Engineering Physics Handnotes

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Unit and Dimension

Physics → the study of nature and its laws
↳ Mathematics is the language of Physics.

Physical Quantities → The quantities which we can measure.

Ex: Height, weight, distance → measurable
happiness, sadness → can't measure.

→ Two types of physical quantities ⇒
(a) Fundamental Physical Quantity
(b) Derived Physical Quantity.

They do not depend on any other physical quantity for their measurement.
Ex → Mass, Length, time.

They depend on other physical quantities for their measurement. And expressed as some mathematical function of fundamental physical quantities.

Ex:

$$\text{Velocity} = \frac{\text{Length}}{\text{time}}$$

$$\text{Force} = \text{mass} \times \text{acceleration}$$
$$= \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

Units → It is an internationally accepted reference standard which is used to measure a physical quantities.

Fundamental Units → The units for the fundamental physical quantities.

Derived Units → The units of other physical quantities ~~can be~~ are expressed in terms of fundamental units. These are known as derived units.

System of units → A complete set of both fundamental & derived units.

(a) C.G.S. System → Based on Centimeter, Gram, Second as fundamental units of length, mass and time.

(b) M.K.S. System → Based on meter, kilogram and second as fundamental units.

(c) SI Unit → Formed by the international Bureau of weights and measures in 1971. It is based on 7 fundamental units and 2 supplementary units.

7 fundamental Units

Mass → Kilogram (Kg)
Length → Meter (m)
Time → Second (s)
Temperature → Kelvin (K)
Elect. current → Ampere (A)
Luminosity → Candela (cd)
Amount of substance → Mole (mol.)

Supplementary Units:
Angle → Radian (Rad)
Solid Angle → Steradian (Sr.)

Dimensions → These are the powers to which the fundamental quantities / base quantities are raised to represent a physical quantity.

→ denoted by putting '[ ]' around a quantity.

Ex :- $[\text{Length}] = [L] = [L^1]$

$[\text{Mass}] = [M] = [M^1]$

$\text{Velocity} = \frac{\text{Length}}{\text{Time}} = \frac{[L]}{[T]} = [M^0 L^1 T^{-1}]$

→ Calculation of dimension of physical quantity (Do some examples).

→ Dimensional equation → The eqn obtained by equating a physical quantity with its dimensional formula is called dimensional eqn of that quantity.

Ex:- velocity ⇒ $[v] = [M^0 L^1 T^{-1}]$

Acceleratⁿ ⇒ $[a] = [M^0 L^1 T^{-2}]$

Density ⇒ $[\rho] = [M^1 L^{-3} T^0]$

Dimensional Analysis →

Principle of homogeneity → The dimension of each terms of a dimensional eqⁿ on both sides should be same.

* Conversion from MKS to CGS or viceversa
* Numericals on dimensional correctness of dimensional eqⁿs

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Scalar & Vector

Physical Quantities

Scalar

(Have ~~both~~ only magnitude)

→ Expressed as a number

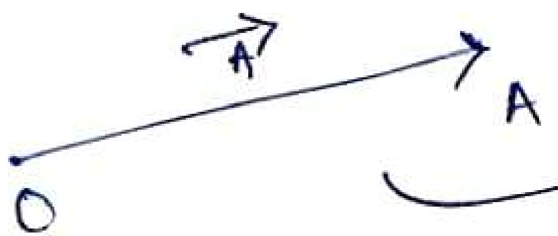
Vector

(Have both magnitude and direction)

→ expressed with an arrowhead above.

Ex: Force vector $= \vec{F}$

Representation of Vector ⇒



→ length of OA is $|\vec{A}|$

magnitude of vector $\vec{A}$

Scalar — Temp, pressure, energy, mass, time

→ Have only magnitude
. a number with an unit.
→ Mathematical operations are same as plane Maths.

Vector

→ has both direction & magnitude.
→ displacement vector
→ They added by triangle law of vector addition or parallelogram law of vector addition.

(Equal vector – same magnitude & same direction)

Representation of vector ⇒

Null vector → zero magnitude / arbitrary direction
→ a point or dot.
→ Adding with gives same.
→ Dot product & cross product ⇒ zero.
⇒ Unit vector →
Magnitude = 1
gives the direction of vector.
written with cap
$\hat{i}, \hat{j}, \hat{u}$

Colinear vectors –
- Parallel.
- Antiparallel.

Perpendicular vector.

~~Negative~~ Equal vectors.

Negative vectors ⇒
→ Same magnitude & opposite direction
→ All negative $\vec{A}$ are antiparallel.

Coinitial

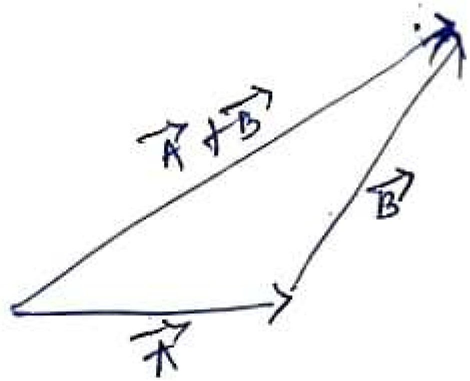
Coplanar

Position vectors –
Indicate the position of a point in a coordinate system.

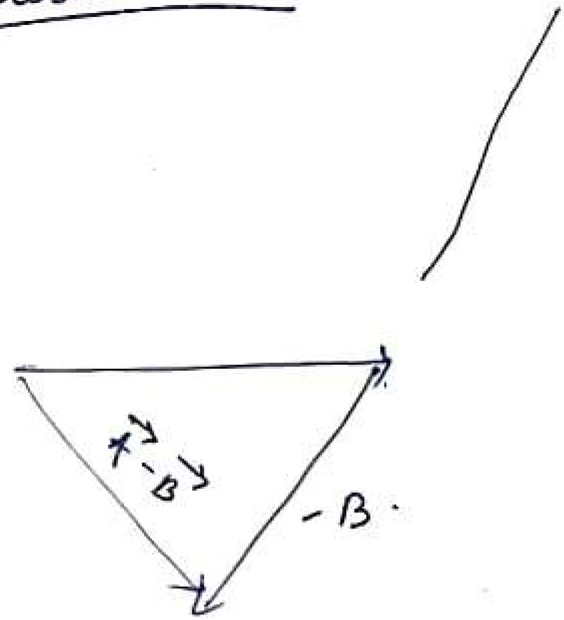
Vector Addition

Parallel shift of vector ⇒
If we shift a vector parallely, it will remain unchanged.

Graphical Method / Triangle law / Head-to-Tail method.

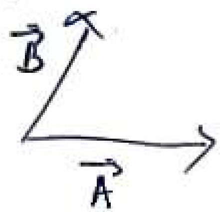


Subtraction

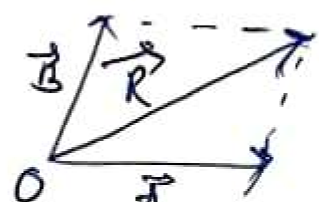


Parallelogram law :-

→ Join the tails



→ Draw ~~two~~ a lines, from head of $\vec{B}$ & $\vec{A}$ parallel to $\vec{A}$ & $\vec{B}$ respectively

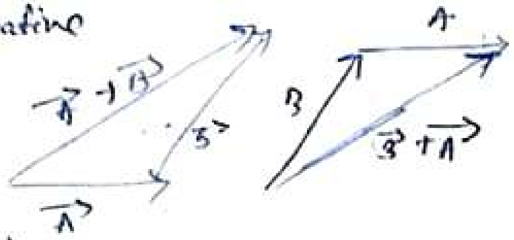


• Parallelogram law is ∥ to triangle law ⇒
The dotted lines (drawn ∥) are vectors
$\vec{A}$ & $\vec{B}$.

Properties

1) Vector addition is commutative

$\vec{A}+\vec{B} = \vec{B}+\vec{A}$

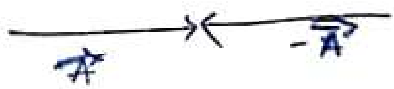


2) Associative

$(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$

Null Vector / Zerovector

$\vec{0} = \vec{A}+(-\vec{A})$



$\vec{0} = \vec{A}\times\vec{0}$

$\vec{A} = \vec{A}+\vec{0}$

Multiplication with a scalar →
A vector multiplied ~~wit~~ by a +ve scalar it will give a vector in same direction.

$\lambda\times\vec{A} = \lambda\vec{A}$.

⇒ Multiplying with -ve scalar, we will get.
$-\lambda\times\vec{A} = -\lambda\vec{A}$
we will get a vector in opposite direction.

Equal Vector

Two vectors having same magnitude & same direct^n are equal vector.
Give examples

Unit vectors

→ Unit = 1.
⇒ length/magnitude is 1.
⇒ Points in one direction & gives direction any vector.
⇒ If we multiply a scalar with an unit vector, we will get a vector.
So any vector can be represented by scalar × unit vector
magnitude of vector   direction

$\hat{n} = \frac{\vec{A}}{|\vec{A}|} = \hat{A}$

→ Unit vectors are dimensionless.

→ Unit vectors are called directional vectors too

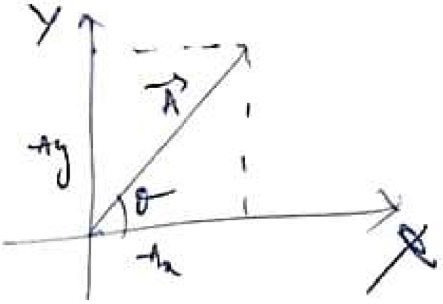
Resolution of Vectors ⇒

We write any vector in terms of their components along X & Y axis.

$\vec{A} = A_x \hat{i} + A_y \hat{j}$

A_x → Projection of A on X-axis

$A_y \hat{j}$ → Projection of A on Y-axis.



$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$

$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$

$A_x^2 + A_y^2 = A^2$ → Pythagoras thm.

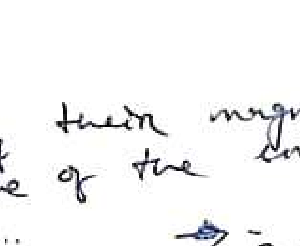
$\Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2}$

Direction of $\vec{A}$ ⇒ $\tan\theta = \frac{A_y}{A_x}$

~~ΔOTQ & ΔPTQ.~~

$\sin\beta$.

~~tan β~~



Dot product →

$\vec{A} \cdot \vec{B}$ = product of their magnitude and the cosine of the smaller angle between them.

$= A\, B \cos\theta$

$= |\vec{A}|\, |\vec{B}| \cos\theta$

$\theta = 0$, $\vec{A} \cdot \vec{B} = AB$, $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\theta = 90°$, $\vec{A} \cdot \vec{B} = 0$; $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

Dot product in rectangular component.

$\vec{A} \cdot \vec{B} = (A_x \hat{i} + - - -) \cdot (B_x \hat{i} + - - -)$.

$= A_x B_x (\hat{i} \cdot \hat{i}) +$

$= A_x B_x + A_y B_y + A_z B_z$ -

sum of product of rectangular components along the coordinate axes.

Q) $\vec{A} = 3\hat{i} + 2\hat{j} + 5\hat{k}$, $\vec{B} = 4\hat{i} + 3\hat{j} + 7\hat{k}$ } $A \cdot \vec{B}$

b) $2\hat{i} + 5\hat{j} + 6\hat{k}$.

$3\hat{i} - 6\hat{j} + \hat{k}$.

Q) $5\hat{i} + 2\hat{j} + 3\hat{k}$.

$2\hat{i} - 3\hat{j}$

Q) $\vec{A} = 3\hat{i} + 2\hat{j}$ & $\vec{B} = 4\hat{i} + 3\hat{j}$.

Cross product

$\vec{A} \times \vec{B} = AB \sin\theta$ -

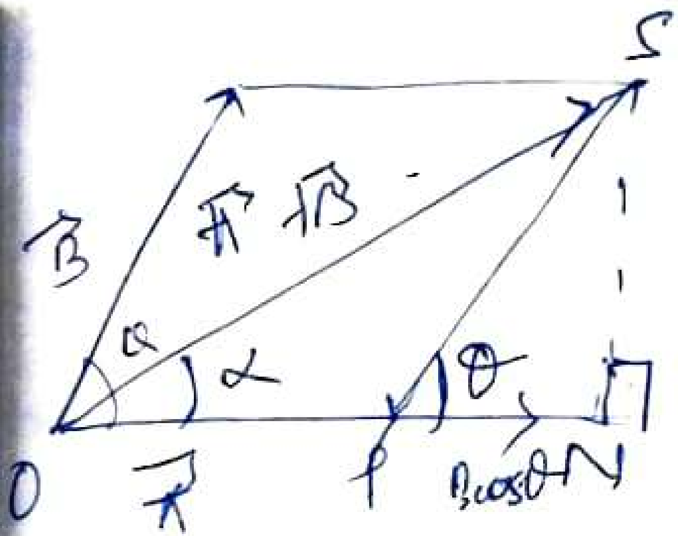
Right hand thumb rule → holding in the Normal to the plane right hand. -

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$.

Cross product using rectangular component



$SN \perp ON$ take

ΔOSN,

$$(OS)^2 = (ON)^2 + (SN)^2$$

$$= (OP + PN)^2 + (SN)^2$$

$$= (A + B\cos\theta)^2 + (SN)^2$$

$$= (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$= A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin$$

$$\Rightarrow R^2 = A^2 + 2AB\cos\theta + B^2$$

$$\Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Direction of $\vec{R}$

$$\tan\alpha = \frac{SN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

KINEMATICS

3. Curvillinear Motion & Kinematic

Curvillinear Motion :- Motion of an object in curved path with variable direction of velocity is called as curvillinear motion.

Projectile :- Any object projected into the space & is moving under the influence of gravity only after projection is called as proje

Trajectory :- The path of the projectile is called as trajectory & the motion of the projectile is called as projectile motion.

Angle of Projection :- The angle at which the projectile being projected is called as angle of projection.

Maximum Height (H) :- It is the maximum displacement travelled by the projectile in vertical direction.

Horizontal Range (R) :- It is the maximum displacement travelled by the projectile along horizontal dir^n.

Time of Flight (T) :- It is the total time taken by the projectile to come back to the same level from which it is projected

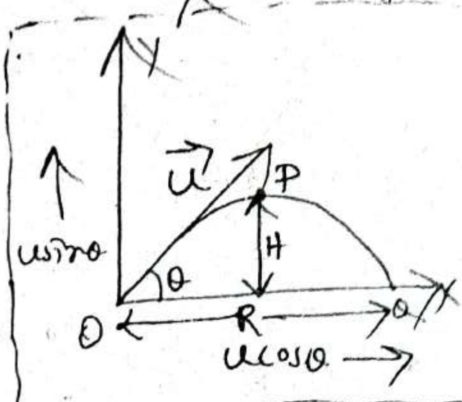
Time of ascent :- It is time taken to reach maximum height in vertical dir^n.

Time of descent :- Time taken to come back from max height to the level of projection

Projectile fired at an angle 'θ' with the horizontal :-

Let a projectile is projected with initial velocity '$\vec{u}$' at an angle 'θ' with the horizontal. Suppose the projectile raised to a height 'P' & then fall to the pt. 'Q' on the same level from ~~projection~~ which it was projected.

As the projectile projected in free space i.e in 2-d space, Hence initial velocity '$\vec{u}$' will be resolve into two components. such as:-



(i) $\vec{u}\cos\theta$ along horizontal dirn which is uniform as in this dirn accln due to gravity has no effect &

(ii) $\vec{u}\sin\theta$ along vertical dirn which is non-uniform as in this dirn accln due to gravity 'g' acts exactly opposite to it.

① Equation of Trajectory :-

According to the ~~ki~~neomatic eqn, the distance travelled by the projectile after time 't' is given by,

$$S = ut + \frac{1}{2}at^2 \quad \text{—— ①}$$

For horizontal direction suppose the distance travelled in 't' time is ~~given by~~ 'x' then in its given by,

$$x = ut + \frac{1}{2}at^2 \quad \text{—— ②}$$

But in horizontal dirⁿ,

$\vec{u} = \vec{u}\cos\theta$, accⁿ $a = 0$ as in this dirⁿ velocity is uniform.

Hence from ② we get, $x = u\cos\theta . t + t\frac{1}{2}.0$.

$$\left.\begin{aligned} &\Rightarrow x = u\cos\theta . t \\ &\Rightarrow t = \frac{x}{u\cos\theta} \end{aligned}\right\} — ③$$

Now for vertical dirⁿ suppose the distance travelled in 't' is y & is given by,

$$y = ut + \frac{1}{2}at^2 \quad — ④$$

But in this vertical dirⁿ, $\vec{u} = \vec{u}\sin\theta$, accⁿ $a = -g$.

Hence from ④ we get, $\Rightarrow y = u\sin\theta . t + \frac{1}{2}(-g)t^2$

$$\Rightarrow y = u\sin\theta . t - \frac{1}{2}gt^2 \quad — ⑤$$

Now putting value of 't' from ③ in ⑤, we get,

$$y = u\sin\theta . \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = \tan\theta . x - \frac{g x^2}{2u^2\cos^2\theta}} \quad — ⑥$$

This eqⁿ is a form of parabola, hence the path of the projectile is parabolic.

② Time of flight :- The total time taken by the projectile to come back to the level of projection. It can be calculated at pt. 'Q' i.e at final point.

Now at 'Q' the vertical displacement covered by the projectile, $y = 0$.
Using this from eqn (5) we can write,

$$0 = u\sin\theta \cdot t - \frac{1}{2} g t^2$$

$$\Rightarrow \frac{1}{2} g t^2 = u\sin\theta \cdot t \Rightarrow \frac{1}{2} g t = u\sin\theta$$

$$\Rightarrow \boxed{t = \frac{2u\sin\theta}{g}} \quad \text{--- (7)}$$

(3) Time of ascent :- It is the time taken by the projectile at a heigh H i.e at point 'P'.

At this point, initial (vertical) velocity, $\vec{u} = \vec{u}\sin\theta$
final (vertical) velocity, $\vec{v} = 0$
distance covered, $s = H$
accl^n, $a = -g$
time (time of ascent), $= t_1$

Now using the kinematic equation,

$$v = u + at$$

we get,

$$\Rightarrow 0 = u\sin\theta + (-g) t_1$$

$$\Rightarrow u\sin\theta - g t_1 = 0 \Rightarrow u\sin\theta = g t_1$$

$$\Rightarrow \boxed{t_1 = \frac{u\sin\theta}{g}} \quad \text{--- (8)}$$

Time of descent :- ~~Here, initial (vertical) velocity $\vec{u} = \vec{u}\sin\theta$~~
~~final (vertical) velocity, $\vec{v} =$~~

As total time taken by the projectile is the sum of time of ascent (t_1) & time of descent (t_2)

Hence time of flight, $t = t_1 + t_2$

$$\Rightarrow \frac{2u\sin\theta}{g} = \frac{u\sin\theta}{g} + t_2$$

$$\Rightarrow t_2 = \frac{2u\sin\theta}{g} - \frac{u\sin\theta}{g} = \frac{u\sin\theta}{g} \quad \text{— (9)}$$

[using eqⁿ (7) & (8)]

Hence time of ascent is equal to time of descent.

(4) Maximum Height (H) :—

The ~~time~~ distance ~~taken~~ taken by the projectile after covering the maximum height 'H' and to reach at point 'p' as shown in fig B. given by, using the kinematic eqⁿ.

$$v^2 = u^2 + 2as.$$

Here at point P,

$$v \rightarrow 0$$
$$u \rightarrow u\sin\theta$$
$$a \rightarrow -g$$
$$S \rightarrow H$$

Hence we get,

$$0 = u^2\sin^2\theta + 2(-g)H$$

$$\Rightarrow 2gH = u^2\sin^2\theta$$

$$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}} \quad \text{— (10)}$$

This is the expression for maximum height of a projectile projected with an angle 'θ' with the horizontal.

(ii) Horizontal Range (R) :- The total horizontal distance covered by the projectile to reach at final point is given by, from eqn (3)

$$X = u\cos\theta \cdot t$$

Now using the value of 't' from eqn (7) we get,

$$X = u\cos\theta \cdot \frac{2u\sin\theta}{g}$$

$$\Rightarrow X = \frac{u^2(2\sin\theta \cdot \cos\theta)}{g}$$

$$\Rightarrow \boxed{X = \frac{u^2\sin 2\theta}{g}} \text{ or } \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

~~it is also denoted~~

This is the required expression for the horizontal range covered by the projectile projected with an angle θ with the horizontal & it is also denoted by 'R'.

NOTE :- Condition for maxm range covered by the projectile :-

maxm range i.e $R \to$ maxm

But for $R \to$ maxm $\Rightarrow \sin 2\theta = 1$

$$\Rightarrow \sin 2\theta = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

Hence the projectile will cover a maxm range when it is projected by an angle of 45° with the horizontal.

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Friction :-

CHAPTER-4

Defⁿ:- Whenever a body in contact or in motion with a surface, then an opposing force comes to play tangentially at the point of contact. This force is called as friction.

Ex:- → Walking of a person on a floor by friction betⁿ feet & floor.

→ Friction enable us to drive & stop the vehicle

Friction are of 4 types. such as

→ Static friction.

→ Kinetic friction. (Dynamic friction)

→ Sliding friction → Fluid friction

① Static Friction:-

→ Defⁿ:- It is the force of friction comes to play when a body is forced to move along a surface but movement doesn't start.

→ The magnitude of static friction remains equal to the applied force & the dirⁿ is always opposite to dirⁿ of motion.

→ Maximum value of static friction is limiting friction after which only the body starts to move.

② Dynamic Friction

→ Defⁿ:- It is the force of friction comes to play when a body just starts moving along a surface.

→ If the magnitude of dynamic friction is lesser than the external applied force, then only the body will move.

→ Maximum value of dynamic friction only is the sliding friction

Eg - Pushing on a wall | Eg - pushing a box at rest on floor. 7

Rolling friction:

Laws of Limiting friction:

(1) The dirⁿ of force of friction is always opposite to the dirⁿ of motion.

(2) The force of limiting friction depends upon the nature & state of polish of the surfaces in contact & it acts tangentially to the interface betⁿ the two surfaces.

(3) The magnitude of limiting friction (F) is directly proportional to magnitude of normal reaction R betⁿ the two surfaces in contact.

i.e $F_l \propto R \Rightarrow F_l = \mu_l R \Rightarrow \mu_l = \frac{F_l}{R}$

Here $\mu_l \rightarrow$ co-efficient of limiting friction.

(4) Magnitude of limiting friction is independent of the area & shape of the surfaces in contact, so long as normal reaction remains same.

Methods of Reducing friction:-

→ By rubbing & polishing, friction force can be reduce

→ By using lubricants on the surfaces in contact, friction may reduce

→ If we convert sliding friction into rolling friction

→ By streamlining the shape of the body, the fluid friction can be reduce.

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Unit 5&6 Gravitation, Planetary Motion & S.H.M

Gravitation :- Whenever an object released in free space, it falls towards the earth. ~~It app~~ Hence it appears that earth attracts everything towards it which is called Gravitation.

Kepler's Laws of Planetary Motion :-

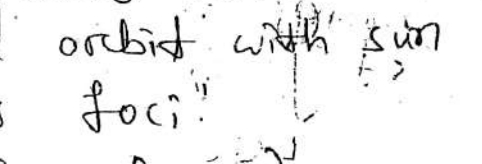
1st Law :- (Law of elliptical orbits)

It states that "a planet moves around the sun in an elliptical orbit with sun situated at one of its foci".

2nd Law :- (Law of areal velocity)

It states that "a planet moves around the sun in such a way that its areal velocity remains constant".

i.e $\boxed{\text{Areal velocity} = \text{constant}}$



$A_1 = A_2 = A_3 = A_4$ (in same time interval)

3rd law :- (The Harmonic law) → Law of Time period

It states that "a planet moves around the sun in such a way that the square of its time period is directly proportional to the cube of ~~the~~ semi-major axis of its elliptical orbit".

i.e $\boxed{T^2 \propto R^3}$

Where, $T \rightarrow$ Time period

& $R \rightarrow$ Semi-major axis.

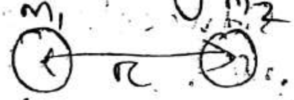
i.e if the distance between the planet & sun is more, hence more time it will take to complete its rotation.

Newton's law of Gravitation:-

Statement:- It states that "every particle of the matter in the universe attracts every other particle with a force which directly proportional to the product of the masses of the two particles & inversely proportional to the square of the distance between them".

$\Rightarrow$ This force of attraction between any two bodies in the universe is known as ~~grav~~ force of gravitation.

Let m_1 & $m_2 \rightarrow$ masses of two bodies



& $F \rightarrow$ Force ~~of~~ attraction betn them

& $r \rightarrow$ distance betn the two bodies,

Then according to Newton's law,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \boxed{F = G\frac{m_1 m_2}{r^2}}$$

where $G \rightarrow$ a constant of proportionality called as universal gravitational constant. ~~& its value is said~~

If $m_1 = m_2 = 1$ unit & $r = 1$ unit

Then, $\boxed{F = G}$

Hence, the gravitational constant 'G' is defined as the magnitude of the force of attraction between two ~~masses~~ Bodies each of unit mass & separated by a unit distance from each oth.

Unit of G :-

S.I.

We know, $F = G\frac{m_1 m_2}{r^2}$

$\Rightarrow G = \frac{FR^2}{m^2}$

Hence in S.I. $= \frac{\text{Newton} \times m^2}{kg^2}$

or $N \cdot m^2 \, kg^{-2}$

& in C.G.S Dyne. cm^2. g^{-2}

Dimension (G.)

$G = \frac{FR^2}{m^2}$,

$\Rightarrow [G] = \frac{[M^1 L^1 T^{-2}][L^2]}{[M^2]}$

$= [M^{-1} L^3 T^{-2}]$.

Acceleration due to gravity 'g' :-

The acc^n produced by weight (gravity) of a body is called as acc^n due to gravity & is denoted by 'g'.

i.e gravity (weight) $= mg$

If m & $M \rightarrow$ mass of a particle & earth respectively

$R \rightarrow$ distance betn the particle placed on the earth surface & centre of earth

Acc. to, Newton's law,

$F = G\frac{Mm}{R^2} \Rightarrow mg = G\frac{Mm}{R^2}$

$$\Rightarrow \boxed{g = G\frac{M}{R^2}}$$

Unit of g :-

In S.I $\rightarrow$ m/s^2

In C.G.S $\rightarrow$ cm/s^2

Dimension of g

$[g] = [M^0 L^1 T^{-2}]$

Difference betⁿ G & g :-

G	g
→ It is called as universal gravitational constant.	→ It is called as accⁿ due to gravity.
→ It's value remains const everywhere, so it is called as universal const of gravitation.	→ It's value changes at diff. place of earth as it depends upon mass & radius of the planet.
→ It's units are $N\,m^2\,kg^{-2}$ (SI) & $dyne\,cm^2\,g^{-2}$ (cgs)	→ Its unit is same as acclⁿ i.e m/s^2 or cm/s^2
→ It's dimension is $[M^{-1}L^{3}T^{-2}]$	→ It's dimⁿ is $[M^0L^1T^{-2}]$
→ $G = \frac{Fr^2}{m^2}$	→ $g = G\frac{M}{R^2}$
→ value of $G = 6.67\times10^{-11}\,Nm^2/kg^2$	→ value of $g = 9.8\ m/s^2$

Variation of 'g' with Altitude :-

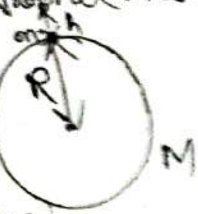
Consider a body of mass 'm' placed on the surface of the earth.

Let M & R ⟹ Mass & radius of earth respectively.

g ⟹ accⁿ due to gravity on the surface of earth.

Then, $g = \frac{GM}{R^2}$, where, G ⟹ Gravitational constant.

If the body is taken to a height 'h' above the surface of earth & the accⁿ due to gravity at this height is g'.



Then, $g' = \frac{GM}{(R+h)^2}$

Now, $\frac{g'}{g} = \left[\frac{GM}{(R+h)^2}\right] \Big/ \frac{GM}{R^2}$

$$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{2h}{R} \Rightarrow g' = g\left(1-\frac{2h}{R}\right) = g - \frac{2gh}{R}$$

$$\Rightarrow g' - g = -\frac{2gh}{R} \Rightarrow \boxed{g - g' = \frac{2gh}{R}}$$

As g & $R \rightarrow$ are constants at a given place on earth i.e.

hence, $g - g' \propto h$

It is concluded that if h increases, g' will decrease to increase $(g-g')$ value as g is constant.

Thus the value of accl^n due to gravity 'g' decreases with [redacted] in height above the earth surface.

<u>Variation of 'g' with depth</u>:-

Let's consider ~~the body of mass~~ earth as a homogeneous sphere of radius 'R', mass 'M' & density 'ρ'.

Let's consider a body lying on the surface of earth where the accl^n due to gravity is 'g' & it is given by,

$$g = \frac{GM}{R^2}$$

We know, mass = vol^n × density

$= \frac{4}{3}\pi R^3 \times \rho$

Now, $g = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2} = \frac{4}{3}\pi G R \rho$

Let the body taken to a depth 'd' bellow the earth surface, where the accl^n due to gravity is 'g'' & is given by,

$$g' = \frac{4}{3}\pi G (R-d)\rho$$

Now, $g'/g = \frac{(4/3)\pi G (R-d)\rho}{(4/3)\pi G R \rho} = \frac{R-d}{R}$

$\Rightarrow \frac{g'}{g} = 1 - \frac{d}{R} \Rightarrow g' = g - \frac{dg}{R}$

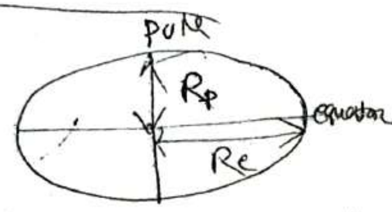
$$\Rightarrow \boxed{g - g' = \frac{d}{R} g}$$

i.e $(g-g') \propto d$

i.e if depth 'd' increase, $(g-g')$ value will increas. But 'g' is constant on the surface, hence to increase $(g-g')$ value with increase of h, 'g'' value will decrease.

Thus the value of accl^n due to gravity 'g' decreases with increase in depth.

Variation of 'g' with latitude :-



The value of acc^n due to gravity changes with ~~altitude~~ due to shape of the earth.

As the shape of the earth is not a perfect sphere. i.e flattened at poles & bulges out at the equator. Thus $R_e > R_p$.

As $g = \frac{GM}{R^2}$

& G, M ~~are~~ constants

Hence, $g \propto \frac{1}{R^2}$

→ As at pole $R \rightarrow$ min^m, hence g is greatest at pole

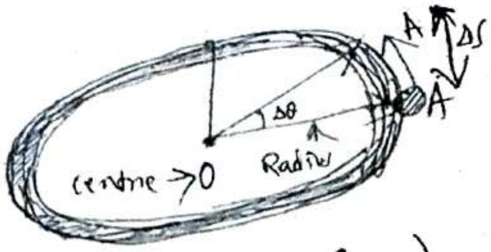
& as at equator $R \rightarrow$ max^m, hence g is least at equator.

Circular Motion :-

Def^n :- The motion of a body is said to be circular if it moves in such a way that its distance from a certain fixed pt always remains the same.

Uniform Circular Motion :-

Def^n :- Circular motion is said to be uniform if the speed of the particle along the circular path remains the same.



Angular displacement :- (θ)

Defn :- It is defined as the angle turned by the radius vector of the particle (undergoing rotational motion).

If $\Delta s \rightarrow$ linear disp.
$r \rightarrow$ radius of the circular path.
$\Delta\theta \rightarrow$ angular disp.

Then, $\boxed{\Delta s = r \times \Delta\theta}$ or $\boxed{\vec{\Delta s} = \vec{r} \times \Delta\theta}$

Angular velocity :- (ω)

Defn :- It is defined as the rate of change of angular disp. with time.

i.e Angular velocity, $\boxed{\omega = \frac{d\theta}{dt}}$

& the relation betn angular velocity & linear velocity is, $\boxed{V = r \times \omega}$ or $\boxed{\vec{V} = \vec{r} \times \vec{\omega}}$

where, $\omega \rightarrow$ Angular velocity.

Angular Acceleration :- (α)

Defn :- It is defined as the rate of change of angular velocity with time.

i.e angular accln, $\boxed{\alpha = \frac{d\omega}{dt}}$

& reln betn angular accln, linear accln is, $\boxed{a = r \times \alpha}$ or $\boxed{\vec{a} = \vec{r} \times \vec{\alpha}}$

Simple Harmonic Motion (S.H.M) :-

Defn :- The motion of a particle is said to be S.H.M if its acceleration is directly proportional to the disp. & is always directed towards the mean position. (Ex :- vibration of simple pendulum, ...)

The eqn for S.H.M is given by,

$$\boxed{y = r \sin(\omega t + \phi)}$$

where,
- $y \rightarrow$ displacement.
- $r \rightarrow$ amplitude of S.H.M.
- $\omega \rightarrow$ angular velocity.
- $\phi \rightarrow$ phase angle.

S.H.M parameters :-

(1) **Amplitude** :- Amplitude of a particle executing in S.H.M is defined as its maximum displacement on either side of the mean position.

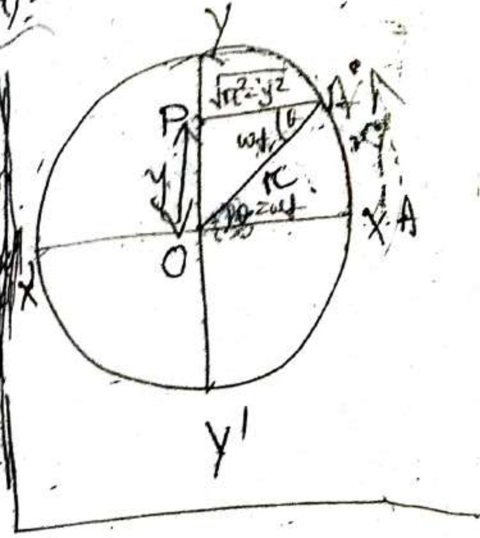
(2) **Frequency** :- The no. of oscillations made in unit time by the oscillating particle is called its frequency.

It is given by,

$$n = \frac{1}{T} = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi}\sqrt{\frac{acc^n}{disp.}}$$

S.H.M as a projection of a uniform circular motion (on any diameter):-



Consider a particle 'A' moving in uniform circular motion in a circular path having XOX' & YOY' as its horizontal & vertical diameter, as shown in fig.

Let 'P' be the projection (foot of the $\perp^{ar}$ drawn from A) of A while it is at 'X'. Let A moves towards Y and after some time it reaches at a point it projection is at 'P' i.e on moving from X to Y along the vertical diameter projection moves from O to Y & again A moves from to Y to X' & projection moves from Y to O.

Thus A completes its journey along the circumference of the circle, its projection move from O to Y, Y to O, O to Y' & Y' to O.

Hence the motion along YOY' is called S.H.M. So S.H.M is defined as the projection of uniform circular motion on the diameter of circle of reference.

Derivation of disp, velocity & Accln:-

(a) Displacement :- (y)

Disp. of a particle vibrating in S.H.M at any point on the circular path is defined as its distance from the mean position at that instant.

In the above fig, 'P' is the projection of particle 'A' at some instant at time 't'

In Δ OAP, $\sin\theta = \frac{OP}{OA} = \frac{y}{r}$

$\Rightarrow y = r\sin\theta$ —— (1)

As, angular velocity, $\omega = \frac{\theta \text{ (angular disp)}}{t \text{ (time)}}$

$\Rightarrow \theta = \omega t$ —— (2)

Using (2) in (1) we get,

$\boxed{y = r\sin\omega t}$ —— (3) $\Rightarrow \boxed{\frac{y}{r} = \sin\omega t}$ P(t)

Special case :-

Disp. y will be max^m when $\sin\omega t$ is max^m & min^m when $\sin\omega t$ is min^m.

Maximum value: y → max^m if $\sin\omega t = 1$

$\Rightarrow y = r$ (radius of circular path / amplitude of vibration)

Min^m value: y → min^m if $\sin\omega t = -1$

$\Rightarrow y = -r$

(ii) Velocity :- (V)

From the defination of velocity we know, velocity is the rate of change of disp.

i.e $v = \frac{dy}{dt}$

But from (3), $y = r \sin \omega t$

Hence, $v = \frac{dy}{dt} = \frac{d}{dt}(r \sin \omega t)$

$$\Rightarrow v = r \frac{d}{dt} \sin \omega t$$

$$\Rightarrow v = r \cos \omega t \frac{d}{dt}(\omega t)$$

$$\Rightarrow v = r \cdot \cos \omega t \cdot \omega$$

$$\Rightarrow \boxed{v = (r\omega) \cos \omega t} \quad \text{—— (4)}$$

~~But, linear velocity = radius x angular velocity~~

~~i.e. $v = r\omega$~~

~~Thus, $v = v \cos \omega t$ —— (4)~~

In Δ OAP, $\cos \omega t = \frac{AP}{OA} = \frac{\sqrt{r^2 - y^2}}{r}$

Using this in (4) we get,

$$v = r\omega \frac{\sqrt{r^2 - y^2}}{r} \Rightarrow \boxed{v = \omega \sqrt{r^2 - y^2}}$$

Spl case:-

(i) At O, $y = 0 \Rightarrow v = \omega \sqrt{r^2} = \omega r = v$

(ii) At Y/Y', $y = \pm r \Rightarrow v = \omega \sqrt{r^2 - r^2} \Rightarrow v = 0$.

Thus a particle executing in S.H.M, passes with maxm velocity through the mean position and is at rest at the extreme position.

Acceleration :-

- As accl^n is the rate of change of velocity,

$$\Rightarrow accl^n = \frac{dv}{dt}$$

$$\Rightarrow a = \frac{d}{dt}(r\omega \cos\omega t) \quad [\text{using } (4)]$$

$$= r\omega \frac{d}{dt}\cos\omega t$$

$$= r\omega(-\sin\omega t)\frac{d}{dt}(\omega t)$$

$$= -r\omega^2 \sin\omega t \text{ — } (5)$$

In the Δ OAP, $\sin\omega t = \frac{y}{r}$ [from ...]

Using this in (5) we get,

$$a = -r\omega^2 \cdot \frac{y}{r}$$

$$\Rightarrow \boxed{a = -\omega^2 y}$$

Special case :-

(i) At O, $y = 0 \Rightarrow a = 0$

(ii) At Y/Y', $y = \pm r \Rightarrow a = \pm\omega^2 r$

Thus a particle vibrating in S.H.M has zero accl^n while passing through mean position and has max^m acceleration while at extreme position.

(d) Time Period :- (T)

It is the time taken by the oscillating particle to complete one ~~vibration~~ oscillation.

It is given by, $T = \frac{2\pi}{\omega}$

where, $\omega \rightarrow$ Angular velocity.

But we know, $accl^n = -\omega^2 y$

$\Rightarrow -\frac{accl^n}{y} = \omega^2$

$\Rightarrow \omega = \sqrt{\frac{accl^n}{disp}}$

Thus, $T = \frac{2\pi}{\sqrt{\frac{accl^n}{disp}}}$

$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{disp}{accl^n}}}$

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X. HEAT & THERMODYNAMICS

Heat → It is a form of energy which produces the sensation of warmth.

Thermodynamics → Thermo + dynamics

Thermo → Means heat

dynamics → Mechanical motion involving wor[k]done

Hence the chapter heat & thermodynamics gives the idea about heat & the workdone in a system due to motion of heat in it.

Thermal Expansion of solid:—

Thermal Expansion:—

When an object heats, whether it is a solid, liquid or gas, it expands. This expansion in the object due to increase in the temp^{re} is called as thermal expansion.

In case of solid there are 3 types of expansion. such as:—

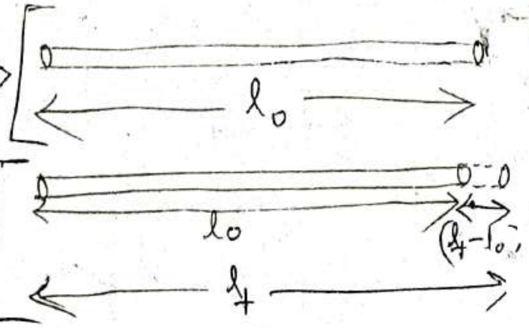
1. Linear Expansion. (Expansion through length i.e 1-d expansion)
2. Superficial / Areal Expansion. (Expansion through area i.e 2-d expansion)
3. Cubical Expansion. (Expansion through volume i.e 3-d expansion)

(1) Linear Expansion:—

Let's consider a long & thin rod, whose length is very large in comparison to its diameter.

Let $l_0 \rightarrow$ Original length at $0^\circ C \rightarrow$

& $l_t \rightarrow$ Final length at $t^\circ C \rightarrow$



Then the change in length $= (l_t - l_0)$

This increase in length $(l_t - l_0)$ depends upon two factors i.e upon original length (l_0) & upon rise of temp^r (t) i.e

$$(l_t - l_0) \propto l_0$$
$$\propto t$$

$$\Rightarrow (l_t - l_0) \propto l_0 t$$

$$\Rightarrow \boxed{l_t - l_0 = \alpha l_0 t} \quad \text{———} (1)$$

Where $\alpha \rightarrow$ constant of proportionality known as co-efficient of linear expansion

Now from (1),

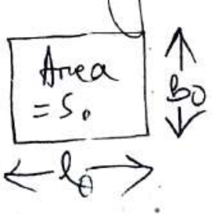
$$\boxed{\alpha = \frac{l_t - l_0}{l_0 t}} \quad \text{———} (2)$$

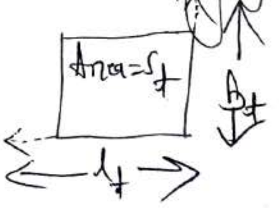
$$\& \quad l_t = l_0 + \alpha l_0 t$$

$$\Rightarrow \boxed{l_t = l_0 (1 + \alpha t)} \quad \text{———} (3)$$

(2) Superficial / Areal Expansion :-

Let's consider a thin sheet, which having some length & breadth but negligible thickness.

At $0^\circ C \rightarrow$ 

At $t^\circ C \rightarrow$ 

Let $S_0 \rightarrow$ original area of the sheet at 0°C

& $S_t \rightarrow$ final area of the sheet at t°C

Then change in area $= (S_t - S_0)$

& this increase in area is depending upon two factors i.e upon original area and upon rise in temp^r.

$$\text{i.e} \quad (S_t - S_0) \propto S_0$$

$$\propto t$$

$$\Rightarrow (S_t - S_0) \propto S_0 t$$

$$\Rightarrow \boxed{S_t - S_0 = \beta S_0 t} \quad \text{——} \ (4)$$

Where $\beta \Rightarrow$ constant of proportionality known as co-efficient of superficial expansion

Now from (4),

$$\boxed{\beta = \frac{S_t - S_0}{S_0 t}} \quad \text{——} \ (5)$$

$$\& \quad S_t = S_0 + \beta S_0 t$$

$$\Rightarrow \boxed{S_t = S_0 (1 + \beta t)} \quad \text{——} \ (6)$$

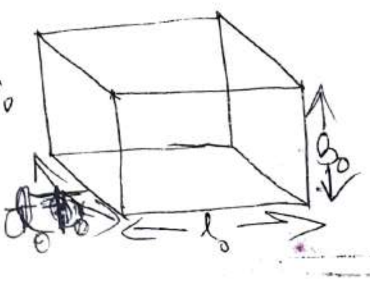
(3) Cubical Expansion :-

Let's consider a cube which having some specified length, breadth & thickness.

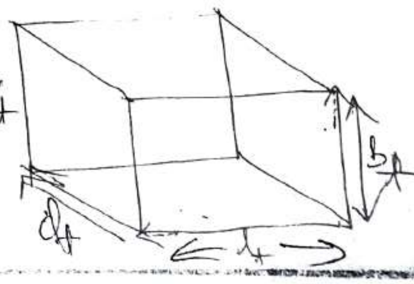
Let $V_0 \rightarrow$ original Volume of the cube at 0°C

$V_t \rightarrow$ Final volume of the cube at t°C

At 0°C
Volume $= V_0$



At t°C
Volume $= V_t$



Then the change in volume $= V_t - V_0$

The increase in volume depends upon two factors i.e. upon original volume and upon the change occur.

i.e $(V_t - V_0) \propto V_0$

$\propto t$

$\Rightarrow (V_t - V_0) \propto V_0 t$

$\Rightarrow \boxed{V_t - V_0 = \gamma V_0 t}$ —— (7)

Where $\gamma \rightarrow$ constant of proportionality known as co-efficient of cubical expansion

From (7), $\boxed{\gamma = \frac{V_t - V_0}{V_0 t}}$ —— (8)

& $V_t = V_0 + \gamma V_0 t$

$\Rightarrow \boxed{V_t = V_0 (1 + \gamma t)}$ —— (9)

Relation between α, β & γ :-

Relation betn α & β :-

We know, $\beta = \frac{S_t - S_0}{S_0 t}$ —— (10)

Again area $=$ length^2

i.e at $0^\circ C \rightarrow$ area, $S_0 = l_0^2$

& at $t^\circ C \rightarrow$ area, $S_t = l_t^2$

Putting this in equn (10) we get,

$$\beta = \frac{l_t^2 - l_0^2}{l_0^2 t}$$

As from eq^n (2) we have, $l_t = l_0(1+\alpha t)$, so putting this in above we get,

$$\beta = \frac{[l_0(1+\alpha t)]^2 - l_0^2}{l_0^2 t} = \frac{l_0^2(1+\alpha t)^2 - l_0^2}{l_0^2 t}$$

$$= \frac{\cancel{l_0^2}[(1+\alpha t)^2 - 1]}{\cancel{l_0^2} t} = \frac{(1+\alpha t)^2 - 1}{t}$$

$$= \frac{\cancel{1} + \alpha^2 t^2 + 2\alpha t - \cancel{1}}{t} = \frac{\alpha^2 t^2 + 2\alpha t}{t}$$

$$= \frac{\cancel{t}(\alpha^2 t + 2\alpha)}{\cancel{t}} = \cancel{\alpha^2 t}^{\,0} + 2\alpha$$

Neglecting the higher ordered term of α we get,

$$\boxed{\beta = 2\alpha} \quad \text{--- (11)}$$

Relation bet^n α & γ :—

We know, $\gamma = \frac{V_t - V_0}{V_0 t}$ — (12)

Again, volume of a cube $= (\text{length})^3$

i.e " " " at $0°C$, $V_0 = l_0^3$

& " " " " " $t°C$, $V_t = l_t^3$.

Putting the value of V_0 & V_t in (12) we get

$$\gamma = \frac{l_t^3 - l_0^3}{l_0^3 t}$$

As from eq^n (2) we have, $l_t = l_0(1+\alpha t)$, so putting this in above eq^n we get,

$$\gamma = \frac{[l_0(1+\alpha t)]^3 - l_0^3}{l_0^3 t} = \frac{l_0^3(1+\alpha t)^3 - l_0^3}{l_0^3 t}$$

$$\Rightarrow \gamma = \frac{\cancel{l_0^3}\left[(1+\alpha t)^3 - 1\right]}{\cancel{l_0^3}\, t} = \frac{(1+\alpha t)^3 - 1}{t}$$

$$\Rightarrow \gamma = \frac{\cancel{1} + \alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2 - \cancel{1}}{t}$$

$$\Rightarrow \gamma = \frac{\alpha^3 t^3 + 3\alpha t + 3\alpha^2 t^2}{t} = \frac{\cancel{t}\,(\alpha^3 t^2 + 3\alpha + 3\alpha^2 t)}{\cancel{t}}$$

$$\Rightarrow \gamma = \cancel{\alpha^3 t^2}^{\,0} + 3\alpha + \cancel{3\alpha^2 t}^{\,0}$$

As α is very small, hence neglecting the higher ordered term of α we get,

$$\boxed{\gamma = 3\alpha} \quad \text{—— (13)}$$

From eqn (11) & (13) we get,

$$\beta = 2\alpha \quad \& \quad \gamma = 3\alpha$$

i.e. $$\boxed{\alpha = \frac{\beta}{2} = \frac{\gamma}{3}} \quad \text{—— (14)}$$

First Law of Thermodynamics:—

Statement:— It states that "if the quantity of heat supplied to a system is capable of doing some work, then the quantity of heat absorbed ~~supplied~~ by the system is equal to the sum of the increase in the internal energy of the system & external work done by it".

i.e $\boxed{dQ = dU + dW}$

Where $dQ \rightarrow$ Amount of heat absorbed
$dU \rightarrow$ change in internal energy.
$dW \rightarrow$ External work done.

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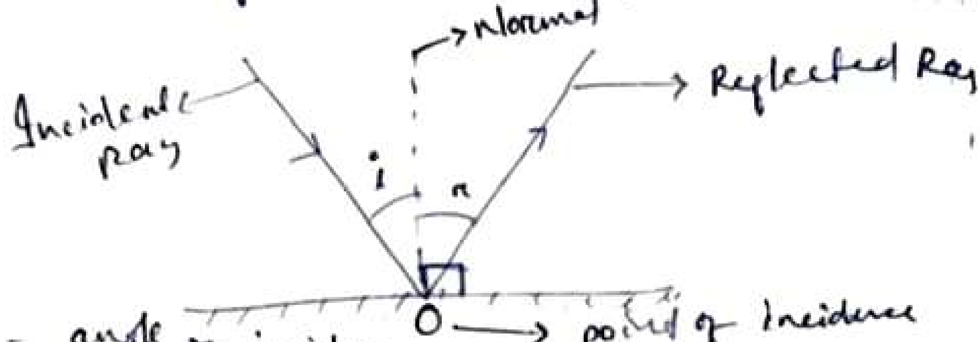
Optics

Optics → Geometrical Optics/Ray optics (Particle nature)
→ Wave optics/Physical optics (Wave nature)

Study of properties & behaviour of light.

Reflection →

The phenomenon in which light travelling from one medium to another returns back to the same medium from the interface.



$\angle i$ = angle of incidence
$\angle r$ = angle of reflection

Laws →

(i) Incident ray, Reflected ray, normal, all lie in one plane and that plane is $\perp$r to the interface.

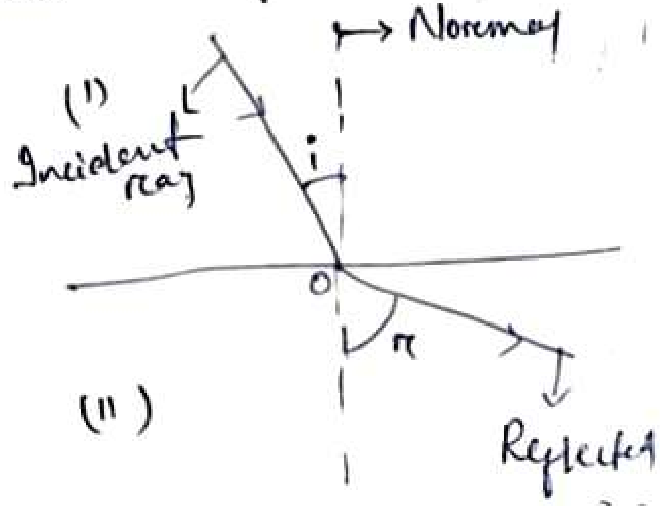
(ii) $\angle i = \angle r$

Refraction →

The phenomenon in which light travelling from one medium to another changes it's path or bends.

$\angle i$ = angle of incidence

$\angle r$ = angle of refraction



Laws →

(i) ~~Reflected~~ Incident ray, Refracted ray, normal, all lie on one plane and that plane is ⊥r to the interface.

(ii) $$\frac{\sin i}{\sin r} = \text{constant} \longrightarrow \text{Snell's law.}$$

$$\Rightarrow \frac{\sin i}{\sin r} = {}^{1}\mu_{2} \longrightarrow$$ Refractive index of 2nd medium w.r.t 1st medium

Refraction Index :

→ Property of medium, that decides how to what extent the direction of light will change in a medium i.e. the speed of light in the medium.

→ R.I. measures the optical density of the medium.

→ R.I. can be related to relative speed of propagation of light in a medium.

Relative speed = V/C

→ $\frac{\sin i}{\sin r} = {}^1\mu_2 \cdot \mu_{2,1} \to$ R.I. of 2nd medium w.r.t. 1st medium

$\mu_{21} > 1$, $\sin i > \sin r$
$\Rightarrow i > r$.

i.e. the ray bends towards the normal. If medium 2 is optically denser.

$\mu_{21} < 1$, $\sin i < \sin r \Rightarrow i < r$

i.e. Ray moves away from normal.

→ Optical density ≠ mass density.

→ It is the measure of absorbance.

More optically denser, light will travel slower.

→ R.I. indicates the no. of times light will be slower in a medium than its in vacuum.

→ $\mu_{21} = 1/\mu_{12}$

→ R.I. is related to relative speed of light in diff. media.

$\mu = \frac{c}{v} \to$ Absolute R.I.

→ ${}^1\mu_2 = \frac{v_1}{v_2}$

$\frac{\sin i}{\sin r} = {}^1\mu_2 \Rightarrow \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{c/v_1}{c/v_2} = \frac{\mu_2}{\mu_1}$ → $\frac{c/v_2}{c/v_1}$

$\frac{v_1/c}{v_2/c} = \frac{1/\mu_1}{1/\mu_2} = \frac{\mu_2}{\mu_1}$

→ higher R.I. → Higher optical ... slower speed of light.

smaller R.I. → optically rarer.

→ R.I. is unitless and dimensionless.

→ Q. R.I. of water with air $= {}^a\mu_w = \frac{4}{3}$

R.I. of glass $= {}^a\mu_g = \frac{3}{2}$

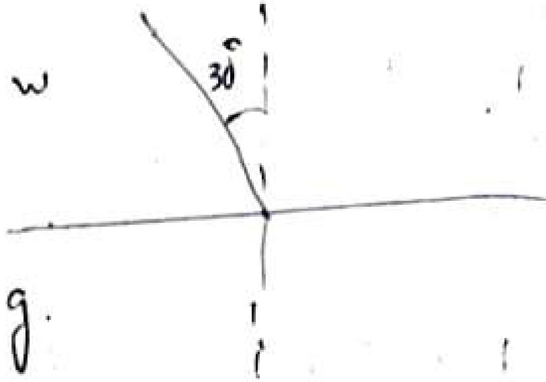
R.I. of glass w.r.t. water.

$$= {}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$$

$\angle i = 30^\circ$

$\mu_g = 3/2$, $\mu_w = 4/3$

$\angle r = ?$



$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3}$$

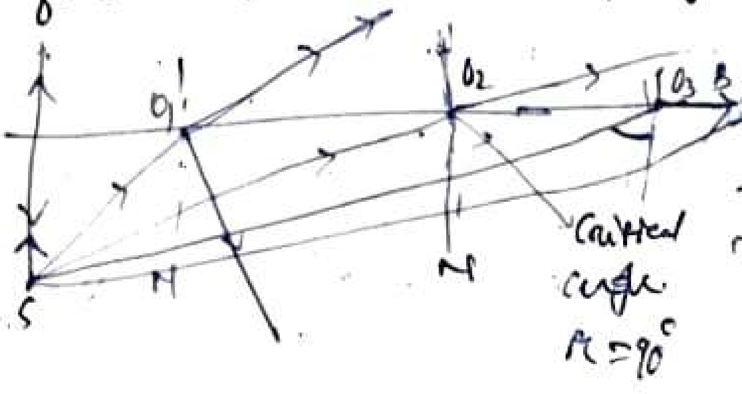
$\Rightarrow \sin 30^\circ = \frac{9}{8} \sin r$

$\Rightarrow \sin r = \frac{8}{9} \times \frac{1}{2} = \frac{4}{9}$

$\Rightarrow r = \sin^{-1}\left(\frac{4}{9}\right)$

Total Internal Reflection →

When light travels from optically denser medium to optically rarer medium, at the interface, it is partly reflected back to the same medium and partly get refracted. This reflection is called internal reflection.



→ At a point when the angle of incidence is further increased, no refraction possible and the incident ray will be totally reflected. It is called total internal reflection.

→ In total internal reflection no transmission of light takes place.

→ The angle of incidence corresponding to the angle of refraction ~~incidence~~ $= 90^\circ$, is called critical angle.

→ If, $\mu < 1$, then $\frac{\sin i}{\sin 90^\circ} = \sin i_c$

$\Rightarrow \cancel{\frac{\sin i}{\sin r} < 1}$ $\Rightarrow \sin i_c = {}^1\mu_2$

$\Rightarrow \cancel{\sin i < \sin r}$.

→ As angle of incidence increases, angle of refraction increases.

$$\boxed{i_c = \sin^{-1}\frac{\mu_2}{\mu_1}}$$

And max^m value of $\sin\theta = 1$, So, ~~μ_2~~ can not exceed μ_1. So, total internal reflection can not occur when light travels from rarer to denser medium.

2 conditions for T.I.R.

① Must travel from denser to rarer medium.

② Angle of incidence must be greater than critical angle.

Relation betn i_c and R.I. →.

Snell's law $\frac{\sin i}{\sin r} = {}^1\mu_2 \Rightarrow \frac{\sin i_c}{\sin 90^\circ} = {}^1\mu_2 \Rightarrow \sin i_c = \mu_{21}$

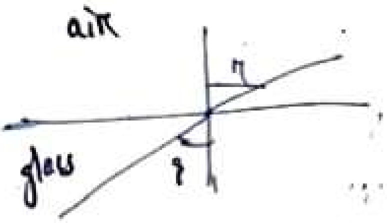
For angle of incidence = C, angle of refraction = 90°

So by Snell's law, R.I. of 1st medium w.r.t. 2nd medium i.e. ${}^2\mu_1$ =

$${}^1\mu_2 = \frac{\sin i}{\sin r} = \frac{\sin C}{\sin 90^\circ} = \frac{\sin C}{1} = \sin C$$

$$\Rightarrow \frac{1}{{}^1\mu_2} = \frac{1}{\sin C}$$

$$\Rightarrow {}^2\mu_1 = \frac{1}{\sin C}$$



If 2nd medium is air,

$${}^2\mu_1 = \mu_1$$

So, $\boxed{\mu_1 = \frac{1}{\sin C}}$ → i.e. Absolute R.I. of a medium is equal to reciprocal of the sine of the critical angle for that medium

Q) The critical angle of incidence of a glass slab immersed in air is 30°. What will be the critical angle when it immersed in medium of R.I. $\sqrt{2}$

Soln

$$\sin C = \frac{1}{{}^a\mu_g} \Rightarrow \sin 30^\circ = \frac{1}{{}^a\mu_g}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{{}^a\mu_g}$$

$$\Rightarrow {}^a\mu_g = 2$$

$$\sin C_m = \frac{1}{{}^m\mu_g} = \frac{1}{{}^m\mu_a \times {}^a\mu_g} = \frac{{}^a\mu_m}{{}^a\mu_g}$$

$$= \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$\Rightarrow C_m = \sin^{-1}\sqrt{2} = 45^\circ.$$

Q) What is the critical angle for a ray going from glass to water? The R.I. of glass and water are 1.62 & 1.32.

Soln:-

$$\sin C = \frac{1}{{}^w\mu_g} = \frac{\mu_w}{\mu_g} = \frac{1.32}{1.62}$$

R. I. of material of prism →.

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(A/2\right)}.$$

Optical Fiber :-

→ They work on the principle of TIR.

→ Cylindrical wave guide consisting of two layers. core & outside cladding. R. I. of core is higher.

→ It's made of high quality transparent, glass or plastic. Successive internal reflection.

→ Study of optical fibers is called fiber optics.

→ No appreciable loss of ~~energy~~ intensity of light signal.

Properties

→ It has higher band width (can contain larger variety of frequency), So, optical fiber have higher information carrying capacity. and permit transmission over long distances than electrical cables.

→ Small in size, light weight, have high tensile strength so flexible. They can be bent or twisted easily.

→ It has hight degree of signal security. As

→ It's free from electrical interference.

→ They does not carry high voltage or current. So, they are safer than electrical cable.

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8.9 Current Electricity & Electromagnetism

1. Kirchoff's Laws:-

(1) Kirchoff's Current Law (KCL) :-

Statement:- It states that "the algebraic sum of all the currents meeting at an electrical junction/node is equal to zero."

or Sum of currents entering is equal to sum of currents leaving at a junction.

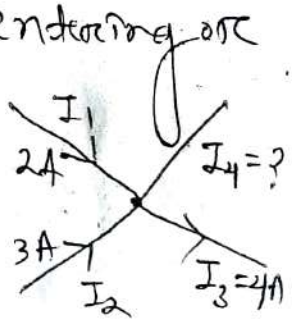
i.e $$\boxed{\sum_k I_k = 0}$$

Sign convention:- To determine the algebraic sum of currents we have to follow the sign conventions given bellow such as:-

→ The current entering at a given junct^n is taken as +ve.

→ The current leaving at a given junct^n is taken as -ve.

Explanation:- Let I_1, I_2, I_3, I_4 are the four currents meeting at a junction where I_1, I_2 are the entering currents & I_3 leaving the junction. We have no idea about 'I_4' i.e it is entering or leaving.



According to KCL, $\sum I = 0$

$\Rightarrow I_1 + I_2 + I_3 + I_4 = 0 \Rightarrow I_4 = -I_1 - I_2 - I_3$

Taking sign convention in account & putting the values of I_1, I_2, I_3 we get

$\Rightarrow I_4 = -(2A) - (3A) - (-4A)$

$\Rightarrow I_4 = -5A + 4A = -1A$

i.e $I_4 = -1A$ & this -ve sign indicating that I_4 current is leaving the junction.

Hence, $I_1 + I_2 = I_3 + I_4$ $\left[\because \begin{array}{l} I_1 + I_2 = 5A \\ I_3 + I_4 = 5A \end{array}\right.$

i.e sum of entering currents = sum of leaving currents

Kirchhoff's Voltage Law (KVL) :-

Statement :- It state that "in a closed circu the algebraic sum of e.m.f/voltage ~~equal to~~ & product of resistances & current is equal to ze

i.e $\boxed{\sum_{n} V_n + I_n R = 0}$

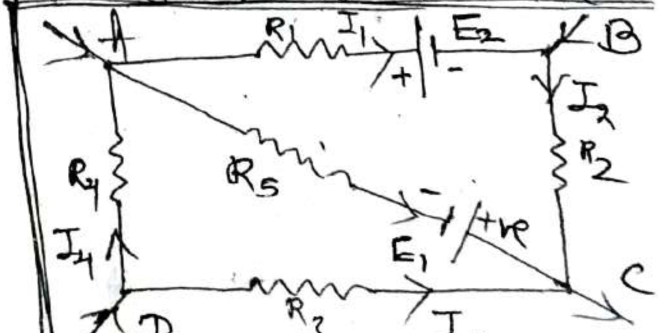
Sign convention :-

→ If the current is flowing from -ve to +ve then the emf is taken as +ve.

→ If the current is flowing from +ve to -ve then the emf is taken as -ve.

→ If the voltage is along the dirn of current then it is -ve & if the voltage is in the opposite dirn to current then it is +ve."

→ Resistance has no sign convention.



Explanation :- In the given closed circuit ABCD, R_1, R_2, R_3, R_4, R_5 are the resistance & I_1, I_2, I_3, I_4, I_5 are the currents in the five arms of the circuit, with two e.m.f source E_1 & E_2.

Applying KVL to mesh/loop ABC we get,

$$I_1 R_1 + I_2 R_2 - I_5 R_5 = E_1 - E_2 \quad \text{(1)}$$

& in mesh/loop ACD, we get.

∵ I_1 & I_2 → +ve
I_5 → -ve

$$I_5 R_5 - I_3 R_3 - I_4 R_4 = E_1 \quad \text{(2)}$$

In general from (1) & (2) we get,

$$\boxed{\sum IR + \sum E = 0}$$

Application of Kirchhoff's law to Wheatstone Bridge :-

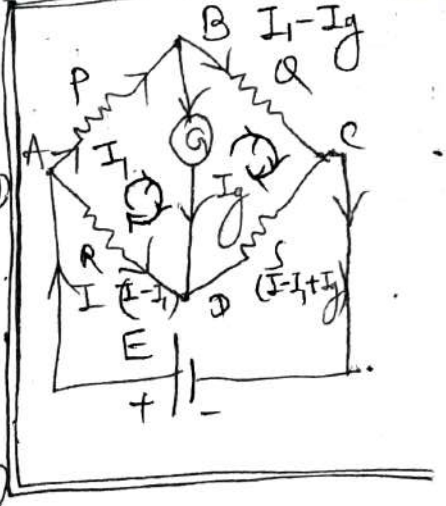
→ Wheatstone bridge is an electrical arrangement used to determine the value of an unknown resistance in an electrical ckt.

→ Let's consider a wheatstone bridge ABCD consisting of 4 resistors P, Q, R, S in four arms and an e.m.f source 'E' connected betn A & C.

~~through~~ → Again one galvanometer 'G' is connected between the terminals B & D, as shown in the given fig.

→ After closing the circuit the resistances P, Q, R, S are so adjusted that the galvanometer shows no deflection, hence the wheatstone bridge is in Balanced condition.

→ Now Using KCL, the current Distribution in the circuit is as follows.



In the mesh ABD, we get

$$I_1P + I_gG - (I-I_1)R = 0 \quad \text{—(1)}$$

& in the mesh BCD, we get,

$$(I_1-I_g)Q - I_gG - (I-I_1+I_g)S = 0 \quad \text{—(2)}$$

→ The R.H.S of both eqn's are zero as there is no e.m.f in the closed cct. ABCD

→ Since the Bridge is balanced, so there is no fluctuation current through galvanometer i.e $I_g = 0$ —(3)

Using (3) in (1) & (2) we get,

$$I_1P - (I-I_1)R = 0 \Rightarrow I_1P = (I-I_1)R \quad \text{—(4)}$$

$$I_1Q - (I-I_1)S = 0 \Rightarrow I_1Q = (I-I_1)S \quad \text{—(5)}$$

Dividing (4) by (5) we get,

$$\frac{I_1P}{I_1Q} = \frac{(I-I_1)R}{(I-I_1)S}$$

$$\Rightarrow \boxed{\frac{P}{Q} = \frac{R}{S}} \quad \text{—(4)}$$

This is the required ~~balan~~ condition for which wheatstone bridge is said to be balanced.

Biot-Savart's Law :-

~~As we all know a current carrying conductor~~

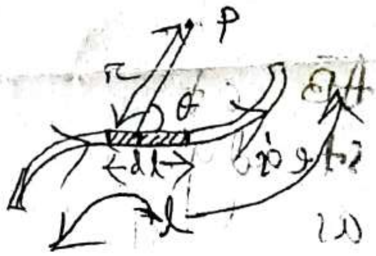
Statement :- It states that "the differential contribution (dB) of a magnetic field around a wire of length 'l' carrying current 'i' at any point is directly proportional to amount of current flow, sine of the angle betn posn vector and current, length of the differential portion but inversely proportional to square of the distance betn observation point & wire.

i.e $dB \propto i$

$\propto dl$

$\propto \sin\theta$

$\propto \frac{1}{r^2}$



$\Rightarrow dB \propto \frac{i\,dl\sin\theta}{r^2} \Rightarrow dB = K\frac{i\,dl\sin\theta}{r^2}$ —— (1)

Where $K \rightarrow$ constant of proportionality and in S.I system $K = \frac{\mu_0}{4\pi}$ & in C.G.S $K = 1$.

Now, using $K = \frac{\mu_0}{4\pi}$ in (1) we get,

$$\boxed{|\vec{dB}| = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}}$$

Special case :-

Case-1 :- If $\theta = 0°/180° \Rightarrow \sin\theta = 0$

Hence, $|\vec{dB}| = 0 \rightarrow$ which is the minm value i.e there is zero magnetic field intensity at any pt.

on the axial line.

Case-2 :- If $\theta = 90^\circ \Rightarrow \sin\theta = 1$

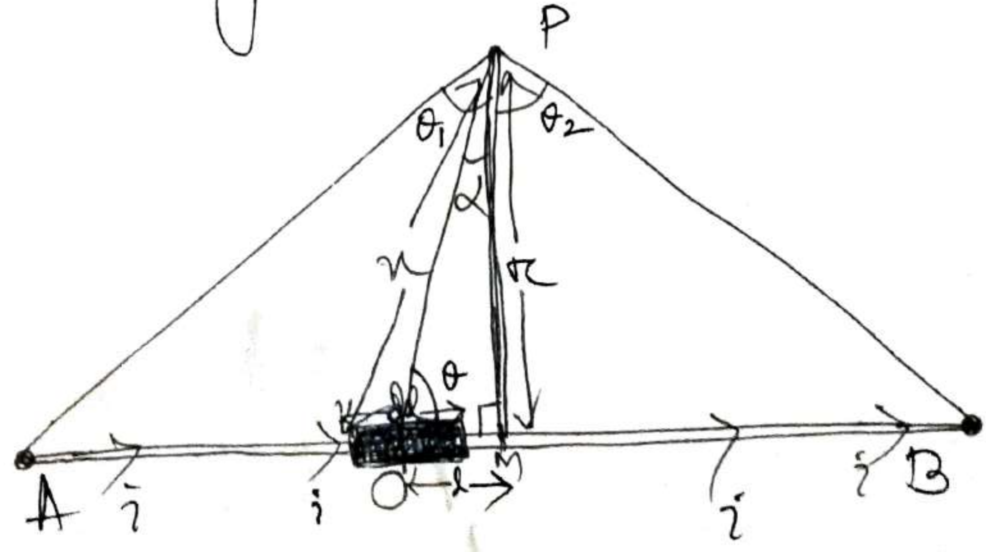
Hence, $dB = \frac{\mu_0}{4\pi}\frac{i\,dl}{r^2} \rightarrow$ maximum value.

i.e the magnetic field intensity at any point lying on a line $\perp^{ar}$ to the length of the wire is maximum.

Applications of Biot-Savart's law:-

(1) Magnetic field intensity due to a straight conductor carrying current:-

Let's consider a long straight conductor AB of finite length which is carrying a steady current 'i' in dir^n A to B as shown in the fig bellow. Now taking the small element 'dl' of the wire AB & let's find out the magnetic field intensity using Biot-savart's law.



→ Let ~~we~~ have to find out the magnetic field intensity at point 'p' which is at a distance 'x' from the small element dl such that 'p' is at a distance 'r' on the line $\perp^{ar}$ to which is centre of $\overline{AB}$.

→ According to Biot-savart's law for the small element 'dl' we can write

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{x^2} \quad \text{—— (1)}$$

→ Now let's findout the value of $\sin\theta$, x^2 & dl.

In the right angle triangle OMP,

$$\alpha + \theta = 90^\circ \Rightarrow \theta = 90^\circ - \alpha \Rightarrow \sin\theta = \sin(90^\circ - \alpha)$$
$$= \cos\alpha \quad \text{—— (2)}$$

& in the same triangle OMP,

$$\cos\alpha = \frac{MP}{OP} = \frac{r}{x} \Rightarrow x = \frac{r}{\cos\alpha} \Rightarrow x^2 = \frac{r^2}{\cos^2\alpha} \quad \text{—— (3)}$$

& in this triangle, $\tan\alpha = \frac{OM}{PM} = \frac{l}{r}$

$$\Rightarrow l = r\tan\alpha \Rightarrow dl = r\sec^2\alpha \, d\alpha \quad \text{—— (4)}$$

Using (2), (3) & (4) in eqn (1) we get,

$$dB = \frac{\mu_0}{4\pi} \frac{i\,(r\sec^2\alpha \, d\alpha)\cos\alpha}{r^2/\cos^2\alpha}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r} \cos\alpha \, d\alpha \quad \text{—— (5)}$$

Now the net magnetic field intensity at Pt. 'p' due to total length of the conductor is given by, $B = \int_0^B dB = \int_{\theta_1}^{\theta_2} \frac{\mu_0}{4\pi} \frac{i}{r} \cos\alpha \, d\alpha$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} \int_{\theta_1}^{\theta_2} \cos\alpha \, d\alpha$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} \left[\sin\alpha\right]_{\theta_1}^{\theta_2}$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{i}{r} (\sin\theta_2 - \sin\theta_1) \quad \text{— (6)}$$

This is the total magnetic field intensity for a finite length of conductor.

For Infinitely long conductor :-

In this case, the angles

$$\theta_1 \to -90^\circ \ \& \ \theta_2 \to 90^\circ$$

Putting the value of θ_1 & θ_2 in (6) we get,

$$B = \frac{\mu_0}{4\pi} \frac{i}{r} \{\sin 90^\circ - \sin(-90^\circ)\}$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r} (\sin 90^\circ + \sin 90^\circ)$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r} 2 \sin 90^\circ$$

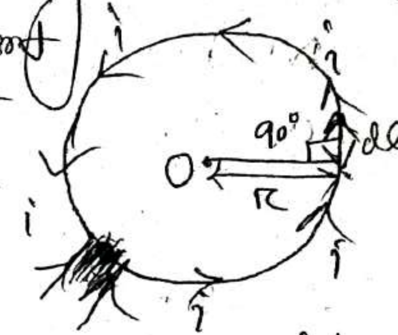
$$= \frac{\mu_0}{4\pi} \frac{i}{r} (2 \times 1) = \frac{\mu_0}{4\pi} \frac{i}{r} \cdot 2$$

$$\Rightarrow \boxed{B = \frac{\mu_0}{2\pi} \frac{i}{r}}$$

Magnetic Field at the centre of a Circular coil

Let's consider a circular coil of radius 'r' carrying current 'i' & having centre at 'O'.

Let's consider a small element 'dl' of the circular coil & findout the magnetic field intensity at the centre of the coil 'O' which is at a distance 'r' from $\vec{dl}$ & making an angle $\theta = 90^\circ$ with current flow dirn through '$\vec{dl}$'.



Now according to Biot-savart's law,

$$|\vec{dB}| = \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$$

But here $\theta = 90^\circ \Rightarrow |\vec{dB}| = \frac{\mu_0}{4\pi} \frac{i\,dl \sin 90^\circ}{r^2}$

$$\Rightarrow |\vec{dB}| = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2}$$

Now the net magnetic field intensity at 'O' dued to total length of the circular coil is given by,

$$B = \int_0^B dB = \int \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} = \frac{\mu_0}{4\pi} \frac{i}{r^2} \int_0^l dl$$

$$= \frac{\mu_0}{4\pi} \frac{i}{r^2} \times 2\pi r$$

[$\because \int dl =$ circumference of the circular coil & $dl = 2\pi r$]

$$= \frac{\mu_0}{4\pi} \frac{2\pi i}{r}$$

$$\Rightarrow B = \frac{\mu_0 i}{2r}$$

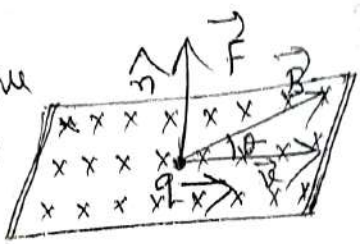
For 'N' no. of turns in coil,

$$\boxed{B = N\frac{\mu_0 i}{2r}}$$

Motion of a charged particle inside a uniform magnetic field:–

Let's consider a charge (q) moving inside a uniform magnetic field '$\vec{B}$' with a velocity $\vec{v}$ such that the dirⁿ of motion of charge makes an angle 'θ' with the dirⁿ of magnetic field as shown in the fig bellow.

Then the charge 'q' will experiences a force '$\vec{F}$' given by,



$$\left.\begin{aligned} \vec{F} &= q(\vec{v}\times\vec{B}) \\ \Rightarrow \vec{F} &= qvB\sin\theta\,\hat{n} \\ \& \ |\vec{F}| &= qvB\sin\theta \end{aligned}\right\} \text{ — (1)}$$

Special case:–

→ **Case-1:–** If $\theta = 0°/180° \Rightarrow \sin\theta = 0 \Rightarrow |\vec{F}| = 0$

i.e no force experienced by a charge moving along a line parallel/antiparallel to the dirⁿ of magnetic field.

→ **Case-2:–** If $\theta = 90° \Rightarrow \sin\theta = 1 \Rightarrow |\vec{F}| = qvB$

i.e maximum force experienced by a charge particle while moving at $\perp$ar dirⁿ to magnetic field.

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19. Electro-Magnetic Induction

Electromagnetic Induction:-

The phenomenon of production/change in electricity due to magnetism is called as electro-magnetic induction. i.e electricity is induced due to magnetic field.

Faraday's Laws of Electromagnetic Induction:-

There are ~~three~~ two laws such that:-

(1) Whenever magnetic flux linked with the circuit changes an e.m.f (electromotive force) is induced in it. and

(2) The indu... e.m.f such that [illegible] the circuit so long as the change in magnetic flux linked with it continues.

(2) The induced e.m.f is equal to the -ve rate of change of magnetic flux linked with the circuit.

i.e e.m.f, $\boxed{E = -\frac{d\phi_B}{dt}}$

Where, $E \to$ e.m.f or electromotive force.

$\phi_B \to$ magnetic flux linked with the circuit.

$\frac{d\phi_B}{dt} \to$ Rate of change of magnetic flux

-ve sign $\to$ Indicating that e.m.f ~~$\propto \frac{d\phi_B}{dt}$~~ tends ~~to~~ oppose ~~the~~ change in ...

If 'N' → no. of coils linked with the,

then, $$\boxed{E = -N\frac{d\phi_B}{dt}}$$

NOTE :-

Electromotive Force :- (e.m.f)

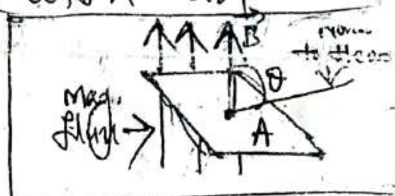
Electromotive force is not a force actually but a potential or voltage measured in volt i.e it is the energy/work per unit charge.

i.e $E = \frac{\text{work}}{\text{charge}} = \frac{W}{q} = \frac{\text{Joules}}{\text{coulomb}} = \text{volt}$.

Hence e.m.f is the energy supplied to an electric charge.

Magnetic Flux :- (ϕ_B)

The magnetic flux linked with an electric circuit is given by,



$$\phi_B = \vec{B}\cdot\vec{A}$$

$\Rightarrow \phi_B = BA\cos\theta$ and for 'N' no. of turns,

$\phi_B = NBA\cos\theta$

where, ϕ_B → magnetic flux.

B → magnetic field intensity.

A → Area of the electric circuit or the area of the conductor/loop.

θ → Angle betn the magnetic field and normal to the area of the conductor.

case-1

If $\theta = 0^\circ \Rightarrow \phi_B = BA$

If $\theta = 180^\circ \Rightarrow \phi_B = -BA$

case-2

If $\theta = 90^\circ$

$\Rightarrow \phi_B = 0$ i.e no mag. flux link

Law:-

atement:- It states that "the direction of induced e.m.f is such that it tends to oppose the very cause which produce it".

$$\text{i.e} \quad E = -\frac{d\phi_B}{dt}$$

→ Here the '-ve' sign indicating that ~~both~~ e.m.f and ~~mag~~ rate of change of magnetic flux both have opposite sign.

→ i.e one is opposing other or the e.m.f opposing the rate of change of magnetic flux.

→ As we all know due to current flow in any region there is influence of magnetic field in this region. Since here the e.m.f / voltage will creat ~~its~~ own magnetic field due to flow of this induced current.

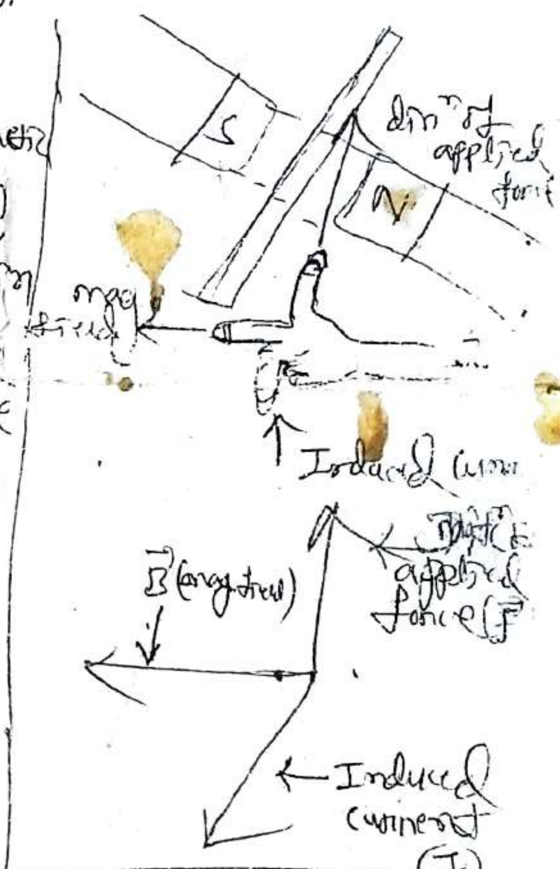
→ So this ~~induced~~ magnetic field created by the induced emf/ ~~voltage~~ /induced current will try to oppose the original magnetic field linked with the original magnetic flux which is the cause of this induced e.m.f

Fleming's Right Hand Rule :-

Statement :- It states that " streching the th first finger & middle finger of right ha in such a way that they are mutual ⊥ to eachothere, if the 1st finger points towards magnetic field dirn, thumb " mot of conductor/applied force dirn then the as ~~and~~ a resnlt middle finger will direct towards induced current".

Explanation :-

In the given fig, magnetic field is in clocuwise dirn then the wire has current in downward dirn & when the magnetic field is in anti-clocwise dirn the wire has current in upward dirn & the force is given by,



$$\vec{F} = \vec{I} \times \vec{B} = IB\sin\theta\,\hat{n}$$

$$\Rightarrow |\vec{F}| = IB\sin\theta$$

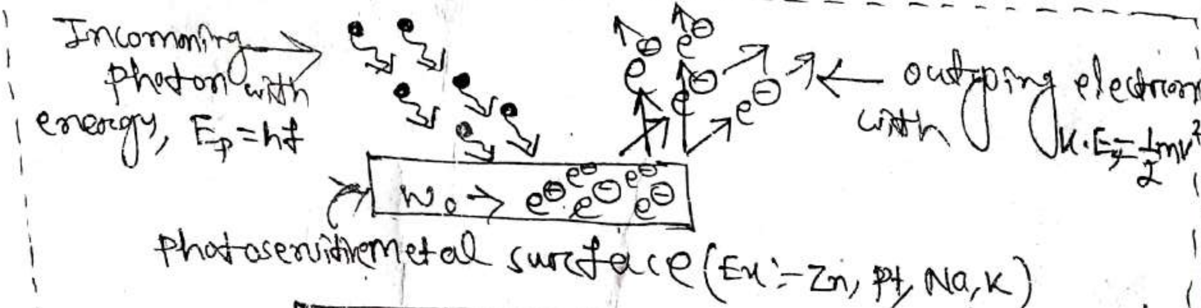
→ Fleming's Right hand rule is applicable for electric generator.

OM SAI/MAA

10. MODERN PHYSICS

Photoelectric Effect :-

Defination :- The phenomenon in which the emission of electrons occurs from the surface of a photosensitive metal due to incidence of light of shorter wavelength, is called as photoelectric effect or photoelectric emission.



Photoelectric Emission

Some terms connected with photoelectric emission :-

① **Photoelectric current / photocurrent (I) :-**

The emitted electrons in the photoelectric process are called as photoelectrons & the current produced due to motion of these electrons is called as photocurrent.

② **Stopping Potential (V_0) :-**

It is the minimum potential/voltage bellow which photoelectric emission stopps or bellow which there is no flow of electron or bellow which the value of photocurrent, $I = 0$.

③ **Threshold Frequency (f_0) :-** It is the certain minimum frequency of incident light, bellow which photoelectric emission stopps or photocurrent, $I = 0$.

④ Work Function (W_0):-

It is the minimum energy required to break the binding energy of the metal to produce free electrons & bellow this minimum energy photoelectric emission stopp. i.e photocurrent, $I = 0$

→ The work function value is different for different metal as the binding energy for different metal is different.

Laws of Photoelectric Effect:-

① It is an instantaneous process. (as it can occure within time 10^{-9} sec)

② The photoelectric current (I) is directly proportional to intensity of incident light & independent of frequency of incident light.

i.e $\boxed{\begin{array}{l} I \propto \text{Intensity of incident light} \\ \& \ I \text{ is independent '' freq. of '' ''} \end{array}}$

③ The stopping potential (V_0) is directly proportional to frequency of incident light & independent of intensity of incident light.

i.e $\boxed{\begin{array}{l} V_0 \propto \text{Frequency of incident light} \\ \& \ V_0 \text{ is independent of intensity incident light} \end{array}}$

④ The photoelectric emission stopps bellow a certain minimum freq. of incident light, is called as threshold frequency (f_0).

Einstein's Equation of photoelectric Effect:-

Einstein explain the photoelectric effect on the basis of planck's theory & acc. to planck's theory, ~~the~~ a beam of light is the collection of discrete wave packets called as photon, each with energy,

$$E_p = hf \quad \text{---} (1)$$

where, $E_p \rightarrow$ Energy of photon.

$h \rightarrow$ planck's constant.

$f \rightarrow$ frequency of incident light.

When we incident light of shorter wavelength (Eg:- UV rays) on the surface of a photosensitive metal, then the energy absorbed from the incident light is utilised in two ways. ~~such as:~~ i-e the minimum energy or the work function to break the binding energy of the metal i-e 'W_0' & the maximum K.E for electron to emitt ~~from the surface.~~ ~~within~~ instantly i-e $\frac{1}{2}mv_{max}^2$ (where, $V_{max} \rightarrow$ max^n velocity).

i-e the total energy for electron,

$$E_{electron} = W_0 + \frac{1}{2}mv_{max}^2 \quad \text{---} (2)$$

According to conservation law of energy,

Energy of incoming photon = Energy of outgoing e^-'s

$$\Rightarrow E_p = E_{electron}$$

Using ① & ② we get,

$$hf = W_0 + \frac{1}{2}mv_{max}^2 \quad \text{— ③}$$

Again, for minimum freq, 'f₀' the minimum energy, $W_0 = hf_0$ $\left(\because \text{Energy}, E = hf\right)$

Using this in ③ we get,

$$hf = hf_0 + \frac{1}{2}mv_{max}^2$$

$$\Rightarrow \frac{1}{2}mv_{max}^2 = hf - hf_0$$

$$\Rightarrow \boxed{\frac{1}{2}mv_{max}^2 = h(f - f_0)} \quad \text{— ④}$$

This equation is called as Einstein's equation of photoelectric effect which shows that the K.E of emitted electrons is directly proportion to frequency of incident light, as 'h' is constant.

Photoelectric cell / Photocell:-

It consists of an evacuated bulb whose innerside is coated with an alkali metal with an window. The bulb is made up of glass if it is to be used for white light & is made up of quartz if it is to be used for UV light.

→ It has ~~coated with~~ an electrode which is given a +ve potential with the help of a battery.

→ Light from a source is focused into the metal with the help of a convex lens & an ammeter connected in the circuit indicates the photoelectric current due to emission of photo-electrons.

...ications of photocell or Appl^n of photoelectric Effect:-

- (1) It plays an important role in television studio.
- (2) It is used for reproduction of sound in films.
- (3) It is used for triggering fire alarm.
- (4) It is used in operating burglar's alarm.
- (5) It is used for automatic switching of street light.
- (6) It is used in electronic counter to count automatically the no. of persons entering or leaving a hall.

LASER:- It is the abbreviation of Light Amplification by Stimulating Emission of Radiation.

L → Light
A → Amplification by
S → Stimulating
E → Emission of
R → Radiation.

Properties / characteristic of LASER:-

- (1) It is highly directional in nature i-e a relatively narrow beam in a specific direction.
- (2) It is coherent in nature i-e it is highly coherent in space & time.
- (3) It is monochromatic in nature i-e it has a single wavelength.

→ ④ It is a highly intense beam as it can be focus over a very small area i.e 10^{-6} m.

→ ⑤ It give huge light power approximately equal to 10 watt/c.m^2.

Application / Use of LASER :-

→ ① In medical field it is used in eye surgery like retina surgery.

→ ② It can cut the flesh & can seal the oozing cells of a human body.

→ ③ Laser beam is using to cut, drills & melt the metals in industry.

→ ④ For high speed photography & as printer also Laser is using.

→ ⑤ In astronomy, it can control/guide the spaceship upto a billion of K.m.

→ ⑥ Fussion is being tried using laser.

LASER PRINCIPLE :-

→ A laser system is consists of an active medium & this active medium is placed inside a resonating cavity having two reflectors at its both ends.

→ Out of these two reflectors, one is total reflector (100%) & the other is partial reflector (90-95%).

→ An electrical or optical pump is there within the resonating cavity to excite the atoms of the medium, as shown in the give fig①.

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