Engineering Mathmatics => 1st Semester

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* MATRIX

matrix is an RA(arrangement) of rows (horizontal line) and coloumb (vertical line) of number functions or any other algebric function.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

sub diagonal

super diagonal

Diagonal element

$m\times n$ → order of matrix

- Order of matrix

Defind no. of row and coloumbs present in that matrix if there are m-rows and n-coloumb then order of the matrix is obtained by $m\times n$.

Types of Matrix

(a) Square matrix

A Matrix is said to be square matrix if the number of rows and no. of coloumbs are equal.

e.g- $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2\times 2}$ $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}_{3\times 3}$

(b) Null Matrix

A matrix is said to be null matrix if all the entries of the matrix are zero

e.g- $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2\times 2}$

$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{2\times 3}$

(C) Row Matrix

A Matrix is said to be row matrix if there is one row and n-number of columbs.

e.g → $A = (1, 2)_{1\times 2}$

$B = (0 \ 5 \ 3)_{1\times 3}$

(d) columb Matrix

A Matrix is said to be columb matrix if there is one columb and n-number of rows.

e.g - $A = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{2\times 1}$ $B = \begin{pmatrix} 0 \\ 5 \\ 3 \end{pmatrix}_{3\times 1}$

(e) Diagonal Matrix

A square matrix is said to be a diagonal matrix if all the diagonal elements are non-zero and rest other elements are zero

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(f) Identity Matrix

A diagonal matrix is said to be an identity matrix if all the diagonal element are unity.

e.g → $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times3}$

• It is denoted by 'I' and it is unique.

*Equality of Matrix

Let A and B be two matrix are said to be equal if they have same order.

If two matrix are equal then its corresponding elements are equal.

eg → $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$

then $a = e$, $b = f$, $c = g$ and $d = h$

*construction of Matrix

e.g → construct a (2x2) matrix where $a_{ij} = |2i - 3j|$

(Ans) $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}_{2\times2}$ Here $a_{11} = 1$ $a_{12} = 4$
$a_{21} = 1$ $a_{22} = 2$

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So $A = \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix}_{2\times 2}$ (Ans)

* operation of Matrix

(a) Addition of Matrix

(b) Substraction of Matrix

(c) Scalar multiplication of Matrix

(d) Multiplication of matrix.

(a) Addition and substraction of Matrix

Let A and B are two Matrix in addition/substraction is possible when they are in same order. Additions are obtained by operations of corresponding elements.

examples

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$A+B = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$

$A-B = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

(C)* Scalar Multiplication of Matrix

Let k be a non-zero scalar then scalar multiplication of a matrix is obtained by multiplication of scalar of each element of matrix.

e.g → $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$ (where k = scalar)

(d)* Multiplication of Matrix

It is possible if and only coloumb of first matrix is equal to row of second matrix.

Let A be a matrix of order $m \times n$ and B is a matrix of order $n \times p$ then order of multiplica of Matrix is $m \times p$.

Multiplication is obtained by i-th of 1st matrix is matrix is ~~i~~ jth coloumb of and marix and resultant martix arranged in row form.

e.g. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

then $AB = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

Home task

(a)* Given $A = \begin{pmatrix} 2 & 8 \\ 7 & 5 \end{pmatrix}_{2\times2}$ $B = \begin{pmatrix} 7 & 0 \\ 5 & 8 \end{pmatrix}_{2\times2}$

So $AB = \begin{bmatrix} 14+40 & 0+64 \\ 49+25 & 0+40 \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 54 & 64 \\ 74 & 40 \end{bmatrix}_{2\times2}$

$BA = \begin{bmatrix} 7 & 0 \\ 5 & 8 \end{bmatrix}_{2\times2} \times \begin{bmatrix} 2 & 8 \\ 7 & 5 \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 14+0 & 56+0 \\ 10+56 & 40+40 \end{bmatrix}_{2\times2}$

$= \begin{bmatrix} 14 & 56 \\ 66 & 80 \end{bmatrix}_{2\times2}$ (Ans.)

(b)* Given $A = \begin{bmatrix} 4 & -2 & -5 \\ 4 & 8 & 0 \end{bmatrix}_{2\times3}$ $B = \begin{bmatrix} 4 & -3 \\ -1 & -7 \\ 5 & 2 \end{bmatrix}_{3\times2}$

then AB

$$= \begin{bmatrix} 16+2-25 & -12+14-10 \\ 16-8+0 & -12-56+0 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} -7 & -8 \\ 8 & -68 \end{bmatrix}_{2\times2}$$

$$BA = \begin{bmatrix} 4 & -3 \\ -1 & -7 \\ 5 & 2 \end{bmatrix}_{3\times2} \times \begin{bmatrix} 4 & -2 & -5 \\ 4 & 8 & 0 \end{bmatrix}_{2\times3}$$

$$= \begin{bmatrix} 16-12 & -8-24 & -20+0 \\ -4-28 & 2-56 & 5+0 \\ 20+8 & -10+16 & -25+0 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 4 & -32 & -20 \\ -32 & -54 & 5 \\ 28 & 6 & -25 \end{bmatrix}_{3\times3}$$ (Ans)

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(C)* Given $A = \begin{pmatrix} 4 & 2 & 1 \\ 7 & -5 & 0 \\ 0 & 5 & 0 \end{pmatrix}_{3\times3}$ $B = \begin{pmatrix} 3 & 2 & -5 \\ 1 & -7 & 6 \\ 9 & 5 & 3 \end{pmatrix}_{3\times3}$

So AB

$$= \begin{bmatrix} 12+2+9 & 8-14+5 & -20+12+3 \\ 21-5+0 & 14+35+0 & -35-30+0 \\ 0+5+0 & 0-35+0 & 0+30+0 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 23 & -1 & -5 \\ 16 & 49 & -65 \\ 5 & -35 & 30 \end{bmatrix}_{3\times3}$$

$$BA = \begin{bmatrix} 3 & 2 & -5 \\ 1 & -7 & 6 \\ 9 & 5 & 3 \end{bmatrix}_{3\times3} \times \begin{bmatrix} 4 & 2 & 1 \\ 7 & -5 & 0 \\ 0 & 5 & 0 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 12+14+0 & 6-10-25 & 3+0+0 \\ 4-49+0 & 2+35+30 & 1+0+0 \\ 36+35 & 18-25+15 & 9+0+0 \end{bmatrix}_{3\times3}$$

$$= \begin{bmatrix} 26 & -29 & 3 \\ -45 & 67 & 1 \\ 71 & 8 & 9 \end{bmatrix}_{3\times3}$$ (Ans)

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* Home task

(a)* Given $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}_{3\times3}$

Here $A^2 = A \times A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}_{3\times3}$$

So $A^2 - 5A + 6I_3$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}_{3\times3} \quad \text{(Ans)}$$

(b)* Given $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}_{3\times3}$

Here $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

LHS

$A^2 - 4A - 5I = 0$

$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$ = RHS (proved)

Determinate 04 September 2023

1x1 matrix for determinate.

e.g - $|a| = a$

2x2 determination.

e.g - $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - cb)$

3x3 determination.

e.g - $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - gf) + c(dh - ge)$

examples

$A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 1(45-48) - 2(36-42) + 3(32-35)$

$= 0$

* Properties of determinate.

(a)* If any row or coloumbs have idential entries in value of determinate is zero.

$$e.g \rightarrow \begin{vmatrix} 1 & 2 & 7 \\ 0 & 5 & 9 \\ 1 & 2 & 7 \end{vmatrix} = 0$$

(b)* If any two row or coloumb are interchange then the sighn of determinate change.

eg → value of $\begin{vmatrix} 4 & 9 \\ 8 & 7 \end{vmatrix} = -44$

but

$$\begin{vmatrix} 8 & 7 \\ 4 & 9 \end{vmatrix} = 44$$

(c)* If a factor addition with any row or any coloumb the determinate is obtained spliting into two sum of matrix.

$$eg. \quad A = \begin{bmatrix} m+a & m+b & m+c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= \begin{vmatrix} m & m & m \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

(d)* If a factor multiplied with any coloumb or any row then determinate obtained by taking common as factor and multiply with the value of determinate.

$$eg \to A = \begin{vmatrix} ma & mb & mc \\ d & e & f \\ g & h & i \end{vmatrix} = m \times \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

(e)* If a row or a coloumb have all ent

*Transpose Matrix ($A^T / A^t / A'$)

It is obtained by inerchanging row into coloumb or coloumb into raw.

$$e.g \to A = \begin{pmatrix} 1 & 2 \\ 7 & -1 \end{pmatrix}, \ A^T = \begin{pmatrix} 1 & 7 \\ 2 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}, \ A^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

* Minor Matrix.

It is obtained by deleting corres conding row and coloumb, If it is 3X3 matrix then after deleting corresponding row or coloumb we get a 2X2 matrix, to required answer find determinate of 2X2 matrix.

→ minor of matrix denoted by "M"

and minors are denoted by m

e.g. → $A = \begin{pmatrix} 2 & 8 \\ 4 & 9 \end{pmatrix}$

$m_{11} = 9$

$m_{12} = 4$

$m_{21} = 8$

$m_{22} = 2$

$m = \begin{pmatrix} 9 & 4 \\ 8 & 2 \end{pmatrix}$

* co-Factor of matrix

$C_{ij} = (-1)^{i+j} m_{ij}$

where C_{ij} = co-factor

m_{ij} = minor

* Adjoint of Matrix

Let [A] be a matrix and its co-factor matrix is denoted by C then adjoint of [A] is denoted by $\boxed{adj A = C^T}$

* Inverse of a matrix

Inverse of a matrix is denoted by

$A^{-1} = \frac{1}{|A|} adj A$

Process to find inverse of a matrix

(a)* Find determinate of given matrix.

case-1
If determinate is equal to zero then inverse doesn't exist.

case-2
If determinat is not equal to zero then procced to process -(2)

(b)* Find minor of the given matrix.

(c)* Find co-factor.

(d)* Find adjoint.

(e)* Find inverse

(f)* To find value of variable put $X = A^{-1}B$ ($\because X$ = Variable)

Home work

* Find inverse of matrix

(a) $A = \begin{pmatrix} 1 & 2 \\ 5 & 8 \end{pmatrix}$

$|A| = 8 - 10 = -2$

$\text{adj } A = \begin{pmatrix} 8 & -2 \\ -5 & 1 \end{pmatrix}$

then $A^{-1} = \frac{1}{(-2)} \begin{bmatrix} 8 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5/2 & -1/2 \end{bmatrix}_{2\times 2}$

(b)* $A = \begin{bmatrix} 3 & -2 \\ 0 & 8 \end{bmatrix}$

$|A| = 24$

$adj = \begin{bmatrix} 8 & 2 \\ 0 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{24} \times \begin{bmatrix} 8 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/12 \\ 0 & 1/8 \end{bmatrix}$ (Ans)

(c)* $A = \begin{bmatrix} 4 & 2 & 8 \\ 3 & 1 & 7 \\ 0 & 0 & 5 \end{bmatrix}$

$|A| = 5(4-6) = -10$

$m_{11} = 5$ $\quad m_{21} = 10$ $\quad m_{31} = -1$

$m_{12} = 15$ $\quad m_{22} = 20$ $\quad m_{32} = 4$

$m_{13} = 0$ $\quad m_{23} = 0$ $\quad m_{33} = -2$

$C_{11} = (-1)^2 \times 5 = 5$ $\quad C_{21} = -10$ $\quad C_{31} = (-1)$

$C_{12} = -15$ $\quad C_{22} = 20$ $\quad C_{32} = (-4)$

$C_{13} = 0$ $\quad C_{23} = 0$ $\quad C_{33} = -2$

$C = \begin{bmatrix} 5 & -15 & 0 \\ -10 & 20 & 0 \\ -1 & -4 & -2 \end{bmatrix}$ $\quad adj A = \begin{bmatrix} 5 & -10 & -1 \\ -15 & 20 & -4 \\ 0 & 0 & -2 \end{bmatrix}$

$$A^{-1} = \frac{1}{(-10)} \times \begin{bmatrix} 5 & -10 & -1 \\ -15 & 20 & -4 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 1 & 1/10 \\ 3/2 & -2 & 2/5 \\ 0 & 0 & 1/5 \end{bmatrix} \quad \text{(Ans)}$$

(d) * $A = \begin{bmatrix} 5 & 1 & 7 \\ 0 & -8 & 1 \\ 3 & -2 & -2 \end{bmatrix}$

$$|A| = 5(16+2) + 3(1+56)$$

$$= (5\times18) + (57\times3)$$

$$= 90 + 171 = 261$$

$m_{11} = 18$ $\quad$ $m_{21} = -2+14 = 12$ $\quad$ $m_{31} = 57$

$m_{12} = -3$ $\quad$ $m_{22} = -10-21 = -31$ $\quad$ $m_{32} = 5$

$m_{13} = 24$ $\quad$ $m_{23} = -13$ $\quad$ $m_{33} = -40$

$c_{11} = 18$ $\quad$ $c_{21} = -12$ $\quad$ $c_{31} = 57$

$c_{12} = 3$ $\quad$ $c_{22} = -31$ $\quad$ $c_{32} = -5$

$c_{13} = 24$ $\quad$ $c_{23} = 13$ $\quad$ $c_{33} = -40$

$C = \begin{bmatrix} 18 & 3 & 24 \\ -12 & -31 & 13 \\ 57 & -5 & -40 \end{bmatrix}$ $\quad$ $\operatorname{adj} A = \begin{bmatrix} 18 & -12 & 57 \\ 3 & -31 & -5 \\ 24 & 13 & -40 \end{bmatrix}$

$$A^{-1} = \frac{1}{261}\begin{bmatrix} 18 & -12 & 57 \\ 3 & -31 & -5 \\ 24 & 13 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\cancel{18}^{2}}{\cancel{261}_{29}} & \frac{-12}{261} & \frac{57}{261} \\ 3/261 & -31/261 & -5/261 \\ 24/261 & 13/261 & -40/261 \end{bmatrix}$$

$$= \begin{bmatrix} 2/29 & -4/87 & 19/87 \\ 1/87 & -31/261 & -5/261 \\ 8/87 & 13/261 & -40/261 \end{bmatrix}_{3\times 3}$$ (Ans)

(e)* $A = \begin{bmatrix} 4 & 2 & 8 \\ 5 & 0 & 7 \\ 7 & 6 & 9 \end{bmatrix}$

$|A| = -5(18-48) - 7(24-2)$

$= (-5 \times -30) + (-7 \times 22)$

$= 150 - 154 = -4$

$m_{11} = -42$

$m_{12} = 38$

$m_{13} = 30$

$m_{21} = -30$

$m_{22} = 24$

$m_{23} = 22$

$m_{31} = 14$

$m_{32} = -12$

$m_{33} = -10$

$M = \begin{bmatrix} -42 & 38 & 30 \\ -30 & 24 & 22 \\ 14 & -12 & -10 \end{bmatrix}$

$C_{11} = -42 \qquad C_{21} = 30 \qquad C_{31} = 14$

$C_{12} = -38 \qquad C_{22} = 24 \qquad C_{32} = 12$

$C_{13} = 30 \qquad C_{23} = -22 \qquad C_{33} = -10$

$$C = \begin{bmatrix} -42 & -38 & 30 \\ 30 & 24 & -22 \\ 14 & 12 & -10 \end{bmatrix}$$

$$\text{adj } A = C^T = \begin{bmatrix} -42 & 30 & 14 \\ -38 & 24 & 12 \\ 30 & -22 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{-4} \begin{bmatrix} -42 & 30 & 14 \\ -38 & 24 & 12 \\ 30 & -22 & -10 \end{bmatrix} = \begin{bmatrix} 21/2 & -15/2 & -7/2 \\ 19/2 & -6 & -3 \\ -15/2 & 11/2 & 5/2 \end{bmatrix}$$

* System of linnear equation.

It is solved by two method i.e given below.

(a) matrix method / inverse matrix method.

(b) crammor's rule

Two Variable

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

3-Variable.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

* $x + 3y = 7$
$x + y = 3$ so

$A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$|A| = 1 - 3 = -2$

$adj(A) = \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj(A) = \frac{1}{-2} \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & 3/2 \\ 1/2 & -1/2 \end{bmatrix}$

$X = A^{-1}B$
$= \begin{bmatrix} -1/2 & 3/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} -7/2 + 9/2 \\ 7/2 - 3/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Rightarrow (x, y) = (1, 2)$ (Ans)

* $x + y = 5$
$x + 2y = 9$ so

$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$

$|A| = 2 - 1 = 1$ $adj(A) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

According to formula $X = A^{-1}B$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$$= \begin{bmatrix} 10-9 \\ -5+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \Rightarrow (x, y) = (1, 4)$$

* $x+y+z=3$
  $x-y-z=1$
  $-x+y-z=-1$ so

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

$|A| = 2 + 2 + 0 = 4$

$m_{11} = 2 \quad m_{21} = -2 \quad m_{31} = 0$

$m_{12} = -2 \quad m_{22} = 0 \quad m_{32} = -2$

$m_{13} = 0 \quad m_{23} = 2 \quad m_{33} = -2$

$$C = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & 2 & -2 \end{bmatrix}$$

$C_{11} = 2 \quad C_{13} = 0 \quad C_{22} = 0 \quad C_{31} = 0 \quad C_{33} = -2$

$C_{12} = 2 \quad C_{21} = 2 \quad C_{23} = -2 \quad C_{32} = 2$

$$adj(A) = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -2 \end{bmatrix}$$

Engineering mathematics

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$= \frac{1}{4}\begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix}$$

According to formula $X = A^{-1}B$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 0 & -1/2 \\ 0 & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 3+1 \\ 3+1 \\ 1+1 \end{bmatrix}} \quad \begin{bmatrix} 3/2 + 1/2 \\ 3/2 + 1/2 \\ -1/2 + 1/2 \end{bmatrix}$$

$$\cancel{\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$(x, y, z) = (2, 1, 0)$ (Ans)!

Engenering mathematics

* $4x + 3y = 7$

  $2x + 8y = 10$

Here $A = \begin{bmatrix} 4 & 3 \\ 2 & 8 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

$|A| = 32 - 6 = 26$

$adj(A) = \begin{bmatrix} 8 & -3 \\ -2 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj(A) = \frac{1}{26} \begin{bmatrix} 8 & -3 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 4/13 & -3/26 \\ -1/13 & 2/13 \end{bmatrix}$

$X = A^{-1}B$

$= \begin{bmatrix} 4/13 & -3/26 \\ -1/13 & 2/13 \end{bmatrix} \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

$= \begin{bmatrix} \frac{28}{13} - \frac{30}{26} \\ -7/13 + \frac{20}{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

hence $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ so $x = 1$ and $y = 1$. (An)

* Cramer's rule :—

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

D = determinant of given matrix

D_1, D_2 and D_3 = Determinant of constant putting matrix

e.g → $11x + 12y = 12$

$13x + 8y = 8$

Here $A = \begin{bmatrix} 11 & 12 \\ 13 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 12 \\ 8 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$B = \begin{bmatrix} 12 \\ 8 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$

$D = 88 - 156 = -68 \neq 0$

30

$D_1 = \begin{vmatrix} 12 & 12 \\ 8 & 8 \end{vmatrix} = 0$

$-\cancel{68}$

$D_2 = \begin{vmatrix} 11 & 12 \\ 13 & 8 \end{vmatrix} = -68$

$\frac{1}{4}$

so $x = \frac{0}{-68} = 0$

$y = \frac{-68}{-68} = 1$

$\Rightarrow (x, y) = (0, 1)$

Engenerring mathematics.

* $x+y+z=-1$

$2x+3y+z=-4$

$3x+2y+2z=-3$

Here $A=\begin{bmatrix}1&1&1\\2&3&1\\3&2&2\end{bmatrix}$ $B=\begin{bmatrix}-1\\-4\\-3\end{bmatrix}$ $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

$D = 1(6-2)-1(4-3)+1(4-9)$

$= 4-1-5$

$= -2$

$D_1 = \begin{vmatrix}-1&1&1\\-4&3&1\\-3&2&2\end{vmatrix} = -1(6-2)-1(-8+3)+1(-8+9)$

$= -4+5+1 = 2$

$D_2 = \begin{vmatrix}1&-1&1\\2&-4&1\\3&-3&2\end{vmatrix} = 1(-8+3)+1(4-3)+1(-6+12)$

$= -5+1+6 = 2$

$D_3 = \begin{vmatrix}1&1&-1\\2&3&-4\\3&2&-3\end{vmatrix} = 1(-9+8)-1(-6+12)-1(4-9)$

$= -1-6+5 = -2$

$x = \frac{D_1}{D} = \frac{2}{-2} = -1$ $y = \frac{D_2}{D} = \frac{2}{-2} = -1$ $z = 1$ Ans

$* \; |A| = -2$

$m_{11} = 6-2 = 4$ $\quad m_{21} = 0$ $\quad m_{31} = -2$

$m_{12} = 4-3 = 1$ $\quad m_{22} = -1$ $\quad m_{32} = -1$

$m_{13} = -5$ $\quad m_{23} = -1$ $\quad m_{33} = 1$

$c_{11} = 4$ $\quad c_{21} = 0$ $\quad c_{31} = -2$

$c_{12} = -1$ $\quad c_{22} = -1$ $\quad c_{32} = 1$

$c_{13} = -5$ $\quad c_{23} = 1$ $\quad c_{33} = -1$

$$C = \begin{bmatrix} 4 & -1 & -5 \\ 0 & -1 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\text{adj } A = C^T = \begin{bmatrix} 4 & 0 & -2 \\ -1 & -1 & 1 \\ -5 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & 0 & -2 \\ -1 & -1 & 1 \\ -5 & 1 & -1 \end{bmatrix}$$

$* \; (x, y, z) = (-1, -1, 1)$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & 1/2 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -2 & 0 & 1 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} -1 \\ -4 \\ -3 \end{bmatrix} = \begin{bmatrix} 2+0-3 \\ -1/2 - 2 + \frac{3}{2} \\ -5/2 + 2 - 3/2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

→ $3x+8y = (-13)$

$x - y = 3$

→ $8x+y = 2$

$4x - 5y = 9$

→ $2x+y+z=4$

$x+3y+7z=11$

$4x-2y-2z=0$

→ $x+3y-z=4$

$-x-3y+z=4$

$x+y+z=2$

→ $7x+8y+9z=9$

→ $x+z=1$

$x=y$

(Both matrix and cramer's rule.)

Home task : —

* $3x+8y = -13$

$x - y = 3$

According to question $A = \begin{bmatrix} 3 & 8 \\ 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -13 \\ 3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$

(a) By matrix method : —

$|A| = -3-8 = -11$

$m_{11} = -1 \quad m_{21} = 8$

$m_{12} = 1 \quad m_{22} = 3$

$C_{11} = -1 \quad C_{21} = -8$

$C_{12} = -1 \quad C_{22} = 3$

so $C = \begin{bmatrix} -1 & -1 \\ -8 & 3 \end{bmatrix}$

$$\text{adj}A = C^T = \begin{bmatrix} -1 & -8 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}A = -\frac{1}{-11}\begin{bmatrix} -1 & -8 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1/11 & 8/11 \\ 1/11 & -3/11 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 1/11 & 8/11 \\ 1/11 & -3/11 \end{bmatrix}\begin{bmatrix} -13 \\ 3 \end{bmatrix} = \begin{bmatrix} -13/11 + \frac{24}{11} \\ -13/11 - 9/11 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Hence $(x,y) = (1,-2)$

(ans)

Engenerring mathematics :

(b) By cramer's rule :

Here $D = -11$, $D_1 = \begin{vmatrix} -13 & 8 \\ 3 & -1 \end{vmatrix} = 13 - 24 = -11$

$D_2 = \begin{vmatrix} 3 & -13 \\ 1 & 3 \end{vmatrix} = 9 + 13 = 22$

so $x = \frac{D_1}{D} = \frac{-11}{-11} = 1$

$y = \frac{D_2}{D} = \frac{22}{-11} = -2$

Hence $(x,y) = (1,-2)$

* $3x + y = 2$
  $4x - 5y = 9$

according to question $A = \begin{bmatrix} 3 & 1 \\ 4 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$

(a) By matrix method :

$|A| = -15 - 4 = -19$

$m_{11} = -5$ $m_{21} = 1$
$m_{12} = 4$ $m_{22} = 3$

$c_{11} = -5$ $c_{21} = -1$
$c_{12} = -4$ $c_{22} = 3$

so $C = \begin{bmatrix} -5 & -4 \\ -1 & 3 \end{bmatrix}$

$\text{adj}A = C^T = \begin{bmatrix} -5 & -1 \\ -4 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{-19} \begin{bmatrix} -5 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 5/19 & 1/19 \\ 4/19 & -3/19 \end{bmatrix}$

$X = A^{-1}B = \begin{bmatrix} 5/19 & 1/19 \\ 4/19 & -3/19 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

$= \begin{bmatrix} 10/19 + 9/19 \\ 8/19 - \frac{27}{19} \end{bmatrix}$

$= \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$(x,y) = (1,-1)$.

(b) By camer's rule :

Here $D = -19$, $D_1 = \begin{vmatrix} 2 & 1 \\ 9 & -5 \end{vmatrix} = -10 - 9 = -19$

$D_2 = \begin{vmatrix} 3 & 2 \\ 4 & 9 \end{vmatrix} = 27 - 8 = 19$

So $x = \frac{D_1}{D} = \frac{-19}{-19} = 1$, $y = \frac{D_2}{D} = \frac{19}{-19} = -1$

Hence $(x,y) = (1,-1)$ (ans)

* $2x + y + z = 4$

$x + 3y + 7z = 11$

$4x - 2y - 2z = 0$

Accoring to question $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 7 \\ 4 & -2 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

(a) By matrix method :

$|A| = 2(-6+14) - 1(-2-28) + 1(-2-12)$

$= 16 + 30 - 14$

$= \cancel{22}\ 32$

$m_{11} = -6 + 14 = 8$ $m_{21} = 0$ $m_{31} = 4$

$m_{12} = -30$ $m_{22} = -8$ $m_{32} = 13$

$m_{13} = -14$ $m_{23} = -8$ $m_{33} = 5$

$c_{11} = 8$ $c_{21} = 0$ $c_{31} = 4$

$c_{12} = 30$ $c_{22} = -8$ $c_{32} = -13$ So $C = \begin{bmatrix} 8 & 30 & -14 \\ 0 & -8 & 8 \\ 4 & -13 & 5 \end{bmatrix}$

$c_{13} = -14$ $c_{23} = 8$ $c_{33} = 5$

Engeneering mathematics

* $x+3y-z=4$
$-x-3y+z=4$
$x+y+z=2$

According to question $A=\begin{bmatrix}1 & 3 & -1\\-1 & -3 & 1\\1 & 1 & 1\end{bmatrix}$ $B=\begin{bmatrix}4\\4\\2\end{bmatrix}$ $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

(a) matrix method :

$|A| = 1(-3-1)-3(-1-1)-1(-1+3)$
$= -4+6-2$
$= 0$ Thus inverse doesn't exist.

(b) Cramer's rule :

$|A|=0$ thus value doesn't exist.

* $7x+8y+9z=9$
$x+z=1$
$x-y=0$

According to question $A=\begin{bmatrix}7 & 8 & 9\\1 & 0 & 1\\1 & -1 & 0\end{bmatrix}$ $B=\begin{bmatrix}9\\1\\0\end{bmatrix}$ $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

(a) matrix method :

$|A| = 7(+1)-1(0+9)+1(8-0)$
$= 7-9+8$
$= 6$

$m_{11}=1$ $m_{21}=9$ $m_{31}=8$
$m_{12}=-1$ $m_{22}=-9$ $m_{32}=-2$
$m_{13}=-1$ $m_{23}=-15$ $m_{33}=-8$

$c_{11} = 1 \quad c_{21} = -9 \quad c_{31} = 8$

$c_{12} = 1 \quad c_{22} = -9 \quad c_{32} = 2$

$c_{13} = -1 \quad c_{23} = 15 \quad c_{33} = -8$

so $C = \begin{bmatrix} 1 & 1 & -1 \\ -9 & -9 & 15 \\ 8 & 2 & -8 \end{bmatrix}$

$$\text{adj}A = C^T = \begin{bmatrix} 1 & -9 & 8 \\ 1 & -9 & 2 \\ -1 & 15 & -8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}A = \frac{1}{6}\begin{bmatrix} 1 & -9 & 8 \\ 1 & -9 & 2 \\ -1 & 15 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 1/6 & -3/2 & 4/3 \\ 1/6 & -3/2 & 1/3 \\ -1/6 & 5/2 & -4/3 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 1/6 & -3/2 & 4/3 \\ 1/6 & -3/2 & 1/3 \\ -1/6 & 5/2 & -4/3 \end{bmatrix}\begin{bmatrix} 9 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9/6 - 3/2 \\ 9/6 - 3/2 \\ -9/6 + 5/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$(x, y, z) = (0, 0, 1)$

(b) Cramer's rule :

$D = 6$, $D_1 = \begin{vmatrix} 9 & 8 & 9 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 9(+1) - 1(+9) = 0$

$D_2 = \begin{vmatrix} 7 & 9 & 9 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 1(9-9) = 0$

$D_3 = \begin{vmatrix} 7 & 8 & 9 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 9(-1-0) - 1(-7-8) = -9+15 = 6$

so $x = 0$
$y = 0$
$z = 1$

* Engenering mathematics :

calculating determinate without expanding by using properties :

(1)* prove that
$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$$

LHS

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

$C_1 \to C_1+C_2+C_3$

$$= \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

Taking $(x+a+b+c)$ as common on C_1

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$R_2 \to R_3 - R_3$

$$= (x+a+b+c)\begin{vmatrix} 0 & -x & 0 \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix}$$

$$= (x+a+b+c)\left[1(x^2-0)\right]$$

$$= x^2(x+a+b+c) = RHS$$ (proved)

(2)* prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

<u>LHS</u>

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$$ Taking common $\frac{1}{a}, \frac{1}{b},$ and $\frac{1}{c}$ on c_1, c_2 and c_3 respectively.

$$= \frac{1}{a}\times\frac{1}{b}\times\frac{1}{c}\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix}$$ Taking abc common on c_3.

$$= \frac{1}{abc}\times abc\begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$c_1 \to c_1 - c_2 \quad c_2 \to c_2 - c_3$

$$= \begin{vmatrix} a^2-b^2 & b^2-c^2 & c^2 \\ a^3-b^3 & b^3-c^3 & c^3 \\ 0 & 0 & 1 \end{vmatrix}$$ ~~Now expand~~ Taking $(a-b)$, ~~co~~ $(b-c)$ on

on both c_1 and c_2 respectively.

$$= (a-b)(b-c) \begin{vmatrix} a+b & b+c & c^2 \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (a-b)(b-c) \left[1 \left((a+b)(b^2+bc+c^2) - (b+c)(a^2+ab+b^2) \right) \right]$$

$$= (a-b)(b-c) \left[ab^2+abc+ac^2+b^3+b^2c+bc^2 - (ba^2+ab^2+b^3 + ca^2+abc+b^2c) \right]$$

$$= (a-b)(b-c) \left[\cancel{ab^2}+\cancel{abc}+ac^2+\cancel{b^3}+\cancel{b^2c}+bc^2 - ba^2 - \cancel{ab^2} - \cancel{b^3} - ca^2 - \cancel{abc} - \cancel{b^2c} \right]$$

$$= (a-b)(b-c) \left[ac^2+bc^2 - ba^2 - ca^2 \right]$$

$$= (a-b)(b-c) \left[ca(c-a) + b(c^2-a^2) \right]$$

$$= (a-b)(b-c)(c-a) \left[ca + bc + ba \right] = (RHS)$$

(proved)