GOVERNMENT POLYTECHNIC BHUBANESWAR

LECTURE NOTE

ON

ANALOG & DIGITAL COMMUNICATION – TH-3

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DIGITAL SIGNALPROCESSING

What is a Signal?

- Anything which carries information is a signal. e.g. human voice, chirping of birds, smoke signals, gestures (sign language), fragrances of the flowers.
- Modern high speed signals are: voltage changer in a telephone wire, the electromagnetic field emanating from a transmitting antenna,variation of light intensity in an optical fiber.
- Thus we see that there is an almost endless variety of signals and a large number of ways in which signals are carried from on place to another place.

Signals: The Mathematical Way

- A signal is a real (or complex) valued function of one or more real variable(s).When the function depends on a single variable, the signal is said to be one-dimensionaland when the function depends on two or more variables, the signal is said to be multidimensional.
- Example of a one dimensional signal:A speech signal,dailymaximumtemperature,annualrainfallataplace
  An example of a two dimensional signal:An image is a two

dimensional signal, vertical and horizontal coordinates representing the two dimensions. Four Dimensions:Our physical world is four dimensional(three spatial and one temporal).

Signal processing

- Processing means operating in some fashion on a signal to extract some useful information e.g. we use our ears as input device and then auditory pathways in the brain to extract the information.

  Thesignalisprocessedbyasystem.

- The signal processor may be an electronic system, a mechanical system or even it might be a computer program.

- The signal processing operations involved in many applications like communication systems, control systems, instrumentation, biomedical signal processing etc can be implemented in two different ways

Analog or continuous time method

Digital or discrete time method..

Analog signal processing

- Uses analog circuit elements such as resistors, capacitors, transistors, diodes etc

- Based on natural ability of the analog system to solve differential equations that describe a physical system
- The solutions are obtained in realtime.

Digitalsignalprocessing

- The word digital in digital signal processing means that the processing is done either by a digital hardware or by a digital computer.
- Reliesonnumerical calculations.
- Themethodmay ormaynotgiveresultsinrealtime.

ApplicationsofDigitalSignal Processing

- Speech Processing
- ImageProcessing
- RadarSignalProcessing
- DigitalCommunications
- OpticalFiberCommunications
- TelecommunicationNetworks
- IndustrialNoise Control

Theadvantagesofdigitalapproachoveranalogapproach

- Flexibility:Same hardware can be used to do various kind of signal processing operation, while in the case of analog signal processing one has to design asystem for each kind of operation
- Repeatability:The same signal processing operation can be repeated again and again giving same results, while in analog systems there may be parameter variation due to change in temperature or supply voltage.
- Accuracy
- Easy Storage
- MathematicalProcessing
- Cost
- Adaptability

Classification ofsignals

We use the term signal to mean a real or complex valued functionofrealvariable(s)anddenotethesignalby x(t)Thevariabletiscalledindependentvariableandthe valuexoftas dependent variable. When ttakes a vales in a countable set the signal is called a discrete time signal.

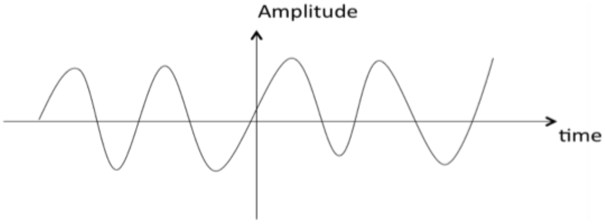
For convenience of presentation we use the notationx[n]to denote discrete time signal.

When both the dependent and independent variables take values in countable sets (two sets can be quite different) the signal is called Digital Signal.

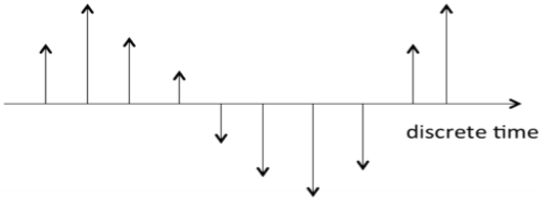
When boththedependentand independentvariable takevalue in continous set interval, the signal is called an Analog Signal.

ContinuousTimeandDiscreteTime Signals

- A signal is said to be continuous when it is defined for allinstants of time.

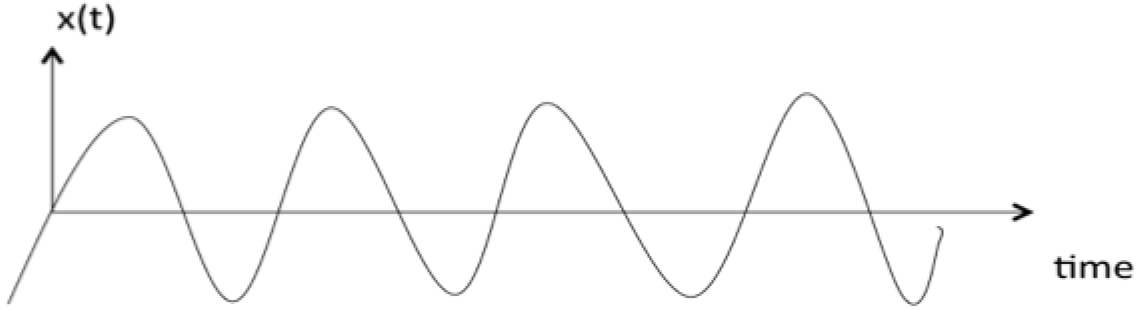


- Asignalissaidtobediscretewhenitisdefinedatonly discrete instants of time.

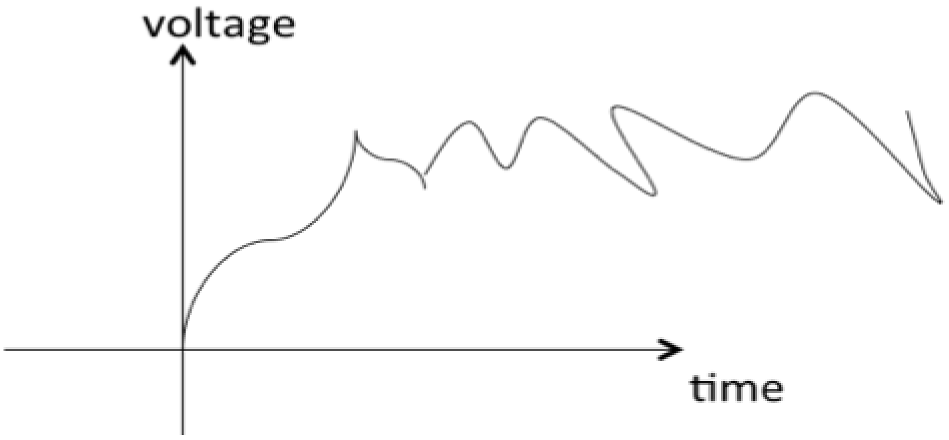


DeterministicandNon-deterministicSignals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.



A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.



EvenandOddSignals

A signal is said to be even when it satisfies the condition x(t) = x(-t)

Example 1:

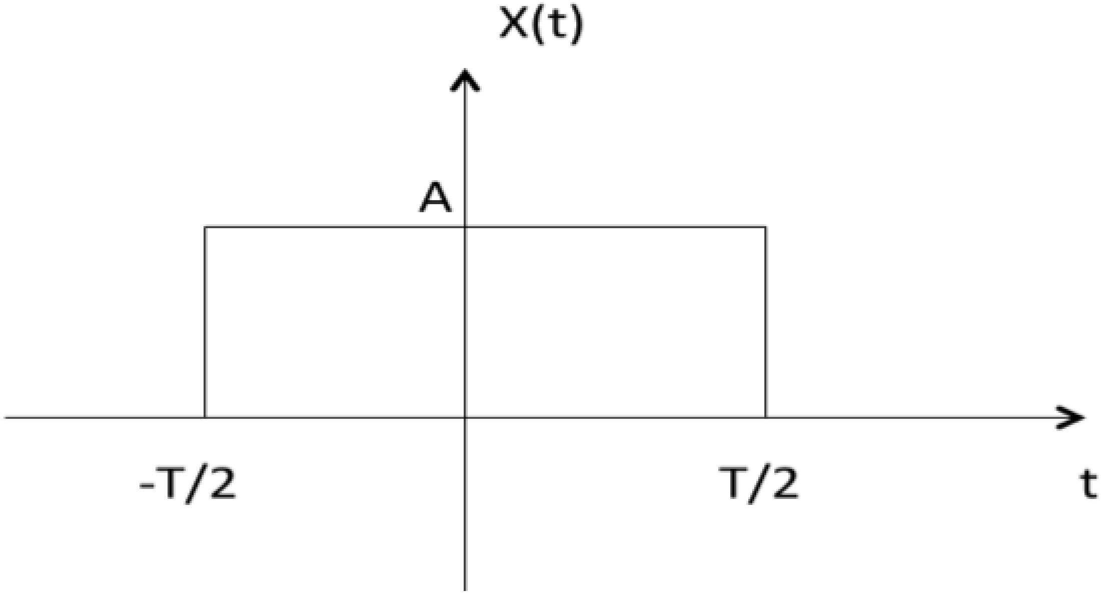
t^2,t^4…cos(t)etc.

Let$x(t)=t^2$

$x(-t)=(-t)^2 =t^2 =x(t)$

Thust^2 isaneven function

Example2:Asshowninthefollowingdiagram,rectangle function x(t) = x(-t) so it is also even function.



A signal is said to be odd when it satisfies the condition x(t) = -x(-t)

Example:t, t^3...And sin(t)

Letx(t)=sint

x(-t) = sin(-t) = -sin t = -x(t)

Thussin(t)isanoddfunction.

Anyfunctionƒ(t)canbeexpressedasthesumofitsevenfunction ƒ$_e$(t) and odd function ƒ$_o$(t).

ƒ(t)=ƒ$_e$(t)+ƒ$_o$(t)

where

ƒ$_e$(t)=½*ƒ(t)+ƒ(-t)]

andƒ$_o$(t)=½*ƒ(t)-ƒ(-t)]

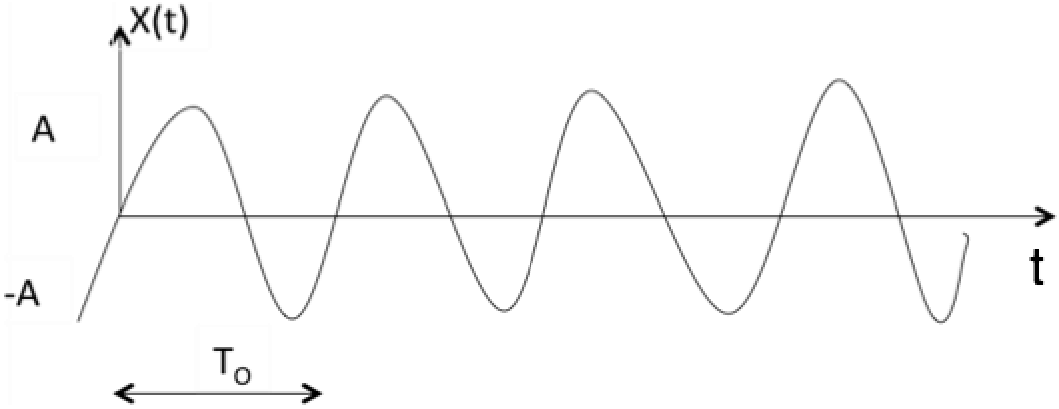
PeriodicandAperiodic Signals

A signal is said to be periodic if it satisfies the condition x(t) =x(t + T) or x(n) = x(n + N).

Where

T = fundamental time

period,1/T=f=fundamentalfrequ



ency.

The above signal will repeat for every time interval T$_0$hence it is periodic with period T$_0$.

EnergyandPower Signals

Asignalissaidtobeenergysignalwhenithasfiniteenergy.

$$\text{Energy,E}= \sum_{n=-\infty}^{+\infty} |x(n)|^2$$

Asignalissaidtobepowersignalwhenithasfinitepower.

$$\text{Power,P=}\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^{2}$$

NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0

Energyofpowersignal=∞

Energy/PowerSignalProblems

Find the Energy and Power of the following signals and find whether the signals are power ,energy or neither energy nor power signals

1. **$x(n)=(1/3)^{n}\ u(n)$**

   Energyofthesignalisgivenby

$$E=\sum_{n=-\infty}^{+\infty}|x(n)|^{2}$$

$$=\sum_{n=-\infty}^{+\infty}[(1/3)^{n}]^{2}$$

$$=\sum_{n=-\infty}^{\infty}(1/9)^{n}$$

$$=\frac{1}{1-(\frac{1}{9})} \qquad 1+a+a^{2}+a^{3}+........+\infty=\frac{1}{1-a}$$

$$=\frac{9}{8}$$

Powerofthesignalisgivenby

$$P=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=0}^{N}\left(\frac{1}{9}\right)^{n}$$

$$=\lim_{n\to\infty}\frac{1}{2N+1}\frac{1-\left(\frac{1}{9}\right)^{N+1}}{1-\left(\frac{1}{9}\right)}$$

=0

So Energy is finite and Power is zero. Therefore the signal is an Energy signal.

2. x(n)=e^{2n}u(n)

$E=\sum_{n=-\infty}^{+\infty}|x(n)|^{2}$

$=\sum_{n=0}^{\infty}e^{4n}$

$=\infty$

$$P=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x(n)|^{2}$$

$$=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}e^{4n}$$

$$=\lim_{N\to\infty}\frac{1}{2N+1}\frac{e^{4(N+1)-1}}{e^{4}-1}$$

$=\infty$

Thissignalisneitherenergynorpower signal.

<u>Periodic/AperiodicSignalProblem</u>

Determinewhetherornoteachofthefollowingsignalsis periodic . If periodic find its fundamental period

1. x(n)=$e^{j6\pi n}$**=**$e^{jw_0 n}$

Sow_0**=6π**

Fundamentalfrequencyismultipleofpi.Sothesignalisperiodic.

Periodofthesignalisgivenby

N=2π $\frac{m}{w_0}$

=2π $\frac{m}{6\pi}$

TheminimumvalueofmforwhichNisanintegeris3

N=2π $\frac{3}{6\pi}$

=1

Thereforethefundamentalperiodis1

2. x(n)=$e^{j3/5(n+1/2)}$

Herew_0**=3/5,whichisnotamultipleofpi.Sosignalisaperiodic.**

3. x(n)=cos(2π/3)n

Herew_0**=2π/3.Soperiodic.**

Thefundamentalperiodis

N=2π($\frac{m}{2\pi/3}$**)**

=3m

Form=1, N=3

Thereforethefundamentalperiodofthesignalis3.

4. x(n)=cos($\frac{\pi}{3}n$)+cos($\frac{3\pi}{4}n$)

Thefundamentalperiodofthesignalcos($\frac{\pi}{3}n$)is

$N_1=2\pi\left(\frac{m}{\pi/3}\right)$

$N_1=6m$

Form=1, $N_1=6$

Thefundamentalperiodofthesignalcos($\frac{3\pi}{4}n$) is

$N_2=2\pi\left(\frac{m}{3\pi/4}\right)$

$N_2=8m/3$

Form=3, $N_2=8$

Now $\frac{N1}{N2}=\frac{6}{8}=\frac{3}{4}$

SoN=$4N_1=3N_2=24$

So N=24

OperationsOn signals

Signalprocessingisagroupofbasicoperationsappliedtoan input signal resulting in another signal as output. The mathematicaltransformationfromonesignaltoanotheris represented as

$$y(n)=T[x(n)]$$

Thebasicsetofoperations are

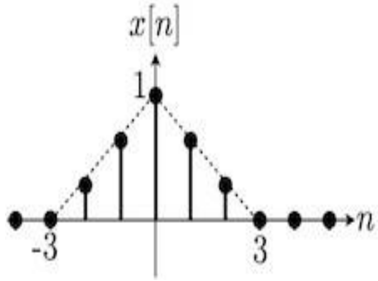
1. **TimeShifting**
2. **TimeReversal**
3. **TimeScaling**
4. **AmplitudeScaling**
5. **Signal Multiplier**
6. **Signal Addition**

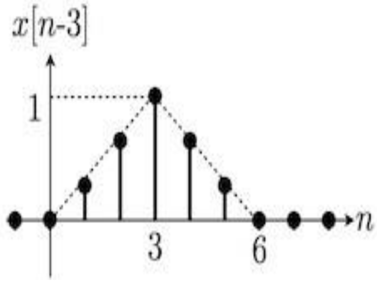
- **TimeShifting**

A signal x(n) may be shifted in time by replacing the independent variable n by n - k, where k is an integer.

y(n)=x(n-k)

- **If kisapositiveinteger,thetimeshiftresultsinadelay of the signal by k units of time.**
- **Ifkisanegativeinteger,thetimeshiftresultsinan advance of the signal by |k| units in time.**
  x(n-3)- Delay (Right Shift)
  x(n+3)–Advance(LeftShift)



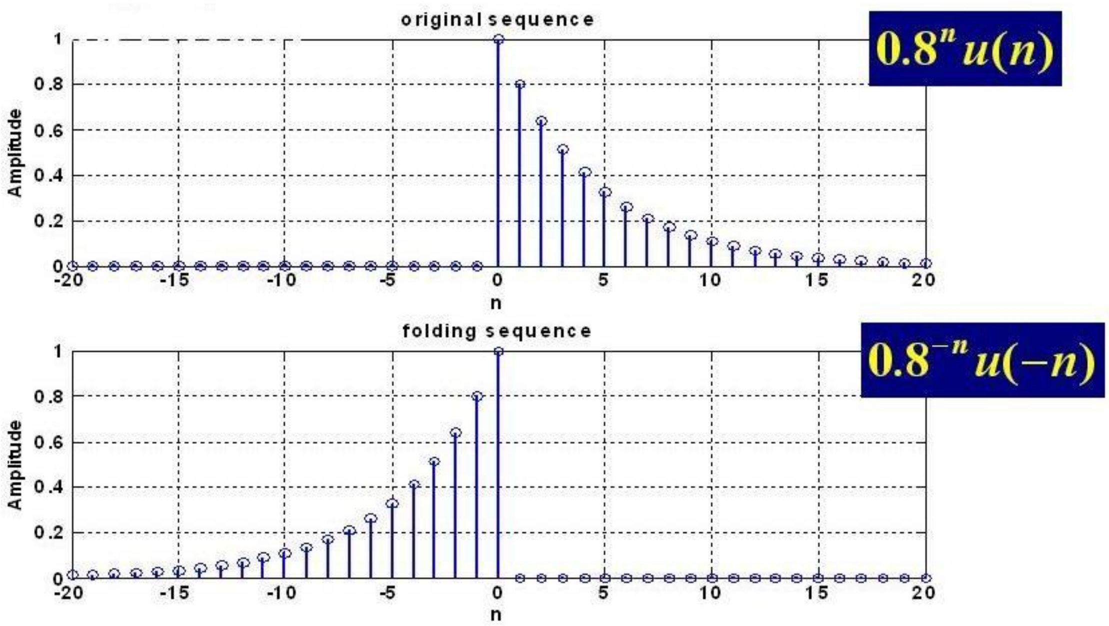


x(n-3)isobtainedbyshiftingx(n)by3unitstowardsright. x(n+3) is obtained by shifting x(n) by 3 units towards left.

- **TimeReversal**

Anotherusefulmodificationofthetimebaseistoreplacethe independent variable n by -n. The result of this operation is a folding or a reflection of the signal about the time originn = 0.

- Itisdenotedasx(-n).



- **TimeScaling**

A third modification of the independent variable involves replacing n by λn, where λ is an integer. We refer to this time-base modification as time scaling or downsampling.

y(n)=x(λn)

Letx(n)={0,0,0.25,0.75,1,0.75,0.25,0,0}

↑

Whenn=-3,x(2n)=x(-6)=0

Whenn=-2,x(2n)=x(-4)=0

Whenn=-1,x(2n)=x(-2)=0.25

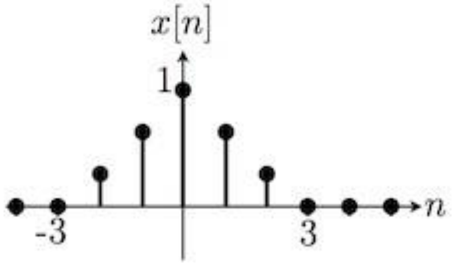
When n=-0, x(2n)=x(0)=1

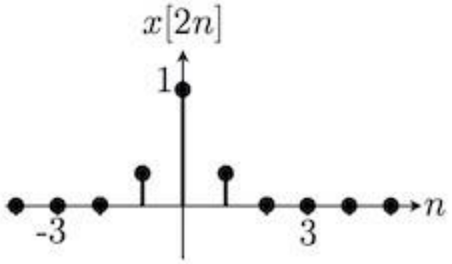
Whenn=1,x(2n)=x(2)=0.25

When n=2, x(2n)=x(4)=0   andsoon....

So x(2n)={0,0,0.25,1,0.25,0,0}

Graphicallywecanrepresentit as





- **AmplitudeScaling**
  - Amplitude modifications include addition, multiplication, and scaling of discrete-time signals.
  - Amplitude scaling of a signal by a constant A is accomplished by multiplying the value of every signal sample by A. Consequently, we obtain y(n) = Ax(n)—∞<n<+∞ e.g.

Letx(n)={ 1,2,3,4,5}

Then2x(n)willbeobtainedsimplybymultiplyingeach sample of x(n) with 2

So 2x(n) ={2,4,6,8,10}

- **Signal Multiplier**

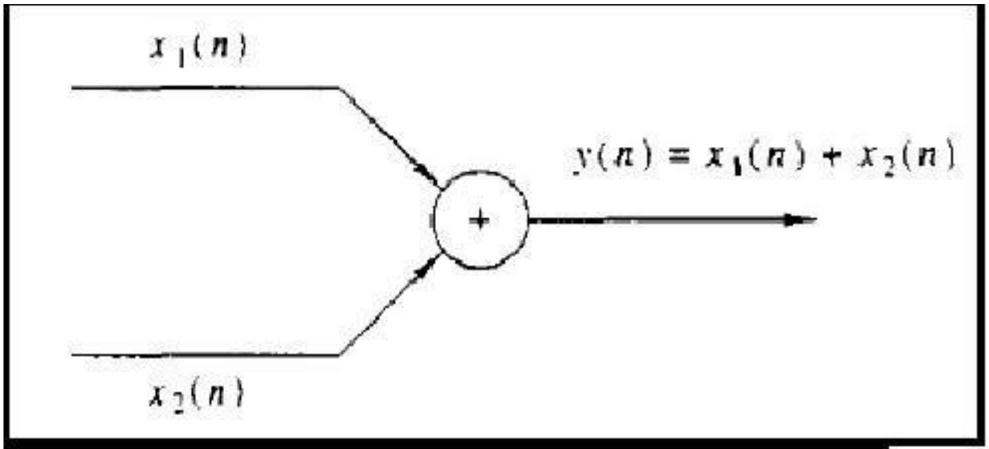
The product of two signals x1(n) and x2(n) is defined on a sample-to-sample basis as

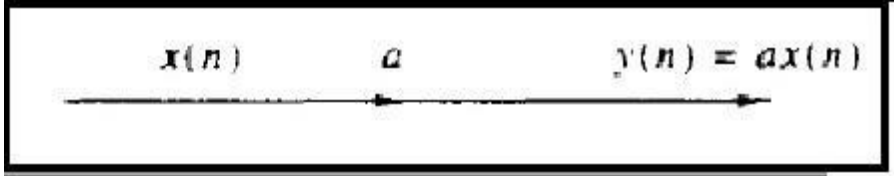
$$y(n)=x1(n)* x2(n) \qquad -\infty<n<+\infty$$

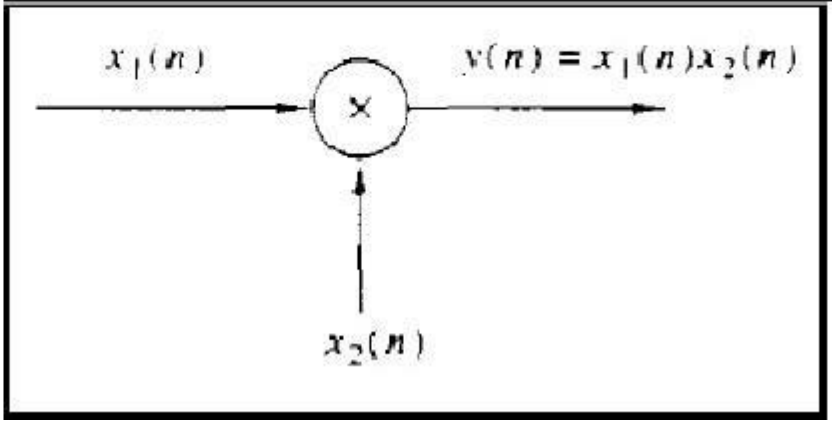
- **Signal Addition**

The sum of two signals x1(n) and x2(n) is a signal y(n), whose value at any instant is equal to the sum of the values of these two signals at that instant, that is

$$y(n)=x1(n)+x2(n). \qquad -\infty<n<+\infty$$







DISCRETE-TIMESYSTEMS

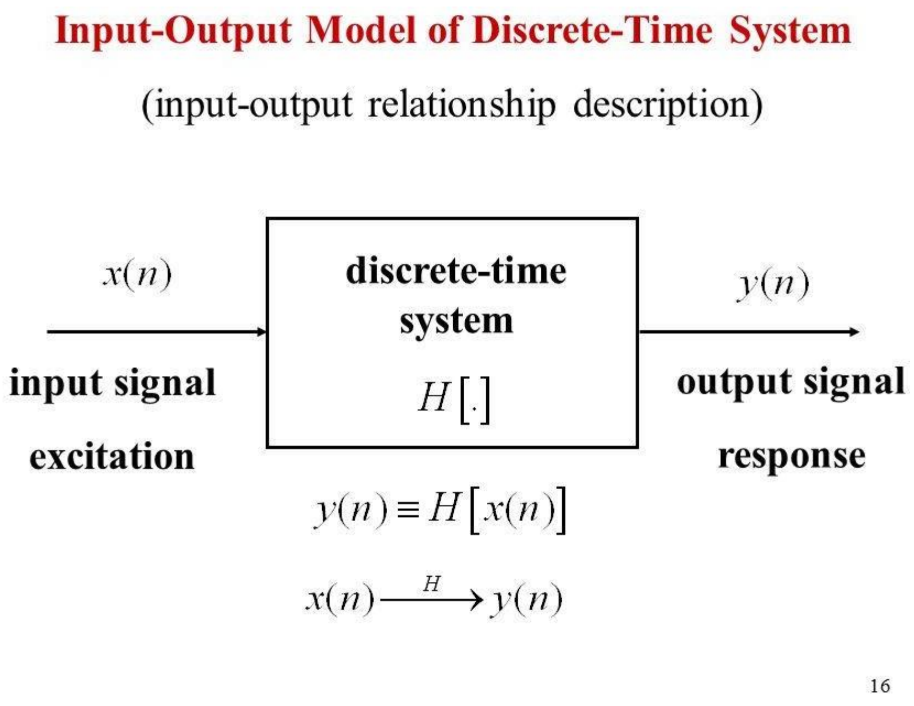
- In manyapplications ofdigitalsignalprocessing we wish to design a device or an algorithm that performs some prescribed operation on a discrete-time signal.
- Suchadeviceoralgorithmiscalledadiscrete-timesystem .
- More specifically, a discrete-time system is a device or algorithm that operates on a discrete-time signal, called the**inputorexcitation**,accordingtosomewell-defined

rule,toproduceanotherdiscrete-timesignalcalledthe **outputorresponse**ofthesystem.

- In general, we view a system as an operation or a set of operations performed on the input signal x(n) to produce the output signal y(n). We say that the input signal x(n) is transformed by the system into a signal y(n), and express the general relationship between x(n) and y(n) as
    - $y(n) \equiv H * x(n) +$

WherethesymbolHdenotesthetransformation(also called an operator), or processing performed by the system on x(n) to produce y(n).

The input output relation of a discrete time system can be shown by the below diagram.



Input-Output Model of Discrete-Time System

(input-output relationship description)

$x(n)$

input signal

excitation

discrete-time system

$H[.]$

$y(n)$

output signal

response

$$y(n) \equiv H\left[x(n)\right]$$

$$x(n) \xrightarrow{H} y(n)$$

16

Question

Determinetheresponseofthefollowingsytemstotheinput signal

1. y(n)=x(n)
   Inthiscasetheoutputisexactlythesameastheinputsignal. Such a system is known as the identity system
2. y(n)=x(n-1)
   Thissystemsimplydelaystheinputbyonesample.
3. y(n)=x(n+1)
   Inthiscasethe system “advances”theinput onesample into the future

BlockDiagramRepresentationofDiscrete-TimeSystems

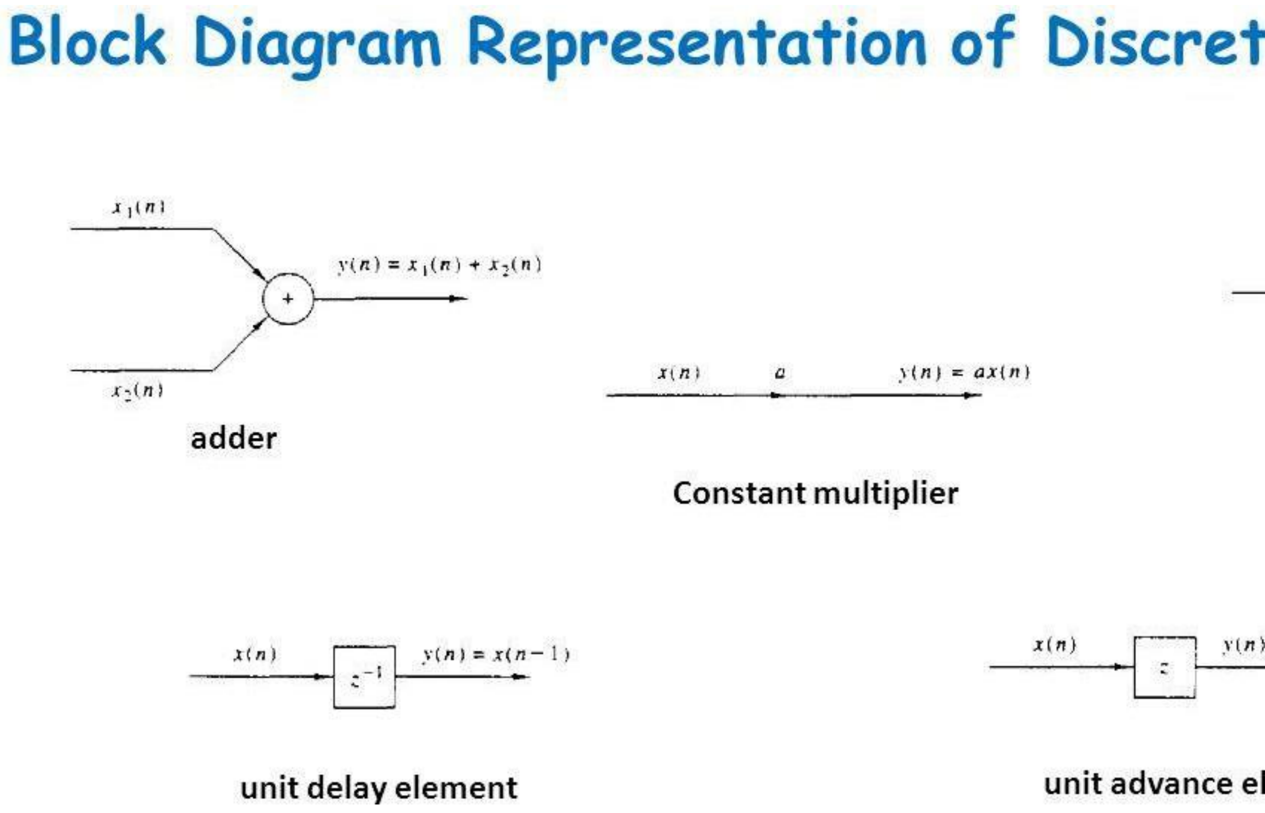
1. **An adder.** a system (adder) that performs the addition of two signal sequences to form another (the sum)sequence, which we denote as y(n).
   It is not necessary to store either one of the sequences in ordertoperformtheaddition.Inotherwords,theaddition operation is memoryless.

2. **A constant multiplier**. This operation represents applyinga scale fact r on the input x(n). This operation is also memoryless.

3. **A signal multiplier**. The multiplication of two signal sequences to form another (the product) sequence y(n). The multiplication operation is also memoryless

4. **A unit delay element**. The unit delay is a special system that simply delays the signal passing through it by one sample. If the input signal is x(n), theoutput is x(n - 1).
   In fact, the sample x(n - 1) is stored in memory at time n -1 and it is recalled from memory at time n to form y(n) = x(n - 1)

5. **A unit advance element**. In contrast to the unit delay, a unit advance moves the input x(n) ahead by one sample in time to yield x(n + 1).



Block Diagram Representation of Discret

adder

Constant multiplier

unit delay element

unit advance el

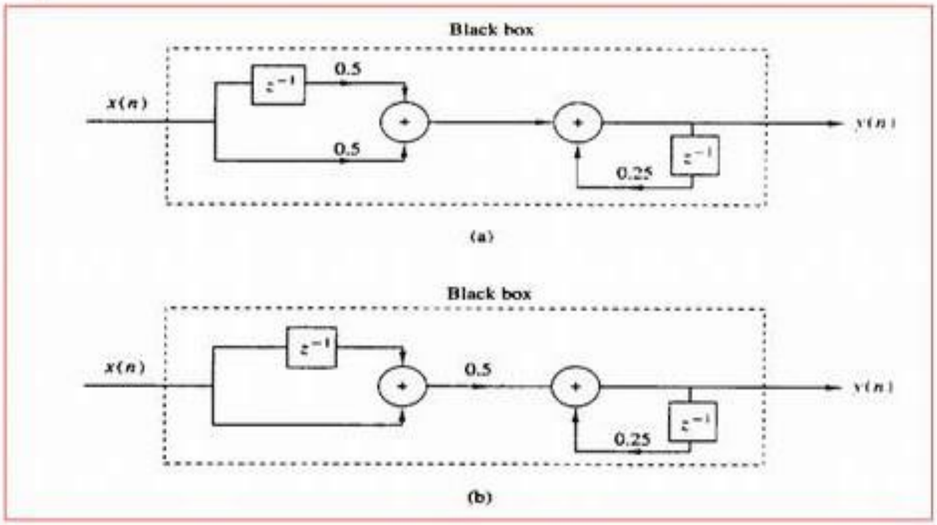
Question

Using basic building blocks introduced above, sketch the block diagram representation of the discrete-time system described by the input-output relation.

y(n)=1/4y(n-1)+1/2x(n)+1/2x(n-1)

Wherex(n)istheinputandy(n)istheoutputofthesystem

$$y(n)=\frac{1}{4}y(n-1)+\frac{1}{2}x(n)+\frac{1}{2}x(n-1)$$



ClassificationofDiscrete-Time Systems

1. Staticversusdynamicsystems
2. Time-invariantversustime-variantsystems
3. Linearversusnonlinear systems.
4. Causalversusnoncausalsystems
5. Stableversusunstablesystems

<u>StaticSystemVsDynamicSystem</u>

- A discrete-time system is called **static or memoryless** if its output at any instant n depends at most on the input sampleatthesametime,butnotonpastorfuture samples of the input.

- Inanyothercase,the systemissaidtobe **dynamic or to have memory**.

The systems described by the following input-output equations are both static or memoryless

$y(n)=ax(n)$
$y(n)=nx(n)+ bx^3(n)$

Ontheotherhand,thesystemsdescribedbythe following input-output relations are dynamic/having memory

$y(n)=x(n)+3x(n- 1)$

$y(n)=x(n)+x(n+2)$

$y(n)= \sum_{k=0}^{n} x(n- k)$ Finite memory

$y(n)= \sum_{k=0}^{\infty} x(n- k)$ Infinite memory

So it can be said that static or memoryless systems are described in general by input-output equations of the form y(n) = T[x(n), n] and they do not include delay elements (memory).

<u>Time-invariantversustime-variantsystems.</u>

- We can subdivide the general class of systems into thetwo broad categories, time-invariant systems and time- variant systems.
- A system is called **time-invariant** if its input-output characteristics do not change with time.
- To elaborate, suppose that we have a system T in arelaxed state which, when excited by an input signal x(n), produces an output signal y(n). Thus we write y(n) =T[x(n)]

Now suppose that the same input signal is delayed by k units of time to yield x(n - k), and again applied to the same system. If the characteristics of the system do not change with time, the output of the relaxed system will be y(n-k). That is, the output will be the same as the response to x(n), exceptthat itwillbe delayed bythe same kunits in time that the input was delayed. This leads us to define a **time-invariant or shift-invariant** system as follows.

A relaxed system T is time invariant or shift invariant if and only if

$$x(n)\xrightarrow{T}y(n)$$

$$⇨ x(n-k) \xrightarrow{T} y(n-k)$$

foreveryinputsignalx(n)andeverytime shiftk.

<u>Linearversusnonlinearsystems</u>.

- The general class of systems can also be subdivided into linear systems and nonlinear systems.
- A **linear system** is one that satisfies the superposition principle.
- Simply stated, the principle of superposition requires that the response of the system to a weighted sum of signalsbe equal to the corresponding weighted sum of the responses (outputs)ofthe system toeach of theindividual input signals. Hence we have the following definition of linearity.
- A relaxed system is linear if and only if $\mathbf{T[a_1\ x_1\ (n) + a_2\ x_2\ (n)] = a_1\ T[x_1(n)] + a_2\ T[x_2(n)]}$for any arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2
- The superposition principle embodied in the relationabove can be separated into two parts.
- First, suppose that $a_2 = 0$. Then the above relation reduces to $T[a_1\ x_1\ (n)\ ]= a_1\ T[x_1(n)] = a_1\ y_1(n)$ where $y_1(n) = T[x_1(n)]$
- The relation above demonstrates the multiplicative or scaling property of a linear system. That is, if the response of the system to the input $x_1\ (n)$ is $y_1(n)$, the response to $a_1\ x_1\ (n)$ is simply $a_1\ y_1(n)$.

- Thus any scaling of the input results in an identical scaling of the corresponding output.
- Second, suppose that a_1= a_2 = 1 . Then T[a_1 x_1 (n) + a_2 x_2(n)] = T[x_1(n)] + T[x_2(n)]=y_1(n)+y_2(n)
- This relation demonstrates the **additivity** property of a linear system. The **additivity** and **multiplicative** properties constitute the superposition principle as itapplies tolinear systems.

Causalversusnoncausalsystems

- A system is said to be **causal** if the output of the system at any time n [i.e., y(n)] depends only on present and past inputs [i.e., x(n), x(n - 1), x(n- 2),...], but does not depend on future inputs [i.e., x(n + 1), x(n + 2),...]. In mathematical terms, the output of a causal system satisfies an equation of the form

$$\mathbf{y(n)=F[x(n),x(n-1),\ x(n-2),...]}$$

- If a system does not satisfy this definition, it is called **noncausal**. Such a system has an output that depends not only on present and past inputs but also on future inputs.

Stableversusunstablesystems

- Stability is an important property that must be considered in any practical application of a system.
- Unstable systems usually exhibit erratic and extreme behavior and cause overflow in any practical implementation.
- An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded output.
- The conditions that the input sequence x(n) and theoutput sequence y(n) are bounded is translated mathematically to mean that there exist some finite numbers,
  say Mx and My such that $|x(n)|<M_x<\infty$and$|y(n)|<M_y<\infty$for all n.
- If for some bounded input sequence ,x(n), the output is unbounded (infinite), the system is classified as **unstable.**

PROBLEMS

Testthefollowingsystemsforlinearity

a) $y(n)=nx(n)$
b) $y(n)=x(n^2)$
c) $y(n)=x^2(n)$
d) $y(n)=e^{x(n)}$

Procedure

1. Let$x_1(n)$and$x_2(n)$betwoinputstosystemHand $y_1(n)$ and $y_2(n)$ be corresponding responses
2. Considerasignal$x_3(n)=a_1x_1(n)+a_2x_2(n)$whichisaweighted sum of $x_1(n)$ and $x_2(n)$.
3. Let$y_3(n)$betheresponsefor$x_3(n)$.
4. Check whether $y_3(n)= a_1\ y_1(n)+a_2\ y_2(n)$.If they are equal then the system is linear, otherwise it is nonlinear.

Solution

a. Consider2signals$x_1(n)$and $x_2(n)$

Let$y_1(n)$and$y_2(n)$betheresponseofthesystemHforinputs$x_1(n)$ and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=nx_1(n)$

$y_2(n)=H\{x_2(n)\}=nx_2(n)$

So$a_1y_1(n)+a_2y_2(n)=a_1nx_1(n)+a_2n\ x_2(n)$

Nowconsideralinearcombinationofinputs $x_3(n)=a_1\ x_1(n)+a_2\ x_2(n)$.

Let$y_3(n)$betheresponseforthislinearcombinationofinputs

$y_3(n)=H\{a_1\ x_1(n)+a_2\ x_2(n)\}=n[a_1\ x_1(n)+a_2\ x_2(n)]= a_1n\ x_1(n)+a_2\ n\ x_2(n)$

Since$y_3(n)=a_1y_1(n)+a_2y_2(n)$,thesystemis**linear**.

b. Consider2signals$x_1(n)$and $x_2(n)$

Let$y_1(n)$and$y_2(n)$betheresponseofthesystemHforinputs$x_1(n)$ and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=x_1(n^2)$

$y_2(n)=H\{x_2(n)\}=x_2(n^2)$

So$a_1y_1(n)+a_2y_2(n)=a_1x_1(n^2)+a_2x_2(n^2)$

Nowconsideralinearcombinationofinputs$x_3(n)=a_1\ x_1(n)+a_2\ x_2(n)$.

Let$y_3(n)$betheresponseforthislinearcombinationofinputs.

$y_3(n)=H\{x_3(n)\}=H\{a_1x_1(n)+a_2x_2(n)\}=a_1x_1(n^2)+a_2x_2(n^2)$

Since$y_3(n)=a_1y_1(n)+a_2y_2(n)$,thesystemis**linear**

c. Consider2signals$x_1(n)$and $x_2(n)$

Let$y_1(n)$and$y_2(n)$betheresponseofthesystemHforinputs$x_1(n)$ and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=x_1^2(n)$

$y_2(n)=H\{x_2(n)\}=x_2^2(n)$

So$a_1y_1(n)+a_2y_2(n)=a_1x_1^2(n)+a_2x_2^2(n)$

Nowconsideralinearcombinationofinputs $x_3(n)=a_1\ x_1(n)+a_2\ x_2(n)$.

Let$y_3(n)$betheresponseforthislinearcombinationofinputs.

$y_3(n)=H\{x_3(n)\}=H\{a_1x_1(n)+a_2x_2(n)\}=x_3^2(n)=[a_1x_1(n)+a_2$ $x_2(n)]^2=a_1^2\ x_1^2(n)+\ a_2^2\ x_2^2(n)+2\ a_1\ x_1(n)\ a_2\ x_2(n)$

Sincey$_3$(n)$\neq$ $a_1y_1(n)+a_2y_2(n)$,thesystemis**non-linear**.

d. Consider2signalsx$_1$(n)and $x_2(n)$

Lety$_1$(n)andy$_2$(n)betheresponseofthesystemHforinputsx$_1$(n) and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=e^{x1(n)}$

$y_2(n)=H\{x_2(n)\}=e^{x2(n)}$

So$a_1y_1(n)+a_2\ y_2(n)=a_1e^{x1(n)}+a_2e^{x2(n)}$

Nowconsideralinearcombinationofinputs $a_1x_1(n)+a_2$ $x_2(n)$.

Lety$_3$(n)betheresponseforthislinearcombinationofinputs

$y_3(n)=\ H\{\ x_3(n)\}=H\{\ a_1\ x_1(n)+a_2\ x_2(n)\}=\ e^{\ x3(n)}=e^{\ [a1x1(n)+a2x2(n)]}=$ $e^{\ a1x1(n)}\ e^{\ a2x2(n)}$

Sincey$_3$(n)$\neq$ $a_1y_1(n)+a_2y_2(n)$,thesystemis**non-linear**

Testthefollowingsystemforlinearity

a) $y(n)=2x(n)+\ \frac{1}{x(n-1)}$

b) $y(n)=x(n)-bx(n-1)$

Solution

a. Consider2signalsx$_1$(n)and $x_2(n)$

Let$y_1(n)$and$y_2(n)$betheresponseofthesystemHforinputs$x_1(n)$ and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=2x_1(n)+\frac{1}{x1(n-1)}$

$y_2(n)=H\{x_2(n)\}=2x_2(n)+\frac{1}{x2(n-1)}$

So$a_1y_1(n)+a_2y_2(n)=a_1[2x_1(n)+\frac{1}{x1(n-1)}]+a_2[2x_2(n)+\frac{1}{x2(n-1)}]$

Nowconsideralinearcombinationofinputs $a_1x_1(n)+a_2$ $x_2(n)$.

Let$y_3(n)$betheresponseforthislinearcombinationofinputs

$y_3(n)=H\{a_1 x_1(n)+a_2 x_2(n)\}= 2[a_1 x_1(n)+a_2 x_2(n)]+\frac{1}{a1x1(n-1)+a2x2(n-1)}$

Since$y_3(n)\neq a_1y_1(n)+a_2y_2(n)$,thesystemis**non-linear**

b. Consider2signals$x_1(n)$and $x_2(n)$

Let $y_1(n)$ and $y_2(n)$ be the response of the system H for inputs$x_1(n)$ and $x_2(n)$ respectively

$y_1(n)=H\{x_1(n)\}=x_1(n)-bx_1(n-1)$

$y_2(n)=H\{x_2(n)\}=x_2(n)-bx_2(n-1)$

So$a_1y_1(n)+a_2y_2(n)=a_1x_1(n)-a_1bx_1(n-1)+a_2x_2(n)-a_2bx_2(n-1)$

Now consider a linear combination of inputsa_1 $x_1(n)+a_2x_2(n)$.

Let $y_3(n)$ be the response for this linear combination of inputs

$y_3(n)$=H{ a_1 $x_1(n)$+a_2 $x_2(n)$}= a_1 $x_1(n)$+a_2 $x_2(n)$-b [a_1 $x_1(n-1)$+$a_2x_2(n-1)$]

=$a_1x_1(n)$-$a_1bx_1(n-1)$+$a_2x_2(n)$-$a_2bx_2(n-1)$

Since$y_3(n)$= $a_1y_1(n)$+$a_2y_2(n)$,thesystemis**linear**

ResponseofLTIDiscreteTimeSysteminTimeDomain

ThegeneralequationgoverninganLTIdiscretetimesystemis

$y(n)=-\sum_{m=1}^{N} a_m y(n-m)+\sum_{m=0}^{M} b_m x(n-m)$

$\sum_{m=0}^{N} a_m y(n-m)=\sum_{m=0}^{M} b_m x(n-m)$with$a_0$=1

The solution of the difference equation is the response y(n) of LTI system, which consists of two parts. In mathematics, the two parts of the solution y(n) are homogeneous solution $y_h(n)$ and particular solution $y_p(n)$

Response,$y(n)=y_h(n)+y_p(n)$

The homogeneous solution is the response of the system when there is no input.

The particular solution is the solution of difference equation for specific input signal x(n) for n≥0

Insignalsandsystems,thetwopartsofthesolutiony(n)are called zero-input response $y_n(n)$ and zero-state response $y_{zs}(n)$

zero-inputresponse

- The zero input response is mainly due to initial conditions in the system. Hence zero-input response is also calledfree response or natural response.
- The zero input response is given by homogeneous solution with constants evaluated using initial conditions.

zero-state response

- The zero-state response is the response of the system due to input signal and with zero initial condition. Hence the zero state response is called forced response. The zero state response or forced response is given by the sum of homogeneous solution and particular solution with zero

Question

Determinetheresponseoffirstorderdiscretetimesystem governed by the difference equation

$$\mathbf{y(n)=-0.5y(n-1)+x(n)}$$

Whentheinputisunitstep, andwithinitial condition

a) $y(-1)=0$
b) $y(-1)=1/3$

Solution

$y(n)+0.5y(n-1)=x(n)$ ....(1)

Homogeneous Solution

The homogeneous equation is the solution of equation 1 when $x(n)=0$

$y(n)+0.5y(n-1)=0$

Puttingy(n)=λ^nintheaboveequation

$\lambda^n+0.5\ \lambda^{n-1}=0$

$\lambda^{n-1}(\lambda+0.5)=0$

$\lambda+0.5=0$

$\Rightarrow \lambda=-0.5$

Thehomogeneoussolution$y_h(n)$isgivenby

$y_h(n)=C\lambda^n=C(-0.5)^n$forn≥0 .... (2)

Particular Solution

Given that the input is unit step and so the particular solutionwill be in the form,

$y(n)=Ku(n)$

Puttingthisinequation1weget

$Ku(n)+0.5K\ u(n-1)=u(n)$ ....(3)

InordertodeterminethevalueofK,wehavetoevaluatefor n=1 in equation 3

$Ku(1)+0.5K\ u(0)=u(1)$

$K+0.5K=1$ [Asu(1)=1,u(0)=1]

$1.5K=1$

$K=1/1.5=2/3$

Theparticularsolution$y_p(n)$isgivenby

$y_p(n)=K\ u(n)=2/3\ u(n)$ for all n

Total Response

Thetotalresponsey(n)ofthesystemisgivenbysumofhomogeneous and particular solution

Responsey(n)=y_h(n)+y_p(n)

=C(-0.5)^n+2/3u(n)

=C(-0.5)^n+2/3 for n≥0 .... (4)

At n=0, equation 1 becomes

y(n)+0.5y(n-1)=x(n)

y(0)+0.5y(-1)=1

5)

y(0)=1-0.5y(-1)

Atn=0,equation4 becomes

y(0)=C+2/3 .... (6)

Fromequation(5)and(6)weget C+

2/3 = 1-0.5y(-1)

C=1-0.5y(-1)-2/3

C=1/3-0.5y(-1)

PuttingthevalueofCinequation4weget

y(n)=(1/3-0.5 y(-1)) (-0.5)^n +2/3

a) Wheny(-1)=0

y(n)=1/3(-0.5)^n+2/3 for n≥0

b) Wheny(-1)=1/3

y(n)=[1/3-0.5x1/3](-0.5)^n+2/3

y(n)=0.5/3(-0.5)^n+2/3 for n≥0

Determine the response $y(n), n\geq 0$ of the system described bythe second order difference equation

$y(n)-2y(n-1)-3y(n-2)=x(n)+4x(n-1)$ .... (1)

when the input signal is $x(n)=2^n$ u(n) and with initial conditions $y(-2)=0, y(-1)=5$

HomogeneousSolution:

Itisthe solutionwhenx(n)=0

$y(n)-2y(n-1)-3y(n-2)=0$... (2)

Puttingy(n)=λ^ninequation2,weget

$\lambda^n-2\lambda^{n-1}-3\lambda^{n-2}=0$

$\lambda^{n-2}(\lambda^2-2\lambda-3)=0$

Thecharacteristicsequationis

$\lambda^2-2\lambda-3=0$

⇨ $(\lambda-3)(\lambda+1)=0$

Therootsare$\lambda=3,-1$

The homogeneous solution, $y_h(n)=C_1\lambda_1^n+C_2\lambda_2^n=C_1(3)^n+C_2(-1)^n$

ParticularSolution:

Lety(n)=$K2^n u(n)$

Puttingy(n)=$K2^n u(n)$inequation1,weget

$K2^nu(n)-2K2^{n-1}u(n-1)-3K2^{n-2}u(n-2)=$ $2^nu(n)+42^{n-1}u(n-1)$ .... (3)

InordertofindthevalueofK,weputn=2inequation3

$K2^2u(2)-2K2^{2-1}u(2-1)-3K2^{2-2}u(2-2)=2^2u(2)+42^{2-1}u(2-1)$

⇨ $4K-4K-3K=4+4x2$

⇨ $-3K=12$

⇨ $K=-12/3=-4$

Sotheparticularsolution$y_p(n)=K2^nu(n)=(-4)2^nu(n)$

Total Solution:

$y(n)=y_h(n)+y_p(n)$

$y(n)=C_1(3)^n+C_2(-1)$ .... ^{n} $+(-4)$ 2^n $u(n)$ for $n\geq0$. .... (4)

When n=0, equation 1 becomes

$y(0)-2y(0-1)-3y(0-2)=x(0)+4x(0-1)$

$y(0)-2y(-1)-3y(-2)=x(0)+4x(-1)$. ....(5)

Giventhaty(-1)=5,y(-2)=0

$x(n)=2^n\ u(n)$

$n=0,x(0)=2^0\ u(0)=1$

$n=-1,x(-1)=2^{-1}u(-1)=0$

Puttingtheaboveconditionsinequation5,weget

$y(0)-2y(-1)-3y(-2)=x(0)+4\ x(-1)$

$y(0)-2x5-3x0=1+4x\ 0$

y(0)-10=1

y(0)=11

When n=1, equation 1 becomes

y(1)-2y(1-1)-3y(1-2)=x(1)+4x(1-1)

y(1)-2y(0)-3y(-1)=x(1)+4x(0)..(6)

Weknowthaty(0)=11,y(-1)=5

$x(n)=2^n u(n)$

$n=0,x(0)=2^0u(0)=1$

$n=1,x(1)=2^1u(1)=2$

Puttingtheaboveconditionsinequation6,weget

y(1)-2y(0)-3y(-1)=x(1)+4 x(0)

y(1)-2x11-3x5=2+4x1

y(1) 22-15=2+4

y(1)=43

Whenn=0 ,

Equation4becomes

$y(0)= C_1 (3)^0 + C_2 (-1)^0+(-4) 2^0 u(0) =C_1+C_2-4$.................................(7)

$C_1+C_2-4=11$

$C_1+C_2=15$ ....(8)

Whenn=1 ,

Equation4becomes

$y(1)=C_1\ (3)^1+C_2(-1)^1+(-4)2^1u(1)\qquad =3C_1-C_2-8$

$3C_1-C_2-8=43$

$3C_1-C_2=51$ ....(9)

Addingequation8and9weget

$4C_1=66$

$C_1=66/4=33/2$

$C_2==3/2$

Thetotalsolutionis

$y(n)=33/2(3)^n-3/2(-1)^n-4\ 2^nu(n)\qquad$ for $n\geq0$

Response of LTI Systems to Arbitrary Inputs: The Convolution Sum

The formula inthat gives theresponsey(n)of the LTIsystem as a function of the input signal x(n) and the unit sample (impulse) response h(n) is called a convolution sum.

$$y(n)=\sum_{=-\infty}^{\infty}\ ()(-)$$

Wesaythattheinputx(n)isconvolvedwiththeimpulseresponse h(n) to yield the output y(n).

If x(n) has N_1 samples and h(n) has N_2 samples, then the output sequence y(n) will have“N_1+N_2-1 “ samples

Theconvolutionrelationcanbesymbolicallyexpressedas

y(n)=x(n)*h(n)= h(n)*x(n)

Procedureforevaluatinglinearconvolution

Theprocessofcomputingtheconvolutionbetweenx(k)and h(k) involves the following four steps.

1. **Folding** Foldh(k)aboutk=0toobtain h(-k).
2. **Shifting** Shift h(-k) by n_0 to the right (left) if n_0 is positive(negative), to obtain h(n_0 - k).
3. **Multiplication** Multiply x(k)byh(n_0-k)toobtainthe product sequence vn_0 (k) = x(k)h(n_0 - k).
4. **Summation** Sumallthevaluesoftheproductsequence vn_0 (k)to obtain the value of the output at time n = n_0.

The above procedure results in the response of the system at a single time instant, say n = n_0.

In general, we are interested in evaluating the response of the system over all time instants -∞ < n < ∞. Consequently, steps 2 through 4 in the summary must be repeated, for all possible time shifts -∞ < n < ∞.

PropertiesofLinear Convolution

Thediscreteconvolutionwillsatisfythefollowing properties

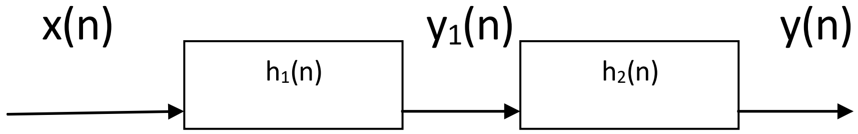
1. CommutativeProperty:**$x_1(n)*x_2(n)=x_2(n)* x_1(n)$**
2. Associative Property : **$[x_1(n) * x_2(n)] * x_3(n)= x_1(n)* [x_2(n) * x_3(n)]$**
3. Distributive Property : **$x_1(n) *[x_2(n)+ x_3(n)]= [x_1(n)* [x_2(n)]+ [x_1(n)* x_3(n)]$**

InterconnectionofDiscreteTimeSystems

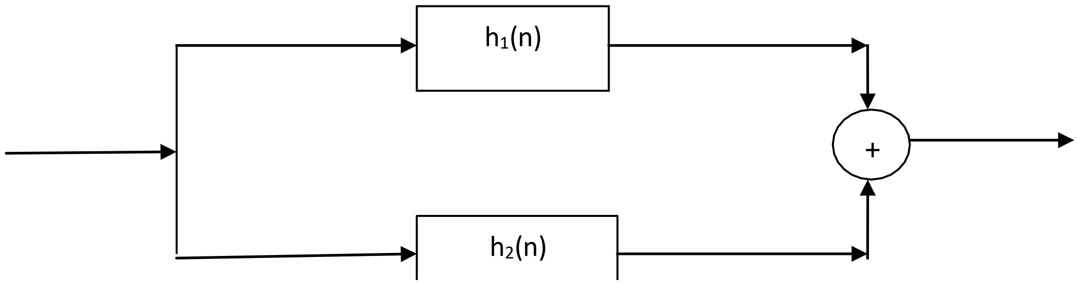
Smaller discrete time systems may be interconnected to form larger systems. Two possible basic ways of interconnection are Cascade connection and Parallel Connection.

The cascade and parallel connections of two discrete time systems with impulse responses $h_1(n)$ and $h_2(n)$ are givenbelow.

Cascade Connection



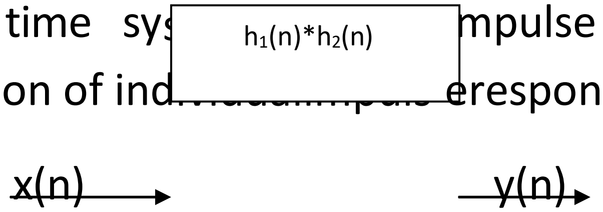
ParallelConnection



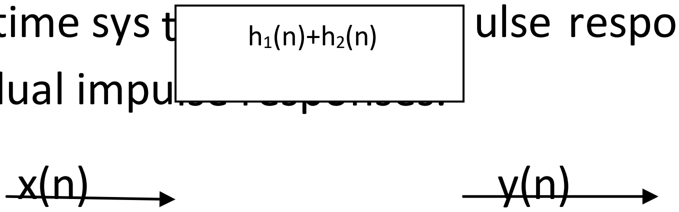
x(n)

y(n)

Twocascadeconnecteddiscretetimesystemwithimpulse responseh$_1$(n)andh$_2$(n)canbereplacedbyasingleequivalent discrete time sys mpulse response is given by convolution of ind eresponses.

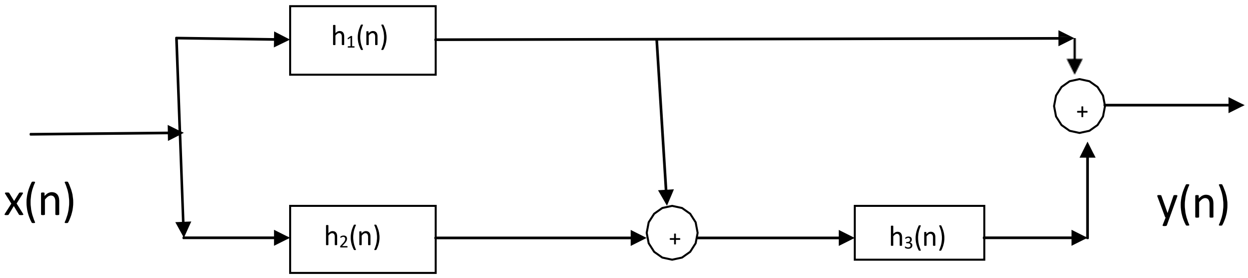


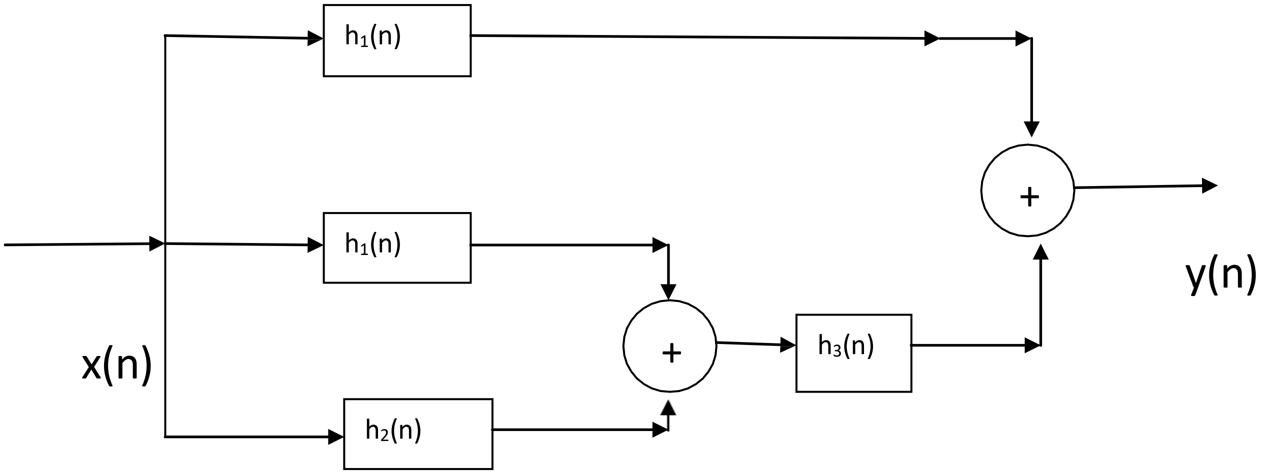
Two parallel connected discrete time system with impulse response h$_1$(n) and h$_2$(n) can be replaced by a single equivalent discrete time sys t ulse response is given by sum of individual impu

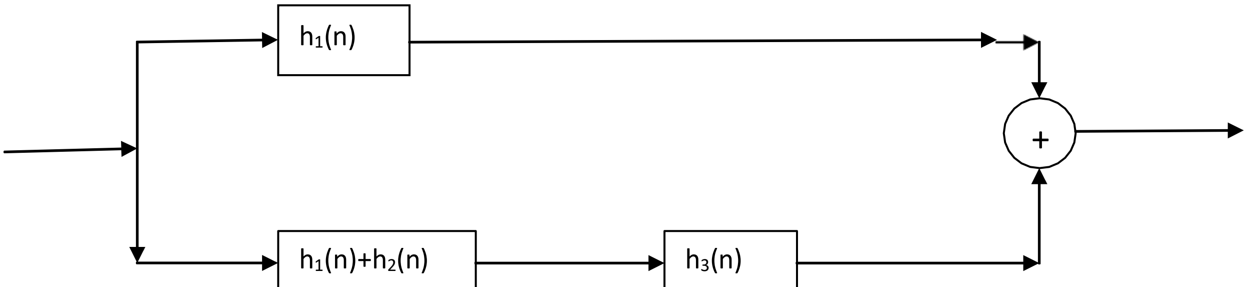


Question

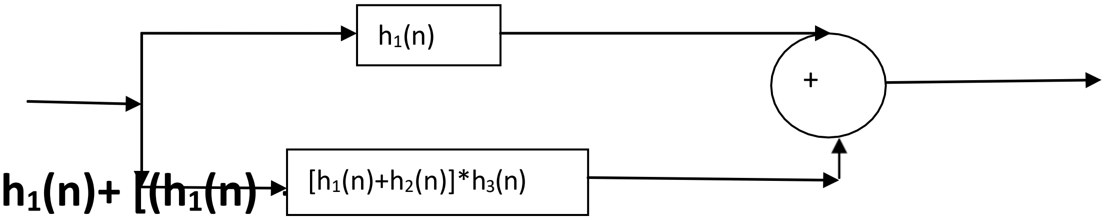
Simplifytheoverallimpulseresponseoftheinterconnected discrete time system shown below







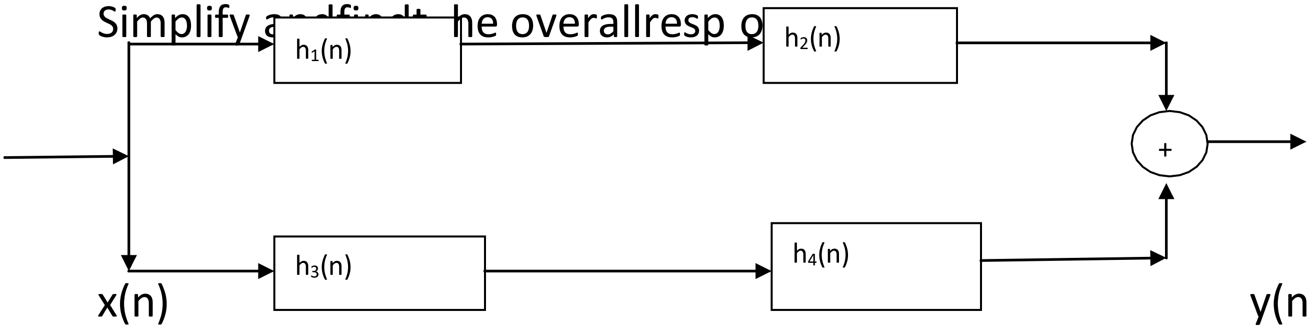
$x(n)$ $y(n)$



$h(n)=$

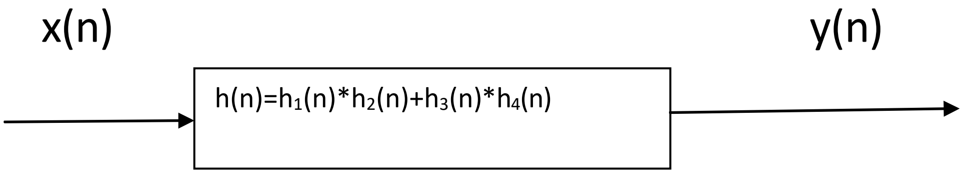
$h_1(n)$+ [($h_1(n)$

Simplify andfindt he overallresp o



$x(n)$ $y(n)$

$h(n)=h_1(n)*h_2(n)+h_3(n)*h_4(n)$



Correlation

Correlation is a measure of similarity between two signals. The general formula for correlation is

$$\int x_1(t)x_2(t-\tau)dt$$

Therearetwotypesofcorrelation:

- Autocorrelation
- Cross correlation

Auto CorrelationFunction

It is defined as correlation of a signal with itself. Auto correlation function is a measure of similarity between a signal&itstimedelayedversion.ItisrepresentedwithR(ττ).

CrossCorrelationFunction

Cross correlation is the measure of similarity between two different signals.

The impulse response of an LTI system is h(n)={ 1,2 ,1,-1}.Find the response of the system for the inputx(n)={ 1,2 ,3,1}

The response y(n) of the systemis given by convolution of x(n) and h(n)

y(n)=x(n)*h(n)..................................(1)

ByconvolutiontheoremofFouriertransform,weknowthat

F {x(n) * h(n)}= X(e^{jw}) . H(e^{jw}) .... **(2)**

From equation (1) and (2) we can write

F{y(n)}=X(e^{jw}).H(e^{jw}) Let

F{y(n)}=Y(e^{jw})

Y(e^{jw})= X(e^{jw}) . H(e^{jw})

y(n)=F^{-1}{X(e^{jw}).H(e^{jw})}

x(n)={1,2,3,1}

X(e^{jw})=$\sum_{=-\infty}^{\infty}\ ()^{-}$

X(e^{jw})=$\sum_{=?}^{?}\ ()^{-}$

X(e^{jw})=$?(?)?^{-???}+?(?)?^{-???}+?(?)?^{-???}+?(?)?^{-???}$**X**

(e^{jw})=1+2$?^{-??}$**+3**$?^{-???}$**+**$?^{-???}$

h(n)={1,2,1,-1}

H(e^{jw})=$\sum_{?=-\infty}^{\infty}\ ?(?)?^{-???}$

H(e^{jw})=$\sum_{?=?}^{?}\ ?(?)?^{-???}$

$H(e^{jw})=?(?)?^{-???}+?(?)?^{-???}+?(?)?^{-???}+?(?)?^{-???}$H

$(e^{jw})=1+2?^{-??}+?^{-???}-?^{-???}$

$X(e^{jw}).H(e^{jw}) \quad =(1+2?^{-??}+3?^{-???}+?^{-???}).(1+2?^{-??}+?^{-???}-?^{-???})$

$=1+2?^{-??}+?^{-???}-?^{-???}+ \quad 2?^{-??}(1+2?^{-??}+?^{-???}-?^{-???})+3?^{-???}(1+2?^{-??}+?^{-???}-?^{-???})+?^{-???}(1+2?^{-??}+?^{-???}-?^{-???})$

$Y(e^{jw})=1+4\ e^{-jw}+8\ e^{-j2w}+\ 8\ e^{-j3w}+\ 3e^{-j4w}-2\ e^{-j5w}\ -\ e^{-j6w}$ ....(3)

BydefinitionofFouriertransformweget,

$Y(e^{jw})=\sum_{?=-\infty}^{\infty} ?(?)?^{-???}$

$=1+4e^{-jw}+8e^{-j2w}+8e^{-j3w}+3e^{-j4w}-2e^{-j5w}-e^{-j6w}$

$=y(0)e^{0}+y(1)e^{-jw}+y(2)e^{-j2w}+y(3)e^{-j3w}+y(4)e^{-j4w}-y(5)e^{-j5w}-y(6)e^{-j6w}$ .... (4)

Comparingequation(3)&(4)weget

y(n)={1,4,8,8,3,-2,-1}

DiscreteFourierTransform(DFT)&FastFourierTransform(FFT)

- Discrete time Fourier transform(DTFT) is used to represent a discrete time signal in frequency domain and to perform frequency analysis of DT signals.
- Drawbacks:
  - Its frequency domain representation is a continuous function of ω
  - Cannotbe processedbydigitalsystem.

DiscreteFourierTransform(DFT)

- ItisobtainedbysamplingDTFTofasignalatuniform frequency intervals.
- It converts continuous function of ω to a discrete function of ω
- Sofrequencyanalysisispossiblebydigitalsystems.
- $X(e^{j\omega})$bediscretetimeFouriertransformofthediscrete time signal x(n).
- The DFT of x(n) is obtained by sampling one period of theDTFT $X(e^{j\omega})$ at a finite number of frequency points.
- ThisisdoneatNequallyspacedfrequencypointsinthe period $0 \le \omega \le 2\pi$
- Thesamplingfrequencyaredenotedbyω_k

$$\omega_k = \frac{2\pi k}{N}$$

fork=0,1,2,3,……….,N-1

The sampling of $X(e^{j\omega})$ is mathematically expressed as

$X(k)=X(e^{j\omega})|\omega=\frac{2\pi k}{N}$

fork=0,1,2,3,……….,N-1

Generally the DFT is defined along with number of samples and is called N-point DFT

DefinitionofDFT

Letx(n)=DiscretetimesignaloflengthL

X(k)=DFT of x(n)

TheN-pointDFTofx(n),whereN Lisdefinedas

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi kn}{N}}$$

;fork= 0,1,2,3,……….,N-1

Symbolically the N-point DFTof x(n) can be expressed as DFT{x(n)}=X(k)

InverseDFT

TheinverseDFT(IDFT)ofthesequenceX(k)oflengthNisdefined as

$$x(n)= 1/N \sum_{k=0}^{N-1} X(k)e^{\frac{j2\pi kn}{N}}$$

; forn=0,1,2,3,……….,N-1

<u>PropertiesofDFT</u>

- Linearity
- Periodicity
- Circulartime shift
- Timereversal
- Multiplication
- Circularconvolution

Linearity

Let DFT{$x_1(n)$}= $X_1(k)$ and DFT{$x_2(n)$}= $X_2(k)$ then by linearity property,

DFT{$a_1x_1(n)+a_2x_2(n)$}=$a_1X_1(k)+a_2X_2(k)$ wherea$_{1,}$a$_2$ are constants

Periodicity

Ifasequencex(n)isperiodicwithperiodicityof Nsamplesthen N-point DFT,X(k) is also periodic with periodicity of N samples

Hence ,if x(n) and X(k) are N point DFT pair then,

$x(n+N)=x(n)$;foralln

$X(k+N)=X(k)$;forallk

Circulartimeshift

This property says if a discrete time signal is circularly shifted in time by m units then its DFT is multiplied by $e^{\frac{-j2\pi km}{N}}$

i.e.ifDFT{x(n)}=X(k),thenDFT{$x((n-m))_N$}=X(k) $e^{\frac{-j2\pi km}{N}}$

Timereversal

This property says reversing the N-point sequence in time is equivalent to reversing the DFT sequence

i.e.ifDFT{x(n)}=X(k),thenDFT{x(N-n)}=X(N-k)

Multiplication

This property says that the DFT of product of two discrete time sequences is equivalent to circular convolution of the DFTs of the individual sequences scaled by a factor 1/N

i.e. if DFT{x_1(n)}= X_1(k)andDFT{x_2(n)}= X_2(k) ,then DFT{x_1(n) x_2(n)}= 1/N[X_1(k) * X_2(k)] * circular convolution

Circularconvolution

ThispropertysaysthattheDFTofcircularconvolutionoftwo sequences is equivalent to product of their individual DFTs

Let DFT{x_1(n)}= X_1(k) and DFT{x_2(n)}= X_2(k) then by convolution property

DFT{x_1(n)*x_2(n)}=X_1(k)X_2(k)

RelationshipbetweenDFTandZ-transform

TheztransformofN-pointsequence x(n)isgivenby, Z{x(n)}=X(z)= $\sum_{n=0}^{N-1} x(n)\, z^{-n}$

Let us evaluate X(z) at N equally spaced points on unit circle i.e.

at z= $e^{\frac{-j2\pi k}{N}}$

x(z)= $\sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}}$

Wecan conclude that the N-point DFT of a finite duration sequence can be obtained from the Z-transform of the sequence ,by evaluating the Z-transform of the sequence at N equally spaced points around the unit circle.

Computethe4-pointDFTofthesequence

x(n)=1/3 ; 0≤n≤2

=0 ;otherwise

x(n)={1/3,1/3,1/3}

4-pointDFT(i.e.N=4)

X(k)= $\sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}}$

X(k)= $\sum_{n=0}^{3} x(n)\, e^{-\frac{j2\pi kn}{4}}$ = $\sum_{n=0}^{2} x(n)\, e^{-\frac{j\pi kn}{2}}$

=x(0)e^0+x(1) $e^{\frac{-j\pi k}{2}}$ + x(2) $e^{-j\pi k}$

=1/3 +1/3 $e^{\frac{-j\pi k}{2}}$ +1/3 $e^{-j\pi k}$

=1/3*1+cosπk/2-jsinπk/2+cosπk-jsinπk+

SoX(k)=1/3*1+cosπk/2-jsinπk/2+cosπk-jsinπk+

- Whenk=0; X(0)= 1/3[1+cos 0-jsin0 +cos0-jsin0]=1/3 [1+1+1]=1
- Whenk=1; X(1)= 1/3*1+cosπ/2-jsinπ/2+cosπ-jsin π+=1/3 *1+0-j-1-j0]=-j/3
- When k=2; X(2)= 1/3* 1+cos π-jsin π+cos 2π-jsin 2π+=1/3*1-1-j0+1-j0]=1/3
- When k=3; X(3)=1/3* 1+cos 3π/2-jsin 3π/2+cos 3π-jsin 3π+=1/3*1+0+j-1-j0]=j/3

X(k)={1,-j/3,1/3,j/3}

INTRODUCTION TO FAST FOURIER TRANSFORM(FFT) ALGORITHM

DirectComputationofDFT

- DFT of a sequenceis obtained by direct computation. But this requires large number of computations which leads to greater processing time.
- DFTofaN-pointsequencex(n),n=0,1,2…………..,N-1is givenby

  X(k)=$\sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$ where k=0,1,2… ……… ,N-1.

- AnN-pointsequenceyieldsanN-point transform.
- X(k)canbeexpressedasaninnerproduct:
- X(k)= $[1 \quad e^{-\mathrm{j}2\pi k/N} \quad e^{-\mathrm{j}2\pi k2/N}e^{-\mathrm{j}2\pi k3/N}$ ………… $e^{-\mathrm{j}2\pi k(N-1)/N}]$

  $x(0)$

  $x(1)$

  .

  .

  .

  $x(N-1)$

- Notation:W_N=$e^{-\mathrm{j}2\pi/N}$
- Hence,
- X(K)=[$1 \quad W_NK$ WN^{2K}WN^{3K}…………WN $^{(N-1)K}$]

  $x(0)$

  $x(1)$

  .

  .

  .

  $x(N-1)$

- By varying k from 0 to N − 1 and combining the N inner products,X = Wx W is an N × N matrix, called as the “DFT Matrix”.
- EachinnerproductrequiresNcomplexmultiplications .There are N inner products Hence we require N^2multiplications.

- Each inner product requires N − 1 complex additions. There are N inner products Hence we require N(N − 1) complex additions.
- IfNislargethenthenumberofcomputationswillgointo lakhs.ThisprovesinefficiencyofdirectDFTcomputation.

Computationallyefficientalgorithm:FFT

- FastFourierTransform”(FFT) exploitsthe2important property (symmetry & periodicity) of W_N^K .
- FFT:Basedonthefundamentalprincipleofdecomposing thecomputationofDFTofasequenceoflengthNintosuccessively smaller DFTs.
- While calculating DFT, we have discussed N can be factorisedas

  N=r_1r_2 r_3………r_v(Everyrisa prime)

  Ifr$_1$=r$_2$=r$_3$=……=r$_v$=r thenN=r^v

  riscalledtheradix(base)ofFFTalgorithmandvindicates number of stages in FFT algorithm.
- **Ifr=2,itis calledradix-2FFTalgorithm.**

  e.gifN=8=2^3

For8pointDFTthereare3stagesofFFT algorithm.

- TypesofFFTalgorithm

  1. Radix-2DecimationInTime(DIT)algorithm
  2. Radix-2DecimationInFrequency(DIF)algorithm

DIT ALGORITHM

Decimatemeansto“breakintoparts”.DITindicatesdividing (splitting) the sequence in time domain.

Firststageof Decimation

From{x(n)}formtwosequencesasfollows:f1(n)=x(2n)and f2(n) =x(2n+1)

f1(n)containstheeven-indexedsamples,whilef2(n)contains the odd-indexed samples

$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$ where k=0,1,2… .... ,N-1

$$X(k)=\sum_{n=0}^{N-1} x(n)W_N^{kn}$$

$$X(k)=\sum_{r=0}^{\frac{N}{2}-1} x(2r)W_N^{2r*k} + \sum_{r=0}^{\frac{N}{2}-1} x(2r+1)W_N^{(2r+1)*k}$$

$$X(K)=\sum_{r=0}^{\frac{N}{2}-1} f1(n)W_N^{2r*K} + W_N^{K}\sum_{r=0}^{\frac{N}{2}-1} f2(n)W_N^{(2r)*K}$$

$$W_N^{(2r)*k}=e^{-j2\pi k2r/N}=e^{-j2\pi kr/(N/2)}=W_{N/2}^{r*k}$$

Hence

$$X(k)=\sum_{r=0}^{\frac{N}{2}-1} f1W_{N/2}^{r*k}+W_N^{K}\sum_{r=0}^{\frac{N}{2}-1} f2(n)W_{N/2}^{r*k}$$

$=f1(k)+W_N^{k}f2(k)$ where k=0,1,2… ....,N-1.

f1(k)andf2(k)areN/2pointDFTs

W_N^{k}iscalled“**Twiddlefactor**”(NthrootofUnity)

- TheN/2pointDFTsf1(k)andf2(k)areperiodicwithperiod N/2

  f1(k+N/2)=f1(k) (PeriodicityProperty)

f2(k+N/2)=f2(k) (Periodicity Property)

and$W_N^{k+N/2}$=-W_N^k (Symmetry Property)

Hence,ifX(k)=f1(k)+W_N^kf2(k),thenX(k+N/2)=f1(k)-W_N^kf2(k),

f1(k) f1(k)+W_N^Kf2(k)

f2(k) f1(k)-W_N^kf2(k)

Second Stage Decimation

Repeat the process for each of the sequences f1(n) and f2 (n).

f1(n)andf2(n)willcontaintwoN/4pointsequences each.

Let,

v_{11}(n)=f1 (2n)&v_{12}(n)=f1 (2n+1)

wheren=0,1,2,.......,N/4-1

v_{21} (n)= f2(2n)& v_{22} (n)= f2 (2n+1)

Likeearlieranalysiswecanshowthat,

F1(k)=V_{11} (k) + $W_{N/2}^k V_{12}$ (k)

F2(k)=V_{21}(k)+$W_{N/2}^k V_{22}$(k)

Hence the N/2 point DFTs are obtained from the results of N/4 point DFTs

SummaryofStepsofradix2DIT-FFT algorithm

- Theno.ofinputsamples,N=2^M,whereMisaninteger.
- Theinputsequenceisshuffledthroughbitreversal.
- Theno.ofstagesinthe flowgraphisgivenbyM=$\log_2$N
- EachstageconsistsofN/2butterflies.

- Thenoofsetsorsectionsofbutterfliesineachstageis given by 2^{M-m} .
- The twiddle factor exponents are given by $k=N*t/2^{m}$ wheret=0,1,2,.......2^{m-1}-1·
- Drawtheflowgraphtaking NinputsamplesandM stages.
- Calculate the DFT values using basic butterfly operations.

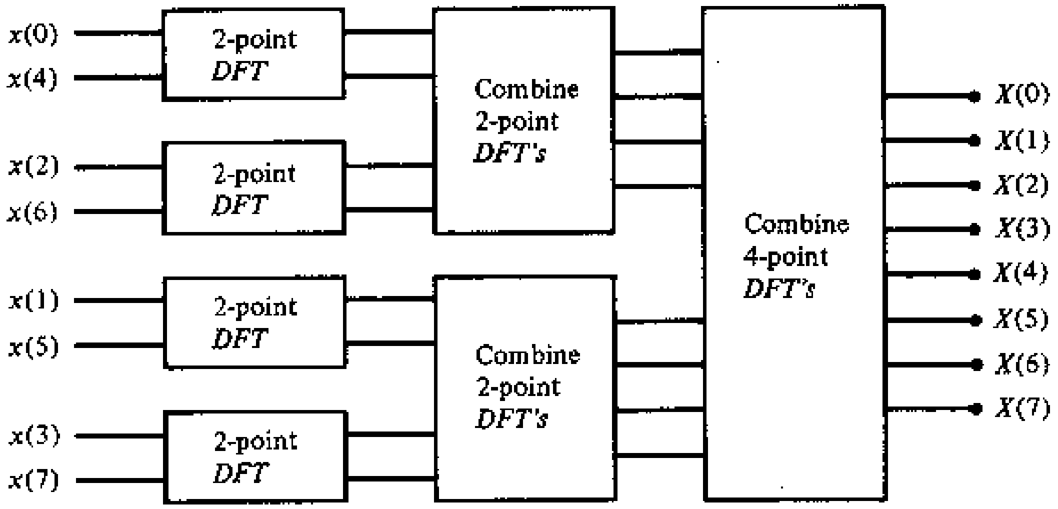
Recallthat,forN=8,thefirstsplitrequiresthedatatobe arranged as follows: x0, x2, x4, x6, x1, x3, x5, x7

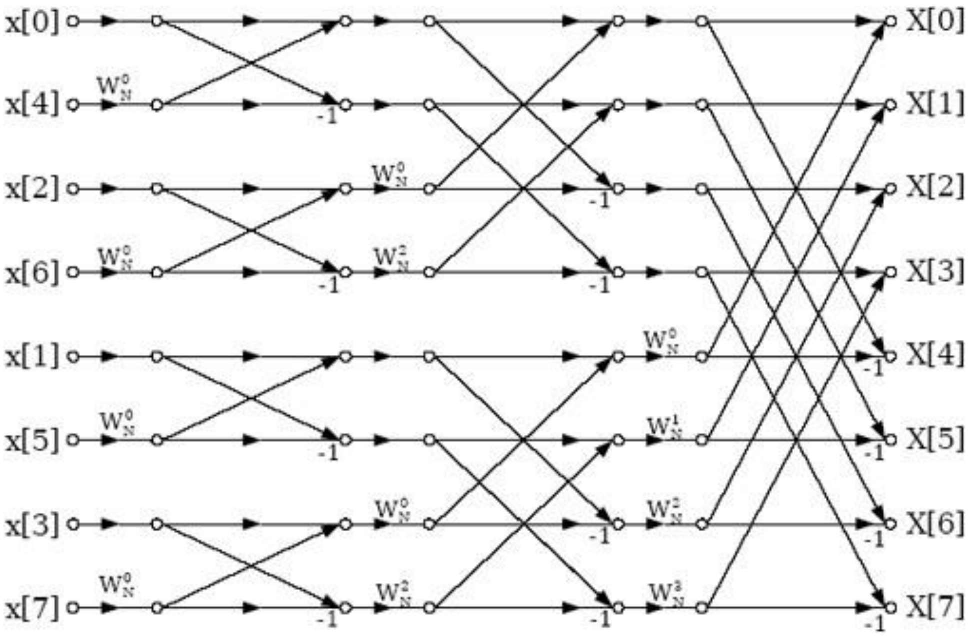
In the second and final split, the data appear in the following order: x0, x4, x2, x6, x1, x5, x3, x7

Thefinalorderissaid tobein“bitreversed” form:

Original	Binary Form	Reversed Form	Final
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

3stagecomputationofa8pointDFT





IntroductiontoDigitalFilters(FIRFilters)

- DigitalFiltersarethediscretetimesystemsusedmainlyfor filtering of arrays or sequences.
- Thearraysorsequences.areobtainedbysamplingthe inputanalogsignals.

- Digital filters mainly performs frequency related operationssuchasLowPass,HighPass,Bandreject,Band pass and All pass.
- Thedesignspecificationsincludecut-off frequency, samplingfrequency,stopbandattenuationetc.
- Digitalfiltersmayberealisedthroughhardwareor software

ImplementationofDigital Filters

- Representedbydifferenceequations,implementedins/w like ‘C’ or assembly language.
- Suchlanguagesarecompiledandanexecutablecodefor theprocessoris prepared.
- Thiscoderunsonthememory,databus,shiftregisters, counters and ALU etc. to give required output.
- DigitalFiltersmayalsobeimplementedbydedicated hardwarewhichisadigitalcircuitconsistingofcounters, shift registers, flip-flops, ALU etc.
- Disadvantage:wecanperformonly onetype of filtering operation.

TypesofDigitalFilters

1. FiniteImpulseResponse(FIR)filters(non-recursivetype)
2. InfiniteImpulseResponse(IIR)filters(recursivetype)

- Basically ,digital filters are LTI systems which are characterisedbyunitsampleresponse.TheFIRsystemhas finite duration unit sample response i.e.

  h(n)=0forn<0forn>=M

- SimilarlyIIRsystemhasinfinitedurationunitsample response i.e.

  h(n)=0 for n<0

IntroductiontoDigitalSignal Processor

- Microprocessorsdesignedspecificallyfordigitalsignal processing applications.
- Containsspecialarchitectureandinstructionsetto executeDSPalgorithmefficiently.
- Types:1.GeneralPurposeDSPs2.SpecialPurposeDSPs
- GeneralPurposeDSPs:Highspeedmicroprocessorwith architecture and instruction sets optimized for DSP operation. e.g. Texas Instruments TMS320C5x, TMS320C54x & Motorola DSP563x etc.
- SpecialPurposeDSPs:Containsh/wdesignedfor specific DSP algorithms such as FFT, PCM, Filtering etc.e.g. FFT processorPDSP16515A,TM44/66,FIRfilterUPDSP16256 etc

Assignment Questions

1. Computethe8pointDFTofthegivensequenceusingradix-2 DIT FFT algorithm x(n)={1,3,1,2,1,3,1,2}.

2. Computethe4pointDFTofthegivensequenceusingradix-2 DIT FFT algorithm x(n)={1,1,2,2}.

2.Whatisphasefactorortwiddlefactor?

3.DrawandexplainthebasicbutterflydiagramofDITradix-2 FFT.

4. Howmanymultiplicationsandadditionsareinvolvedin radix-2 FFT?

ReferenceBooks:

1.SignalandSystemsbyANagoorKani

2.DigitalSignalProcessingbyP.Ramesh Babu

3.DigitalSignalProcessingbySanjaySharma.

Reference site:

http://www.ee.iitm.ac.in/~csr/teaching/pg_dsp/lecnotes/fft.pdf

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