Lecture Notes

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Sub : Strength of Material

Som

Introduction :~

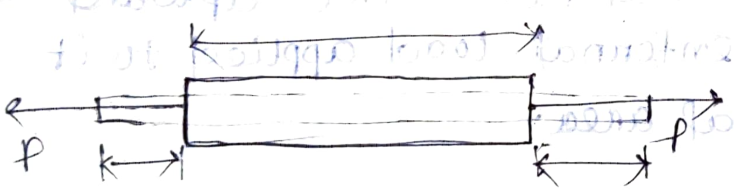
Strength of material is a branch of mechanics, which deals with the study of solid objects to stresses and strains. The study of strength of material often refers to various methods of calculating the stress and strain in structural members such as beam, columns and shafts

or.

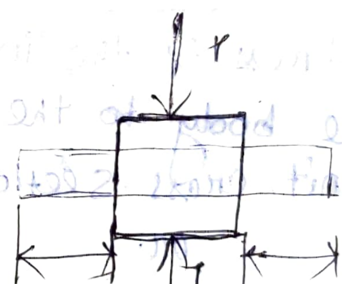
Study of various types loading or force and their internal effects on deformation bodies. To determine stress, strength and deflation by loads.

Deformation :~

When a body is acted open by some external load or forces it undergoes deformation that is change in shape, size and dimension.



(Tensile strength)



(compressive strength)

Classification of loads :-

A load may be define as the combined effect of external forcess acting on a deformable body.

→ The loads are classified as :-

i) Dead load
ii) Flactuating or live load.
iii) Inertial loads of forces.
iv) Centrifugal load.

→ other classification are :-

i) Tensile Load

ii) Compressive Load

iii) Tersonal and Twinsting Load.

iv) Bending Load

v) Shearing Load.

→ Load May be ------

i) **point Load or concentrated Load :-**

point Load consider to act at a point.

ii) **Distributed Load or uniformally verying Load :-**

Distributed load spreed or distributed over the length of the beam.

★ **Stress (σ) :-**

Stress is the internal resistance upward by the body to the enternal load applied to it per unit cross sectional area.

or.

The internal opposite force to the enternal loading ~~appli~~ per unit area is known as Stress.

mathematically, $\text{stress} = \dfrac{\text{Load}}{\text{unit cross sectional area}}$

or

$\text{Stress} = \dfrac{\text{Force applied}}{\text{Area of original cross-sectional (A)}}$

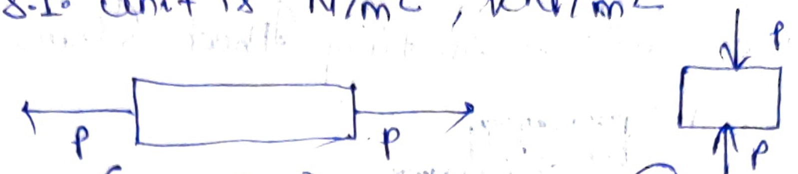
$$\sigma = \frac{P}{A}$$

Where, P = Load/force

A = unit cross-sectional area.

→ Stress are normal to the plane to which they are tensile or compressive in nature.

→ S.I. unit is N/m^2 , kN/m^2



(Tensile) (Compressive)

$1\ N/m^2 = 1\,Pa$

$1\ kN/m^2 = 1000\ Pa$ or N/m^2 or 10^3.

$1\ MN/m^2 = 10^6\ N/m^2$ or Pa.

$1\ GN/m^2 = 10^9\ N/m^2$ or Pa.

Types of Stress :-

Simple / Direct Stresses :- (It developed under direct loading condition) It is three types such as.

(1) Tensile stress (σ_t)

(2) compressive Stress (σ_c)

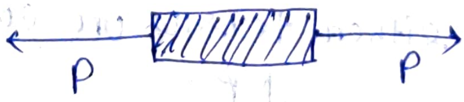
(3) Shear Stress (τ)

(4) volumetric Stress

(5) Longitudinal Stress.

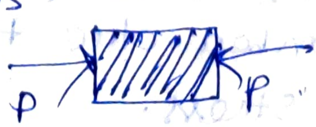
(i) Tensile Stress :- $\left(\frac{P}{A}\right)$

If Applied force on loads tends to increase the length of the solid body the stress induced is called Tensile stress.



(ii) compressive stress :- $\left(\frac{P}{A}\right)$

If the applied force tends to decrease the length of the solid body the stress reduce is called Compressive stress.



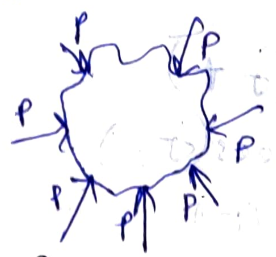
(iii) Shear Stress :-

Stress cast by force acting along or parallel to the area resisting that force.

(iv) Volumetric Stress :-

When a body is acted upon by forces in such a manner that.

→ The force at any point is normal to the surface
→ The magnitude of the force on any small area is proportional to the area
→ The force per unit area is then called the volumetric stress



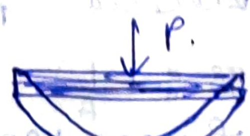
Indirect Stress :-

(i) Bending Stress.
(ii) Torsion Stress.

(i) ~~Bed~~ Bending Stress :-

The stress developed in a member due to bending action of transverse load.

→ It may be either tensile or compressive.

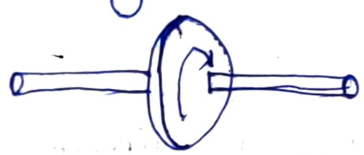


(ii) Torsion Stress :-

The stress developed against the deformation in a member due to torque is known as Torsion stress.

(e) Combined Stress :- (include direct and indirect Stress)

Stress developed against the deformation due to combined action of bending twisting or direct loading is called combined stress



★ Strain :- Strain is measure of deformation

it is denoted by (e) or (E)

→ Mathematically strain $E = \frac{\text{change in Dimension}}{\text{original Dimension}}$

★ Strain is unit less.

→ When a body subjected to stress is said to be strained hence strain is the deformation produced by stress.

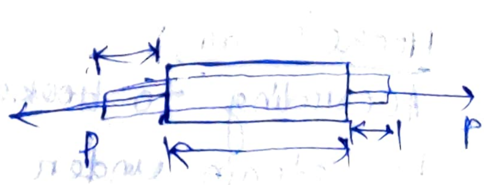
Types of Strain (e_T) :-

There are basically three type of strain such as :
(i) Tensile strain (e_T)
(ii) Compressive strain (ε_c)
(iii) Shear strain (ϕ)

(i) Tensile Strain (ε_T) :-

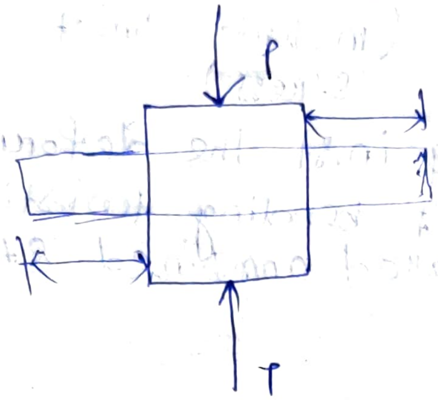
The deformation due to ~~about~~ direct action of Tensile stress is known as Tensile strain.

$$\varepsilon = \frac{\delta l}{l}$$



(ii) Compressive Strain :-

The deformation due to direct action of Compressive stress is known as compressive strain.

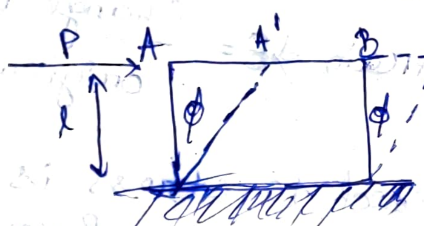


$$\varepsilon_c = \frac{\sigma l}{l}$$

(iii) Shear Strain :-

The deformation due to direct action of shear stress is known as shear strain.

* Strain is an angular deformation.



$AA' = Al$ or σl

$BB' = \sigma l$

in $\triangle BB'C$.

$$\tan\phi = \frac{BB'}{BC} = \frac{\sigma l}{l} \quad [\because \tan\phi = \phi]$$

Volumetric Strain (ε_v) :-

It is define as the ratio between change in volume to its and originated volume when a body is subjected to different mode of stress

→ mathematically → $\varepsilon_v = \frac{\sigma v}{V}$

where, σv = change in volume

V = original volume.

Hooke's Law :-

According to Hooke's law stress is directly proportional to strain under elastic limit, when mathematically

$\sigma \propto \varepsilon \Rightarrow \sigma = E \cdot \varepsilon$

↳ young modulus of elasticity.

$E = \frac{\sigma}{\varepsilon}$.

Young modulus of elasticity :-

It is the ratio between tensile stress and tensile strain or compressive stress and compressive strain

→ It is denoted by 'E'

$$E = \frac{\sigma}{\varepsilon} \left[= \frac{\sigma_T}{\varepsilon_T} \text{ or } \frac{\sigma_c}{\varepsilon_c} \right]$$

Modulus of Rigidity or shear modulus (G or C) denoted

It is the ratio between shear stress and shear strain

→ mathematically $\boxed{G = \frac{\tau}{\phi}}$

where, τ = shear stress

ϕ = shear strain.

→ It is also called as shear modulus of elasticity.

→ It indicates the behaviours of a meterial.

Bulk modulus (K) :-

Concept of Bluk modulus can be used in case of hydrostating loading.

→ Hydrostating loading is that loading in which the solid body is subjected to 3D normal. Stress of same magnitude and of same natu

→ It is the ratio between normal stress to volumetric strain. $\boxed{K = \frac{\sigma}{\varepsilon_v}}$

Poissions Ratio (μ) :-

It is the ratio between lateral strain to longitudinal strain that is.

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1}{m}$$

m → constant

→ $-1 \le \mu \le 0.5$ (universal range)

→ $0 \le \mu \le 0.5$ (maximum Range)

→ longitudinal strain is that strain along with force or stress is applied.

Ex:-1 A steel rod 1m long and 20mm × 20mm in cross-section is subjected to a tensile force of 40 kN. Determine the elongation of the rod, if modulus of elasticity for the rod.

Sol^n Given Length (l) = 1m = 1×10^3 mm

Cross-sectional area (A) = $20 \times 20 = 400\,mm^2$

Tensile force (P) = 40 kN = 40×10^3 N.

young modulus of elasticity (E) = 200 GPa

$= 200 \times 10^3\ N/mm^2$

We know that elongation of the rod

$$\delta l = \frac{P \cdot l}{A \cdot E} = \frac{(40 \times 10^3) \times (1 \times 10^3)}{400 \times (200 \times 10^3)}$$

$= 0.5$ mm

Ex-3// A hollow cylinder 2 m long has an out side diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.

Sol^n Given; Length (l) = 2 m = 2×10^3 mm.

Out side diameter (D) = 50 mm.

inside diameter (d) = 30 mm.

Load (P) = 25 kN = 25×10^3 N.

young modulus of elasticity (E) = 100 GPa

= 100×10^3 N/mm²

Stress in cylinder

we know that cross-sectional area of the hollow cylinder

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4} \times [50^2 - 30^2]$$

$$= 1257 \text{ mm}^2$$

and stress in cylinder,

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{1257} = 19.9 \text{ N/mm}^2$$

$$= 19.9 \text{ MPa}$$

Deformation of cylinder

we also know that deformation of cylinder,

$$\delta l = \frac{P \cdot l}{A \cdot E} = \frac{(25 \times 10^3) \times (2 \times 10^3)}{1257 \times (100 \times 10^3)}$$

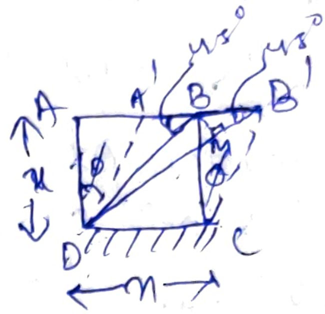
$$= 0.4 \text{ mm}$$

book ans

Calculator 0.39.

Relation between three elastic constant

1) Relation between modulus of Elasticity and rigidity :- (E & G)



$$\text{Strain}(e) = \frac{DB' - DB}{DB}$$

$$(e) = \frac{MB'}{DB} \quad \text{---- (1)}$$

Consider $\Delta BB'M$

$$\cos 45^\circ = \frac{MB'}{BB'}$$

$$\Rightarrow MB' = \frac{1}{\sqrt{2}} BB' \quad \text{---- (2)}$$

ΔABD,

$$\cos 45^\circ = \frac{AB}{DB}$$

$$\Rightarrow DB = \sqrt{2} AB \quad \text{---- (3)}$$

put eqn ~~Relation~~ (2) & (3) in eqn (1)

$$e = \frac{\frac{BB'}{\sqrt{2}}}{\sqrt{2} AB} = \frac{1}{2}\left(\frac{BB'}{AB}\right)$$

negligible

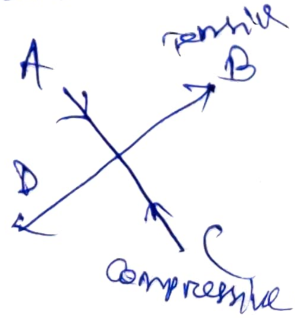
$$= \frac{1}{2}\left(\frac{BB'}{BC}\right) = \frac{1}{2}\tan\phi$$

$$\Rightarrow e = \frac{\phi}{2} \quad \left[\because \frac{BB'}{BC} = \tan\phi \simeq \phi\right] \quad \text{---- (4)}$$

→ Now we will calculate the strain due to applied shear stress (τ)

→ across diagonal BD → Tensile stress
AC → compressive stress

Shear strain along diagonal $BD = \frac{\tau}{E}$



$$AC = -(-\mu)\frac{\tau}{E} = \mu\frac{\tau}{E}$$

Total strain along BD,

$$\Rightarrow e = \frac{\tau}{E} + \mu \frac{\tau}{E}$$

$$\Rightarrow e = \frac{\tau}{E}(1+\mu) \quad \text{--- (5)}$$

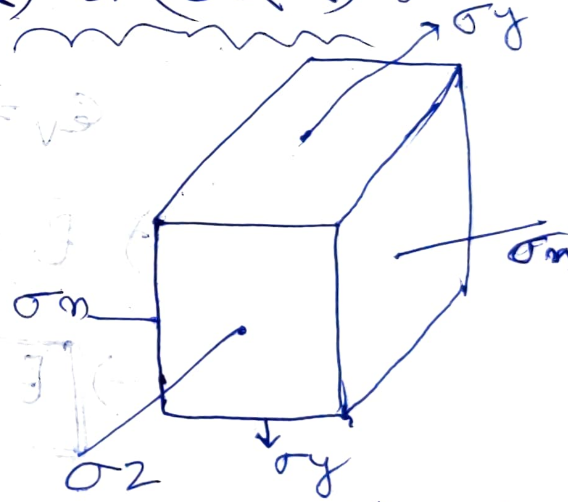
equating eqn (4) & (5)

$$\frac{\phi}{2} = \frac{\tau}{E}(1+\mu)$$

$$\Rightarrow \boxed{E = 2\left(\frac{\tau}{\phi}\right)(1+\mu)}$$

$$\Rightarrow \boxed{E = 2G(1+\mu)}$$

2) Relationship between modulus of elasticity (E) and Bulk modulus (K) or (E & K) :-



let consider a cube which is subjected to normal tensile stress (σ_N) on all the faces

$$\sigma_n = \sigma_x = \sigma_y = \sigma_z$$

The direct strain on x axis $\varepsilon_x = \frac{\sigma_x}{E}$ (tensile)

$$\left.\begin{aligned} \varepsilon_y &= \frac{\mu \sigma_y}{E} \\ \varepsilon_z &= \frac{\mu \sigma_z}{E} \end{aligned}\right\} \text{(Compressive)}$$

Total strain $\Rightarrow \epsilon_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E}$

$$= \frac{\sigma_x}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_x}{E}$$

$$= \frac{\sigma_x}{E}(1 - \mu - \mu)$$

$$\epsilon_x = \frac{\sigma_x}{E}(1 - 2\mu)$$

$$= \frac{\sigma_x}{E}\left(1 - \frac{2}{m}\right)$$ (Tensile due to x dir)

Net change in volume due to all stress $\sigma_x, \sigma_y, \sigma_z$ is $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$$= \frac{\sigma_x}{E}\left[1 - \frac{2}{m}\right] + \frac{\sigma_x}{E}\left(1 - \frac{2}{m}\right) + \frac{\sigma_x}{E}\left(1 - \frac{2}{m}\right)$$

$$\epsilon_v = 3\frac{\sigma_x}{E}\left(1 - \frac{2}{m}\right)$$

$$\Rightarrow E = \frac{3\sigma_x}{\epsilon_v}(1 - 2\mu)$$

$$\Rightarrow \boxed{E = 3K(1 - 2\mu)}$$

Relation between three elastic constant E, K & G

from Relation between E & G

$$E = 2G(1 + \mu) \longrightarrow (1)$$

from Relation between E & K

$$E = 3K(1 - 2\mu) \longrightarrow (2)$$

Now eliminate μ (poissions ratio) so that we can establised the Relation between E, K & G

from eqⁿ ①, $E = 2G(1+\mu)$

$$\Rightarrow \mu = \frac{E}{2G} - 1$$

$$\Rightarrow \mu = \frac{E-2G}{2G} \quad \text{--- ③}$$

put the value of μ in equation ②

$$\Rightarrow E = 3K\left[1 - 2\left(\frac{E-2G}{2G}\right)\right]$$

$$\Rightarrow E = 3K\left(1 - \frac{E-2G}{G}\right)$$

$$\Rightarrow E = 3K\left(\frac{G-E+2G}{G}\right)$$

$$\Rightarrow E = 3K\left(\frac{-E+3G}{G}\right)$$

$$\Rightarrow E = 3K\left(\frac{3G}{G} - \frac{E}{G}\right)$$

$$\Rightarrow E = 3K\left(3 - \frac{E}{G}\right)$$

$$\Rightarrow \frac{E}{3K} = \frac{3G-E}{G} \Rightarrow \frac{EG}{3K} = 3G-E$$

$$\Rightarrow \frac{EG}{3K} + E = 3G$$

$$\Rightarrow \frac{EG+3KE}{3K} = 3G$$

$$\Rightarrow E(3K+G) = 9KG$$

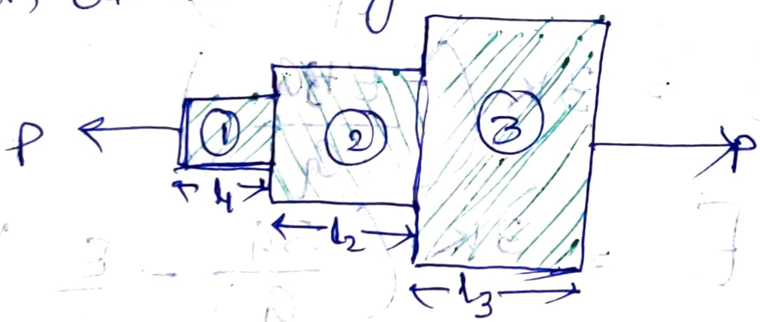
$$\Rightarrow \boxed{E = \frac{9KG}{3K+G}}$$

Principle of supper position :-

According to this principle if member is subjected to various loading then resutant (deformation) will be equal to the algebric sum of the deformation caused by the indivitual forces acted on the member. this principle is valid when their is linear relation between force and stress function.

or.

If two equal, opposite and colinear forces are added to or removeval from the system then their will be no change in the system and position of the body.



Total deflection = sum of total alongation.

$$\boxed{\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3}$$

From Hock's law . $\sigma \propto \epsilon$

$$\Rightarrow \sigma = E\epsilon$$

$$\Rightarrow \frac{P}{A} = E\frac{\delta l}{l}$$

$$\Rightarrow \delta l = \frac{Pl}{AE}$$

★ If materials are same than

$$\Delta l = \Delta l_1 + \Delta l_2 + \Delta l_3$$

$$\Delta_l = \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

★ $$\Delta_l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

★ If materials are different than.

$$\Delta_l = P\left(\frac{l_1}{A_1E_1} + \frac{l_2}{A_2E_2} + \frac{l_3}{A_3E_3}\right)$$

Stress in composite Bars

The bar make up several different meterials, joint to gethon is called to a composite bar.

→ when it is subiected to Tension or Compression each materical entents or constract equaly.

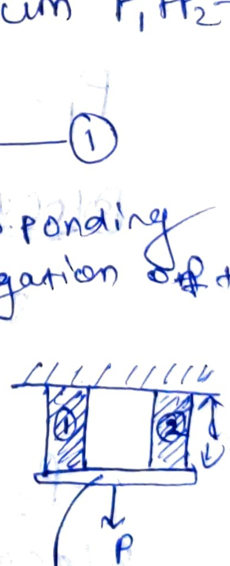
→ also the total enternal load on the bar is equal to the sum of the ~~nodes~~ loads carried by the different materials

ex ! → consider a composite bar made of two different meteriais

Considering equation of equilibrium $P_1 + P_2 = P$

$$\Rightarrow \boxed{\sigma_1 A_1 + \sigma_2 A_2 = P} \text{ — } (1)$$

where σ_1 and σ_2 are corresponding stresses in the bar seens the alongation of the bar are same



cross bar.

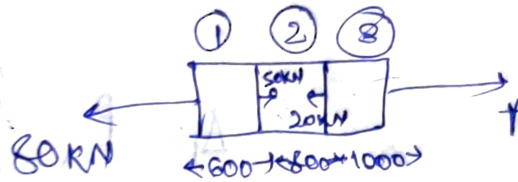
$$\delta l_1 = \delta l_2$$

$$\boxed{\frac{P_1 \cancel{l}}{A_1E_1} = \frac{P_2 \cancel{l}}{A_2E_2}} \text{ — } (2)$$

$$\frac{P_1}{A_1E_1} = \frac{P_2}{A_2E_2} \Rightarrow \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \boxed{e_1 = e_2}$$

Q-1) A steel bar is loaded axialy as shown in fig find the unknown force P and deformation of bar having 100 mm² cross sectional area consider young's modulus of elasticity is

$200 \times 10^3 \, N/mm^2$



$E = 200 \times 10^3 \, N/mm^2$

$A = 100 \, mm^2$

$\Rightarrow 80 + 20 = 50 + P$

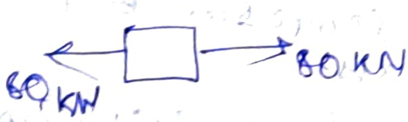
$\Rightarrow P = 50 \, kN.$

$l_1 = 600 \, mm$

$l_2 = 800 \, mm$

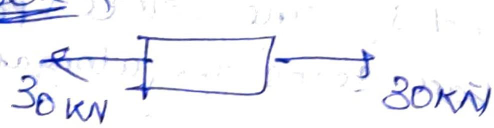
$l_3 = 1000 \, mm$

Block-1



$P_1 = 80 \, kN \, (T)$

Block-2



$+80 - 50 = 30 \, kN$

$P_2 = 30 \, kN \, (T)$

Block-3

$30 + 20 = 50$

$P_3 = 50 \, kN$