'L' i.e,

$$\lim_{x \to a} f(x) = L$$

for any $\varepsilon > 0$ is how even small their exist ('f') a number 'a' where $\delta > 0$, such that

$$|f(x) - L| < \varepsilon$$

Process To Find Limit

i) Left Hand Rule | left Hand limit

* let a function '$f(x)$' and we have to find limit at $x = a$.

Step-1:- $\lim_{x \to a^-} f(x)$

Step-2:- Put, $x = a-h$ and replaced $a-h$ by $h \to 0$ to obtain limit

$$\lim_{h \to 0} f(a-h)$$

Step-3:- Simplify $\lim_{h \to 0}$ by using formula for the given function.

Step-4:- The value obtained step-3 is LHL of $f(x)$ at $x = a$.

D.T.O

(ii) Right Hand Rule | Right hand limit

Step 1:- $\lim_{x \to a^+} f(x)$

Step-2:- Put, $x = a+h$ and replaced by $x \to a+h$ by $h \to 0$ and obtained $\lim_{h \to 0} f(a+h)$

Step-3:- The value obtained in step-3 is the limiting value of $f(x)$ and $x = a$.

Example

Q) $f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$ at $x = 4$

Ans) **LHL**

$\lim_{x \to 4^-} f(x)$

Put $x = 4-h$,

$\lim_{h \to 0} f(4-h)$

$\Rightarrow \lim_{h \to 0} \left(\frac{|4-h-4|}{4-h-4} \right)$

$\Rightarrow \lim_{h \to 0} \frac{|-h|}{-h}$

$\Rightarrow \lim_{h \to 0} (-1) = (-1)$

RHL

$\lim_{x \to 4^+} f(x)$

Put $x = 4+h$

$\lim_{h \to 0} f(4+h)$

$\Rightarrow \lim_{h \to 0} \left(\frac{|4+h-4|}{4+h-4} \right)$

$\Rightarrow \lim_{h \to 0} \left(\frac{h}{h} \right)$

$\Rightarrow \lim_{h \to 0} (1) = 1$

So LHL $\neq$ RHL

so limit doesnot exist.

P.T.O

Q) $f(x) = \begin{cases} 1+x^2 , & 0 < x \le 1 \\ 2-x , & x > 1 \end{cases}$ at $x=1$ show that limit doesnot exist.

Ans) LHL

$\lim_{x \to 1^-} f(x)$

Put $x = 1-h$

$\lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} (1+(1-h)^2)$

$= \lim_{h \to 0} (1+1+h^2-2h)$

$= \lim_{h \to 0} (2+h^2-2h)$

$= 2+(0-2\times 0)$

$= 2$

RHL

$\lim_{x \to 1^+} f(x)$

Put $x = 1+h$

$\lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} (2-(1+h))$

$= \lim_{h \to 0} (1-h)$

$= 1-0$

$= 1$

So LHL $\neq$ RHL , so limit doesnot exist.

Q) $f(x) = \begin{cases} \frac{x-|x|}{x} , & x \neq 0 \\ 2 , & x = 0 \end{cases}$ at $x=0$ show that the limit doesnot exist.

Ans) LHL

$\lim_{x \to 0^-} f(x)$

$\Rightarrow$ Put, $x = 0-h$

$\Rightarrow \lim_{h \to 0} f(-h)$

$\Rightarrow \lim_{h \to 0} \left(\frac{-h-|-h|}{-h}\right) = \lim_{h \to 0} \left(\frac{2\cancel{h}}{\cancel{h}}\right) = \lim_{h \to 0} (2) = 2$

RHL

$\lim_{x \to 0^+} f(x)$

Now,

Put $x = 0+h = h$

$$\lim_{h \to 0} f(h) = \lim_{h \to 0} \left(\frac{h - |h|}{h}\right) = \lim_{h \to 0} \left(\frac{0}{h}\right) = 0$$

$\therefore$ LHL $\neq$ RHL, so limit doesnot exist.

Algebra of limit

* let,

$\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = m$

then,

i) $\lim_{x \to a} (f \pm g)(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = L \pm m$

ii) $\lim_{x \to a} (fg)(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = L \cdot m$

iii) $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{L}{m}$

iv) $\lim_{x \to a} (x \cdot f)(x) = x \lim_{x \to a} f(x) = xL$ ($\because$ K is any constant)

Evolution of limit

* There are three types of limit. They are:

P.T.O

i) Algebric ~~unit~~ limit

ii) Trignometric ~~unit~~ limit

iii) Exponential limit

Algebric Unit

* let $f(x)$ be an algebric function and a and b any real number then $\lim_{x \to a} f(x)$ is known as algebric unit.

* Algebric limit are of four types.

  i) Direct substitution Method

  ii) Factorisation Method

  iii) Rationalisation Method

  iv) Using some standard Method, when $x \to \infty$

Direct Substitution Method

* Direct substitution Method of the point in the given expression we get a finite number then thea number is obtained is the limit of the given expression.

* Example:- i) $\lim_{x \to 1} (3x^2+4x+5)$

Ans) $\lim_{x \to 1} (3 \times (1)^2 + 4 \times 1 + 5)$

$= \lim_{x \to 1} (3+4+5)$

$= \lim_{x \to 1} (12) = 12$

P.T.O

iii) $\lim_{x\to 0}\left(\frac{\cos x}{\sin x+1}\right)$

Ans) $\lim_{x\to 0}\left(\frac{\cos x}{\sin x+1}\right) = \lim_{x\to 0}\left(\frac{1}{0+1}\right)$

$= \lim_{x\to 0}\left(\frac{1}{1}\right) = 1$

Factorisation Method

* $\lim_{x\to a}\frac{f(x)}{g(x)}$ by putting $x=a$ in the rational function $f(x)/g(x)$ takes the form $\frac{0}{0}, \frac{\infty}{\infty}$ or any thing then we find a factor for $x-a$ of both $f(x)$ and $g(x)$ in such case we factorise the numerator and denominator. and cancel the factor after cancelling we again put the point in the rational function produced further.

Example

Q) $\lim_{x\to 2}\frac{x^3-6x^2+11x-6}{x^2-6x+8}$

Ans) $\lim_{x\to 2}\frac{x^3-6x^2+11x-6}{x^2-6x+8}$

$= \lim_{x\to 2}\frac{(x-2)(x^2-4x+3)}{(x-2)(x-4)}$

$= \lim_{x\to 2}\left(\frac{x^2-4x+3}{x-4}\right) = \frac{(2)^2-4\times2+3}{2-4} = \frac{4-8+3}{-2} = \frac{-1}{-2} = \frac{1}{2}$

P.T.O

Q) ii) $\lim_{x \to 3} \dfrac{x^3 - 7x^2 + 15x - 9}{x^4 - 5x^3 + 27x - 27}$

Ans) $\lim_{x \to 3} \dfrac{\cancel{(x-3)}(x^2 - 4x + 3)}{\cancel{(x-3)}(x^3 - 2x^2 - 6x + 9)}$

$= \lim_{x \to 3} \dfrac{x^2 - 4x + 3}{x^3 - 2x^2 - 6x + 9}$

$= \lim_{x \to 3} \dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}(x^2 + x - 3)}$

$= \lim_{x \to 3} \dfrac{x-1}{x^2 + x - 3}$

$= \dfrac{3-1}{9+3-3} = \dfrac{2}{9}$

Rationalisation

* when the numerator and denominator contains some irrational form and direct substitution form give $\frac{0}{0}$ form then we apply rationalisation. In this method we rationalise the irrational function to element to $\frac{0}{1}$ form.

Example : i) $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$

Ans) $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1}$

P.T.O

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1}-1)(\sqrt{x+1}+1)}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1}+1)}{(\sqrt{x+1})^2-(1)^2}$$

$$= \lim_{x \to 0} \frac{\cancel{x}(\sqrt{x+1}+1)}{\cancel{x}+\cancel{1}-\cancel{1}}$$

$$= \lim_{x \to 0} \sqrt{x+1}+1$$

$$= \sqrt{0+1}+1$$

$$= 1+1 = 2$$

iii) Evaluate $\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

Ans) $\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

$$= \lim_{x \to 0} \frac{((\sqrt{1+x})-(\sqrt{1-x}))(\sqrt{1+x}+\sqrt{1-x})}{2x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1+x})^2-(\sqrt{1-x})^2}{2x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x \to 0} \frac{1+x-(1-x)}{2x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \lim_{x \to 0} \frac{2x}{2x(\sqrt{1+x}+\sqrt{1-x})}$$

$$= \frac{1}{\sqrt{1+0}+\sqrt{1-0}} = \frac{1}{2}$$

Chapter ⇒ Derivatives

25/04/2023

* Here in figure 3.1,

$y = f(x)$

$s =$ be any curve

$h =$ small increment

$f(x+h) =$ small encrement in function

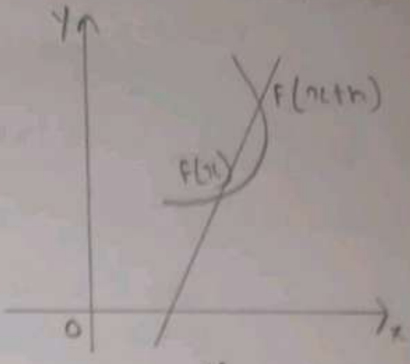

Figure 3.1

Principles of Derivatives

i) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Q) Find the derivative of $f(x) = \sin x$ by using first principle of derivative.

Ans) Here,

$f(x) = \sin x$

Now,

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\frac{x+h+x}{2} \cdot \sin\frac{x+h-x}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2}\cos\frac{2x+h}{2} \cdot \sin\frac{h}{2}}{\cancel{2} \times \frac{h}{2}}$$

$$= \lim_{h \to 0} \cos\frac{2x+h}{2} \cdot \left(\lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= \lim_{h \to 0} \cos\frac{2x+0}{2}$$

$$= \lim_{h \to 0} (\cos x) = \cos x.$$

Q) Find the derivative of $\cos x$ by using first principle of derivative.

Ans) Here,

$f x = \cos x$

Now,

$$f'(x) = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{2\left(\sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2}\right)}{h} = \frac{2 \sin \frac{x+h+x}{2} \cdot \sin \frac{x-x-h}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2} \sin \frac{2x+h}{2} \cdot \sin \frac{-h}{2}}{\cancel{2} \times \frac{h}{2}}$$

$$= - \lim_{h \to 0} \frac{\sin \frac{2x+h}{2} \cdot \sin \frac{h}{2}}{\frac{h}{2}}$$

$$= - \lim_{h \to 0} \sin \frac{2x+h}{2} \cdot \left(\lim_{h \to 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)$$

$$= - \lim_{h \to 0} \sin \frac{2x+h}{2} \cdot 1$$

$$= - \sin \frac{2x+0}{2}$$

$$= - \sin \frac{\cancel{2}x}{\cancel{2}}$$

$$= - \sin x$$

Algebra of Derivate

26/4/23

* let $f(x)$ and $g(x)$ be two function and its $\frac{d}{dx}$ exists.

Q6) i) $\frac{d}{dx} x^5$

Ans) $\frac{d}{dx} x^5 = 5x^{5-1} = 5x^4$

ii) $\frac{d}{dx} x^{-9}$

Ans) $\frac{d}{dx} x^{-9} = -9x^{-9-1} = -9x^{-10}$

iii) $\frac{d}{dx} (7x^{-7})$

Ans) $\frac{d}{dx} 7x^{-7} = 7\left[-7x^{-7-1}\right]$

$= 7(-7x^{-8})$

$= -49x^{-8}$

iv) $\frac{d}{dx} (-8x^5)$

Ans) $\frac{d}{dx} (-8x^5) = -8\left[5x^{5-1}\right]$

$= -8(5x^4)$

$= -40x^4$

v) $\frac{d}{dx} (-x)$

Ans) $\frac{d}{dx} (-x) = -\frac{dx}{dx} = -1(x^{1-1}) = -x^0 = -1$

P.T.O

vii) $\frac{d}{dx}(-7x^{-9})$

Ans) $\frac{d}{dx} -7x^{-9} = -7\left[-9x^{-9-1}\right]$

$= +63x^{-10}$

viii) $\frac{d}{dx}(ax)$

Ans) $\frac{d}{dx}(ax) = a\frac{d}{dx}x = a\left[1 \cdot x^{1-1}\right] = ax^0 = a \cdot 1 = a$

ix) $\frac{d}{dx}(bx^2)$

Ans) $\frac{d}{dx}(bx^2) = b\frac{d}{dx}x^2 = b\left[2x^{2-1}\right]$

$= b2x$

$= 2bx$

x) $\frac{d}{dx}(a+bx^9)$

Ans) $\frac{d}{dx}(a+bx^9) = \frac{d}{dx}(a) + \frac{d}{dx}bx^9$

$= 0 + b\left[9x^{9-1}\right]$

$= 9bx^8$

xi) $\frac{d}{dx}(\sqrt{x})$

Ans) $\frac{d}{dx}(\sqrt{x}) = x^{\frac{1}{2}}$

$= \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}} = \frac{x^{\left(-\frac{1}{2}\right)}}{2}$

xii) $\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$

Ans) $\frac{d}{dx}\left(x^{-\frac{1}{2}}\right) = \left(-\frac{1}{2}x^{-\frac{1}{2}-1}\right)$

$= \left(\frac{-x}{2}\right)^{-\frac{3}{2}}$

Derivative of composite function

* **Definition:-** A function formed by composition of more than one function.

  Ex:- $\sin x^2$

Chain Rule

* Let $y = f(u)$, where u is a function of x defined by $u = g(x)$, Then

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Generalisation of Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dv} \cdot \frac{dv}{dz} \cdot \frac{dz}{dt} \cdots \frac{d}{dt}$$

* **Example :-** i) $y = \sin x^2$

Ans) $\frac{dy}{dx} = \frac{d}{dx} \sin x^2$

Now, put $x^2 = u$.

$y = \sin u$

$\frac{dy}{du} = \sin u = \cos u$

Now,

$u = x^2$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx} x^2 = 2x$

Now,

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \cos u \cdot 2x = 2x \cdot \cos x^3$

$= \cancel{\cos u \cdot 2x^2}$

ii) $y = \cos x^3$

Ans) $\frac{dy}{dx} = \ldots$

Put $x^3 = u$

$y = \cos u$

$\Rightarrow \frac{dy}{du} = \frac{d}{du} \ldots$

Now,

$u = x^3$

$\Rightarrow \frac{du}{dx} = \ldots$

Now,

$\frac{dy}{dx} = \ldots$

iii) ...

Ans) Now, Put ... and ...

So, $\frac{dy}{du} = \ldots$

$\frac{du}{dx} = \ldots$

Now, $\frac{dy}{dx} = \ldots$

ii) $y = \cos x^3$

Ans) $\frac{dy}{dx} = \frac{d}{dx}\cos x^3$

Put $x^3 = u$,

$y = \cos u$

$\Rightarrow \frac{dy}{du} = \frac{d}{du}\cos u = -\sin u$

Now,

$u = x^3$

$\Rightarrow \frac{du}{dx} = \frac{d}{dx}x^3 = 3x^2$

Now,

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= -\sin u \cdot 3x^2$

$= -3x^2 \cdot \sin x^3$

iii) $y = (x^2 + 2x - 1)^5$

Ans) Now,

Put; $u = x^2 + 2x - 1$

and $y = u^5$

So,

$\frac{dy}{du} = \frac{d}{du}(u^5) = 5u^4$

$\frac{du}{dx} = \frac{d}{dx}(x^2 + 2x - 1) = 2x + 2$

Now,

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = 5u^4 \cdot (2x + 2)$

$= 5(x^2 + 2x - 1)^4 (2x + 2)$

3/5/2023

Derivative of Implicity function

Q) Find $\frac{dy}{dx}$ where $x^2+y^2=2axy$

Ans) $\frac{d}{dx}x^2+\frac{d}{dx}y^2=\frac{d}{dx}2axy$

$\Rightarrow 2x+2y\frac{dy}{dx}=2a\left(\frac{d}{dx}xy\right)$

$\Rightarrow 2x+2y\frac{dy}{dx}=2a\left\{x\frac{dy}{dx}+y\frac{dx}{dx}\right\}$

$\Rightarrow 2x+2y\frac{dy}{dx}=2ax\frac{dy}{dx}+2ay$

$\Rightarrow 2y\frac{dy}{dx}-2ax\frac{dy}{dx}=2ay-2x$

$\Rightarrow \frac{dy}{dx}(2y-2ax)=2ay-2x$

$\Rightarrow \frac{dy}{dx}=\frac{2ay-2x}{2y-2ax}=\frac{ay-x}{y-ax}$

Q2) Find $\frac{dy}{dx}$ where

$e^y\ln x+x\ln y=0$

Ans) $e^y\frac{d}{dx}\ln x+\ln x\frac{d}{dx}(e^y)+x\frac{d}{dx}\ln y+\ln y\frac{d}{dx}x=\frac{d}{dx}(0)$

$\Rightarrow e^y\frac{1}{x}+\ln x\, e^y\frac{dy}{dx}+x\frac{1}{y}\frac{dy}{dx}+\ln y=0$

$\Rightarrow \frac{dy}{dx}\left(\ln x\, e^y+\frac{x}{y}\right)=-\ln y-\frac{e^y}{x}$

$\Rightarrow \frac{dy}{dx}=\frac{-\left(\ln y+\frac{e^y}{x}\right)}{\ln x\, e^y+\frac{x}{y}}$

P.T.O

Differentiation using logarithmic

7/04/2023

The forms

i) $y = [f(x)]^{g(x)}$

ii) $y = [f(x) \cdot g(x)]$

iii) $y = [f(x)]^{g(x)} + [h(x)]^{t(x)}$

Answer / Derivation

i) $y = [f(x)]^{g(x)}$

Applying log on both sides,

$\log y = \log [f(x)]^{g(x)}$

$\Rightarrow \log y = g(x) \cdot \log f(x)$

ii) $y = [f(x) \cdot g(x)]$

Applying log on both sides,

$\log y = \log [f(x) \cdot g(x)]$

$\Rightarrow \log y = \log f(x) + \log g(x)$

iii) $y = [f(x)]^{g(x)} + [h(x)]^{t(x)}$

Applying log on both sides,

$\log y = \log [f(x)]^{g(x)} + \log [h(x)]^{t(x)}$

$\Rightarrow \log y = g(x) \cdot \log f(x) + t(x) \cdot \log (h(x))$

Now Assume,

$u = g(x) \cdot \log f(x)$

$v = t(x) \cdot \log h(x)$

find $\frac{du}{dx}$ and $\frac{dv}{dx}$

then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

P.T.O

Differentiation of Infinite series 8/04/2029

Q) $x^{x^{x^{x\ldots\infty}}}$

Ans) Here, let $x^{x^{x\ldots\infty}} = x^y$ (where $y = x^{x^{x\ldots}}$)

$y = x^y$

Applying log on both sides,

$\log y = y \log x$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}\right) = y \frac{d}{dx}\log x + \log x \frac{d}{dx} y$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}\right) = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\Rightarrow \frac{y}{x} = \frac{dy}{dx}\left(\frac{1}{y}\right) - \log x \cdot \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$ $\Rightarrow$

Q) $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\infty}}}}$

Prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

Ans) Here, $y = \sqrt{\sin x + y}$

$\Rightarrow y^2 = \sin x + y$ (squaring both side

$\Rightarrow y^2 - y = \sin x$

$\Rightarrow \frac{d}{dx}(y^2 - y) = \frac{d}{dx}\sin x$

$\Rightarrow 2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$

$\Rightarrow \frac{dy}{dx}(2y-1) = \cos x$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$

Successive Derivative

* Here,

1st $\frac{dy}{dx}$, y' , $f'(x)$, Dy , y_1

2nd $\frac{d^2y}{dx^2}$, y'' , $f''(x)$, Dy' , y_2

3rd $\frac{d^3y}{dx^3}$, y''' , $f'''(x)$, Dy'' , y_3

4th $\frac{d^4y}{dx^4}$, y^{iv} , $f^{iv}(x)$, Dy''' , y_4

Q) $y = x^4$. find y_1 , y_2 , y_3

Ans) Here,

$$y_1 = \frac{d}{dx} x^4 = 4x^3$$

$$y_2 = \frac{d}{dx} 4x^3 = 4\frac{d}{dx} x^3 = 4(3x^2) = 12x^2$$

$$y_3 = \frac{d}{dx} y_2 = \frac{d}{dx} 12x^2 = 12 \cdot 2x = 24x$$

Q) $y = \sin x$. find y_1 and y_3

Ans) Here,

$$y = \sin x$$

$$y_1 = \frac{d}{dx} \sin x = \cos x$$

$$y_2 = \frac{d}{dx} y_1 = \frac{d}{dx} \cos x = -\sin x$$

$$y_3 = \frac{d}{dx} y_2 = \frac{d}{dx}(-\sin x) = -\cos x$$

Q) $x = at^2$, $y = 2at$. find $\frac{d^2y}{dx^2}$

P.T.O

Differentiation of Parametric function

* let, $x = f(t)$ and $y = g(t)$ if two function and its derivative exist.

Then,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

Example

Q) $x = at^2$, $y = 2bt$. find $\frac{dy}{dx}$

Ans) Here,

$$\frac{dy}{dt} = 2bt = 2b\frac{d}{dt}t = 2b$$

$$\frac{dx}{dt} = at^2 = a\frac{d}{dt}t^2 = 2at$$

Now, $\frac{dy}{dt} \cdot \frac{dt}{dx} = \cancel{2}b \cdot \frac{1}{\cancel{2}at} = \frac{b}{at}$

Q) $y = a\cos\theta$ and $x = a(\theta + \sin\theta)$. find $\frac{dy}{dx}$.

Ans) Here,

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = a\frac{d}{d\theta}\cos\theta = -a\sin\theta$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta} \cdot a(\theta + \sin\theta) = a\frac{d}{d\theta}(\theta + \sin\theta)$$

$$= a(1 + \cos\theta)$$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$

$$= -\cancel{a}\sin\theta \cdot \frac{1}{\cancel{a}(1+\cos\theta)} = \frac{-\sin\theta}{1+\cos\theta}$$

Q) find $\frac{dy}{dx}$, $x = \theta + \sin\theta$, $y = 1 + \cos\theta$, where $\theta = \frac{\pi}{4}$

Homogeneous Function 12/05/2023

* Let $z = f(x,y)$ is said to be homogenous function of x and y of degree 'n' if $f(tx, ty) = t^n f(x,y)$, where n is positive and t = any constant.

* Expression of each term having same degree is also homogeneous function.

* If A Homogenous function of degree n, then its partial derivative is of degree $(n-1)$.

* If a function is in the form of a homogenous function of degree n, then it can be written as $z = x^n \phi\left(\frac{y}{x}\right)$

Q) Find the degree of homogeneous function

i) $f(x,y) = x^4 + x^3y - y^4$

Ans) $f(tx,ty) = (tx)^4 + (tx)^3 (ty) - (ty)^4$

$= t^4x^4 + t^3x^3 ty - t^4y^4$

$= t^4x^4 + t^4x^3y - t^4y^4$

$= t^4 (x^4 + x^3y - y^4)$

$= t^4 f(x,y)$

∴ It is a Homogenous function of degree 4.

ii) $f(x,y) = \sin^{-1}\left(\frac{xy}{x+y}\right)$

Ans) Here,

$u = \left(\frac{xy}{x+y}\right)$

Now,

$u(tx, ty) = \left(\frac{tx\,ty}{tx + ty}\right)$

$= \left(\frac{t^2 xy}{t(x+y)}\right) = t\left(\frac{xy}{x+y}\right) = t^1 u$

∴ It is a homogeneous function of degree 1.