GOVERNMENT POLYTECHNIC BHUBANESWAR

LECTURENOTE

ON

CIRCUIT THEORY TH-2

Department of Electronics and TelecommunicationEngineering

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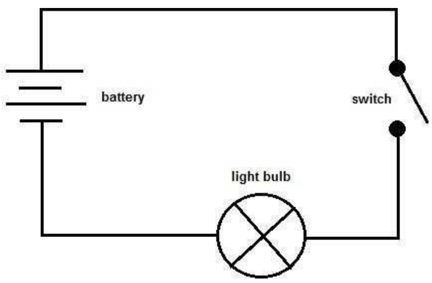
CHAPTER-1

NETWORKELEMENTS

INTRODUCTION:

An electric circuit is an interconnection of electrical elements such as resistors, capacitors, inductors, voltage source etc. In electrical engineering, transfer of energy takes place from one point to another, which requires interconnection of electrical devices. Such interconnection isknown aselectric circuit and each component of the circuit is known as an element.

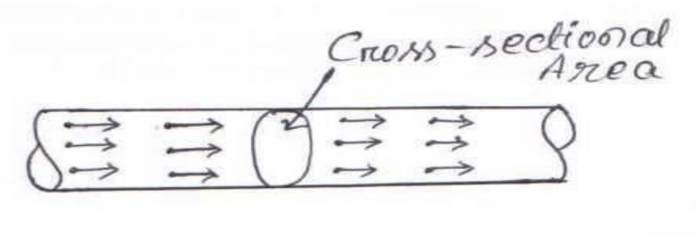
EXAMPLE # Consider an electrical circuit as shown in the figure. This electric circuit consists of four elements abattery,alamp, switch&connectingwires.Circuitand network theorem is the study of the behaviour of the circuit: Its behaviour tells us how does it respond to a given input how do the interconnected elements and devices in the circuit interact?



ELECTRICCURRENT:

Electric current may be defined as the time rate of net motion of an electric charge across a cross sectional boundary as shown in the figure given below. A random motionof electronsin ametal doesnot constituteacurrent unlessthere is anet transfer of charge with time i.e. electric current.

i =Rateoftransferofelectriccharge
=Quantityofelectricchargetransferredduringagiventime duration/ Time duration
$=\frac{dQ}{dt}$



Coulomb is the practical as well as SI unit for measurement of electric charge. Since current is the rate of flow of electric charge through conductor and coulomb is the unit of electric charge, the current may be specified in coulombs per second. In practice the ampere is used as the unit of current. Coulomb is the practical as well as SI unit for measurement of electric charge. Since current is the rate of flow of electric charge through conductor and coulombistheunitof electric charge, the current may be specified in coulombs per second. In practice the ampere is used as the unit of current.

VOLTAGE:

Thevoltageisthepotentialdifferencebetweentwopointsofa conductor carrying a current of one ampere when the power dissipated between thesetwo points is equal to one watt. The practical unit of voltage is volt.

POWER:

Powerisdefinedastherateofdoingworkorrateatwhichitcanperform work. So Power=workdone/Timeinseconds P

$$= \frac{dw}{dt} = \frac{dw\,dq}{dq\,dt} = v\,i$$

Absolute unit of power is watt. One watt is thatpower which is required to perform one joule of work in one second. The practical unitof power is horse power (HP). This value in metric system is 75kg meters per second and in British system is 550 Foot Pounds/second. Therefore

1 HP (Metric) =75Kgmeterspersecond=735.5watt 1
HP (British) = 550 Foot Pound/ second = 746 watt

ENERGY:

Energyofabodyisitscapacityofdoingwork.

$$E=\int_0^t P dt$$

The unit of energy in MKS system is joule and in SI system is KWH. A system can have this energy in various forms, such as electrical, mechanical, heat, chemical, atomic energy etc. Energy of one form can be transformed to other form, but cannot be created nor be destroyed. If one form of energy disappeared, it reappears in another form. This principle is known as law of conservation of energy.

CIRCUITELEMENTS/PARAMETERS:

1. RESISTANCE:

Resistance restricts the flow of electric current through the material. Unit of Resistance(R) is Ohm. From Ohm's law

R=V/I

Whenanelectriccurrentflowsthroughanyconductor,heatisgenerateddueto collision of free electrons with atoms. If I amp is the strength of current for potential difference V volts across a conductor, the power observed by resistor is :

P=VI= (IR).I=I^2Rwatts

Energylostintheresistor informofheatis then

$$E= \int_0^t p.dt = \int_0^t I^2R\,dt = I^2Rt= \frac{V^2}{R}xt$$

2. INDUCTANCE:

It opposes any change of magnitude or direction of electric current passing through the conductor. Unit is Henry (H).When a current will flow through the coils/Inductor an electromagnetic field is created. However in the event of any change of flow on direction ofcurrent,theelectromagnetic field alsochanges.Thischange of field induces a voltage (V) across the coil & is given by

$$V = L.\frac{di}{dt} \quad \text{- ---- (1)}$$

Where'i'iscurrentthroughtheinductor.
Voltageacrossaninductoriszerowhencurrentisconstant. Hence an inductor acts like short circuit to dc.

Powerabsorbedbyinductor

$$P=V\times i = Li\frac{di}{dt} \text{watts} \text{ ------------------------------------ (2)}$$

Energyabsorbed.

$$E=\int_0^t p.dt = \frac{1}{2}Li^2 \text{ ------------------------------------ (3)}$$

From equation (2) & (3): The inductor can store finite amount of energy, even the voltage across it may be nil. A pure inductor does not dissipate energy but can only store it.

3. CAPACITANCE:

Itisthepropertyofcapacitor,whichhavethecapabilitytostoreelectric charge in its electric field established by the two polarities of charges on the two electrodes of a capacitor.

Theamountofchargestorebycapacitoris q = cv

$$i=\frac{dq}{dt} \Rightarrow i=c\frac{dv}{dt}$$

Thereforeifvoltageacrosscapacitorisconstant,currentthroughitis zero. Hence capacitor acts like a open circuit to dc.

Power absorbed $P=V.I=VC\frac{dv}{dt}$

Energy stored $E=\int_0^t p.dt = \frac{1}{2}CV^2$

Acapacitorcanstorefiniteamountofenergy.Evenifthecurrent throughitiszero.It never dissipates energy.

TYPESOFELEMENTS:

ACTIVEANDPASSIVEELEMENT:

An active element has capability to generating energy while passive elements have not.

Ex: ActiveElement: Generators,Batteries,AndAmplifiers.
PassiveElement: Resistor,Inductor,capacitor.

BILATERALANDUNILATERALELEMENT:

If the magnitude of current passing through the element is affected due to change in the polarity of the applied voltage, the element is called unilateral element. And if the current magnitude remains same, it is called as bilateral element.

Ex: Unilateral Element: - Diodes, Transistors.
Bilateral Element: -Resistor, Inductor, Capacitor

LINEARANDNON-LINEARELEMENTS:

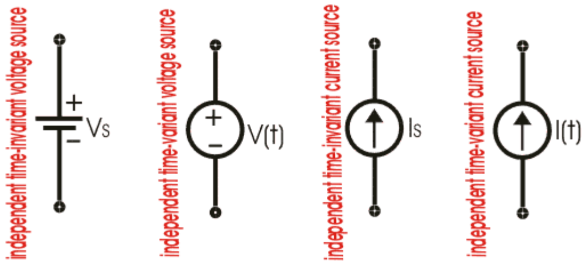
A linear element shows linear characteristics of voltage Vs current. Resistors, Inductor, Capacitor are linear elements and their property does not change in applied voltage on circuit current.

For non-linear elements the current passing through it does not change linearly with the time as change in applied voltage at a particular frequency.

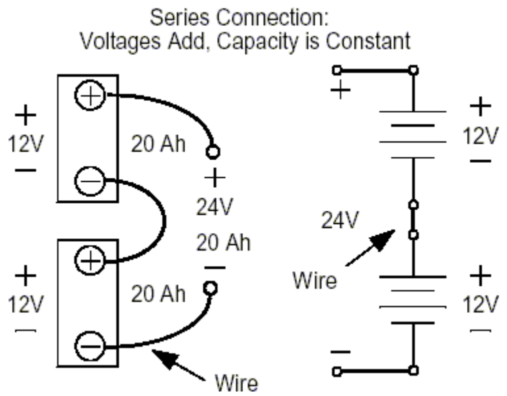
Ex:Semiconductordevices.

ENERGYSOURCES:

Independent Energy sources: The voltage & current sources whose values or strength of voltage and current does not change by any variation in the connected network are called independent sources.

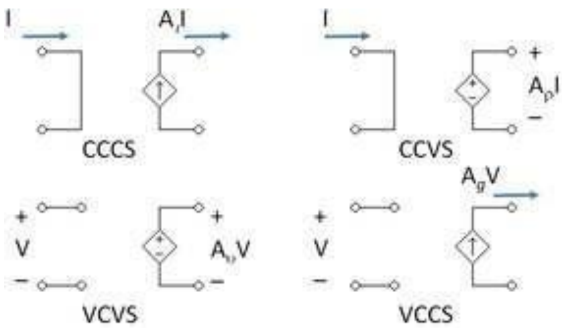


Series connected independent sources: Consider the series connection of two voltage sources as shown in the figure. By KVL the total voltage between the terminals is equal to algebraic sum of individual sources i.e. the voltage sources connected in series may be replaced by a single voltage source whose voltage is equal to the algebraic sum of the individual sources.



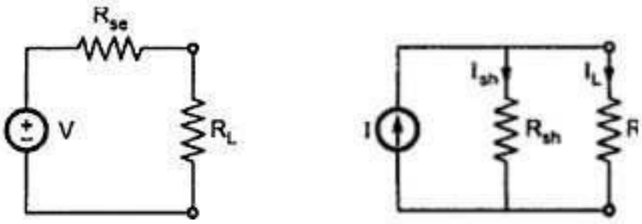
DependentEnergysources:Whenthestrengthofvoltageandcurrentchanges in the sources for any change in the connected network, they are called dependent sources. There four different types of dependent sources

a) Voltagecontrolledvoltagesource(VCVS)
b) Voltagecontrolledcurrentsource(VCCS)
c) Currentcontrolledvoltagesource(CCVS)
d) Currentcontrolledcurrentsource(CCCS)



SOURCETRANSFORMATION:

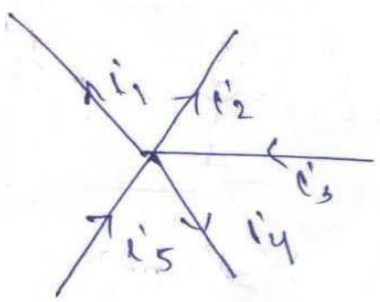
The voltage and current sources aremutually transferable as shown inthe figure below.



KIRCHHOFF'SLAW:

These laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by latter. Kirchhoff's law is oftwotypes,Kirchhoff'scurrentlawand Kirchhoff'svoltagelaw. Kirchhoff'scurrentlaw is used when voltage is chosen as variable while Kirchhoff' voltage law is used when current is chosen as variable.

KCL:According to Kirchhoff's current law the algebraic sum of currents at any node of a circuit is zero. From the figure given below:



$-I_1-I_2+I_3-I_4+I_5=0$

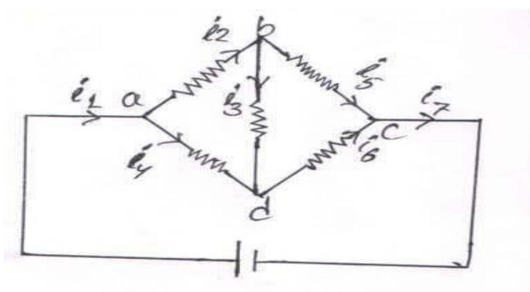
$=>I_1+I_2+I_4=I_3+I_5$

Hence:

Algebraicsumofcurrentsenteringanode= Algebraic sum of current leaving a node.

Example1: Find the magnitude and direction of the unknown current as shownin figure given I_1= 10 A, I_2= 6A, I_5= 4A

Solution: Assume direction ofcurrent in the network



(i) $I_1=I_7=10A$

(ii) $I_1=I_2+I_4=>I_4=I_1-I_2=10-6=4A$

(iii) Atnode b:$I_2-I_3-I_5=0$

$=>6-I_3-4=0=>I_3=2A$

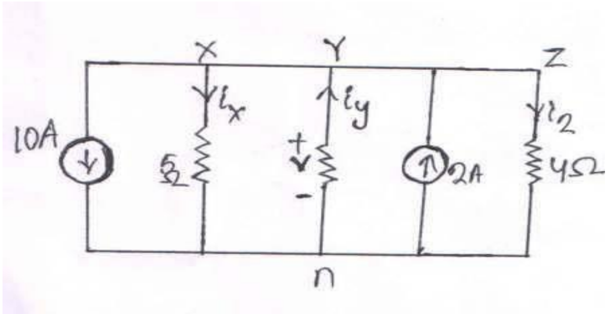
(iv) Atnode d:$I_4+I_3-I_6=0$

$=>4+2-I_6=0$

$=> I_6= 6A$

Assume direction of all current are correct because of their positive magnitude. Assume directions of unknown current are arbitrary and any direction can be taken.

Example2: Find v and the magnitude and direction of the unknown currents in the branch xn, yn and zn as shown in figure.



Solution:

At node y: $10+I_x+I_z=I_y+2$ $I_x - I_y + I_z = -8$

$\frac{V}{5}+\frac{V}{2}+\frac{V}{4}=-8$ [since $I_x = \frac{}{5}$, $I_y=-\frac{V}{5}$, $I_z=\frac{V}{5}$]

$V=-8.42$ volt

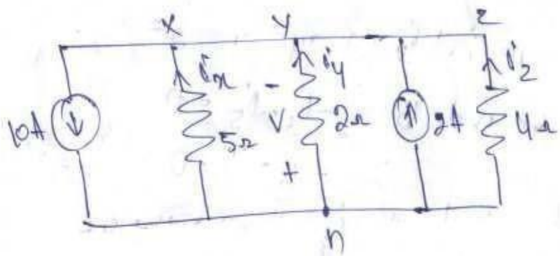
Negative magnitude shows that n to be positive.

Therefore $I_x = -\frac{8.42}{5} = -1.684$ A (i.e. from flowing current n to x)

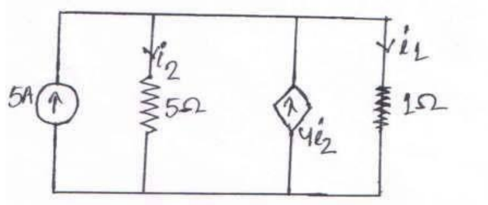
$I_y = -\frac{(-8.42)}{2} = 4.21$ A (ie Current flowing from n to y)

$I_z = \frac{-8.42}{4} = -2.1$ A (ie current flowing from n to z)

The circuit can be redrawn as given below



Example3: Find $i1$ and $i2$ as shown in figure



Solution: The circuit is redrawn in figure

According to KCL: $i1+i2=5+4i2$ --------------------- (1)

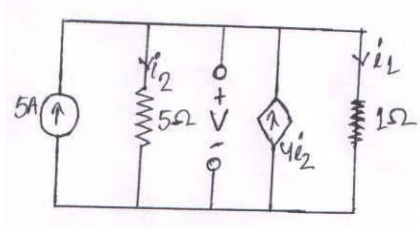
$i1-3i2=5$ ---------------- (2)

Here $i_1=\frac{V}{1}$; $i_2=\frac{V}{5}$

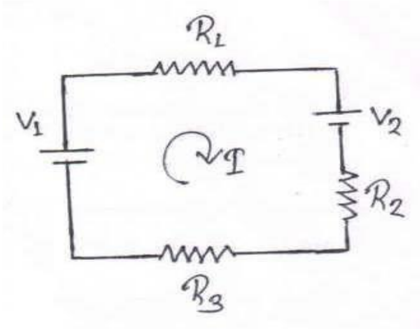
Therefore equation 2: $V-3\frac{V}{5}=5$

$\Rightarrow V=12.5$ volt

Therefore $i_1 = 12.5$ A and $i_2 = 2.5$ A



KIRCHHOFFSVOLTAGELAW:



Thislawcanbestatedas

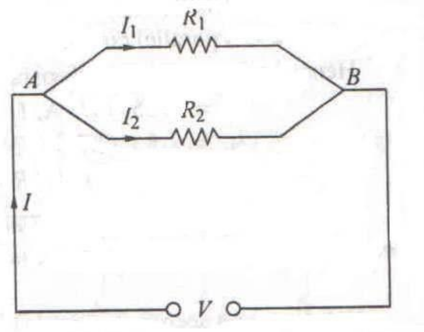
"The algebraic sum of voltage in any closed path of a network that is traversed in single direction is zero."

Explanation:Accordingto KVL

$V_1 - IR_1 - V_2 - IR_2 - IR_3 = 0$ $IR_1 + IR_2 + IR_3 = V_1 - V_2$

$I = \frac{V1-V2}{R1+R2+R3}$

CURRENTDIVISIONRULE:



Two resistors are joined in parallel across avoltageV.Thecurrentineachbranch, asgiven in ohm's law is

$I_1 = V/R_1$ and $I_2 = V/R_2$

Therefore $I_1/I_2 = R_2/R_1 = G_1/G_2$

Hencethedivisionofcurrentinthe branch ofparallel circuit isdirectlyproportional tothe conductanceofthebranchesorinverselyproportional totheirresistances. Wemay also express the branch currents in terms of the total circuit current thus:

Now $I_1 + I_2 = I$

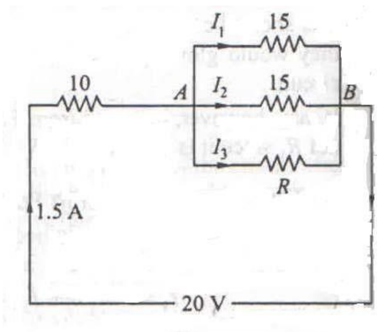
$\Rightarrow I_2 = I - I_1$

Therefore $\frac{I1}{I-I1} = \frac{R2}{R1}$ or $I_1R_1 = R_2(I - I_1)$

Therefore $I_1 = I \frac{R2}{R1+R2}$ and $I_2 = I \frac{R1}{R1+R2}$

Thuscurrentdivisionruleisstatedas

"The current in any of the parallel branches is equal to the ratio of the opposite branch resistance to the total resistance, multiplied by the total current."



Example4: A resistance of 10 ohm is connected in series with two resistances each of 15 ohm arranged in parallel. What resistance must be shunted across this parallel combination so that the total current taken shall be 1.5 A with 20 volt applied?

Solution: The circuit connected in figure

Dropacross10ohmresister=1.5*10=15V

Drop across parallel combination, V_{AB}= 20-15=5V

Hencevoltageacrosseachparallelresistanceis5V.

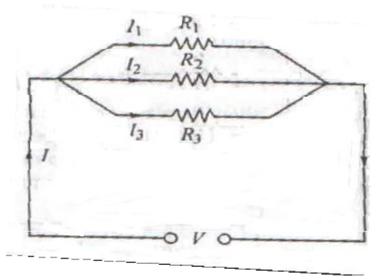
I_1=5/15 =1/3 A

I_2= 5/15=1/3A

I_3=1.5-(1/3+1/3)=5/6A

Therefore I_3R= 5 or (5/6) R=5orR=6ohm

Example5: Calculate thevalue of differentcurrentfor the circuit shown in given figure.



Solution: TotalcurrentI=$I_1+I_2+I_3$

LettheequivalentresistancebeR.

Then V= I R

Also

V=I_1R_1Therefore

I R = I_1R_1

Or $I_1=IR/R_1$ -- (1)

Now (1/R)=(1/R1)+(1/R2)+(1/R3)

$$R=\frac{R1R2R3}{R1R2+R2R3+R3R1}$$

Fromequation 1: $$I_1=\frac{R2R3}{R1R2+R2R3+R3R1}$$

Similarly $$I_2=\frac{R1R3}{R1R2+R2R3+R3R1}$$

$$I_3=\frac{R1R2}{R1R2+R2R3+R3R1}$$

VOLTAGEDIVISIONRULE:

A voltage divider circuit is a series network which is used to feed other networks with a number of different voltages and is derived from a single input voltage source. Figure shows a simple voltage divider circuit whichprovide twooutput voltages V1 and V2. Since no load is connected across the output terminals, it is called an unloaded voltage divider. We may also express the branch voltages in terms of the total circuit voltage thus:

NowV1+V2=V

=>$V_2=V-V_1$

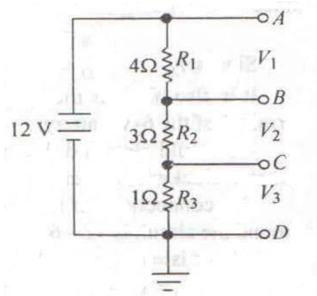
Therefore $\frac{V1}{V-V1}=\frac{R1}{R2}$ or V1 $R_2 = R_1(V-V_1)$

Therefore $V1=V\frac{R1}{R1+R2}$ and $V2=V\frac{R2}{R1+R2}$

ThusVoltagedivisionruleisstated as

"The voltage across a resistor in series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by thetotal resistance of the series elements."

Example9: Find the value of different voltages that can be obtained from a 12 V battery with the help of voltage divider circuit of figure.



Solution:

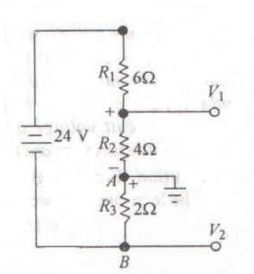
R= R1 + R2 +R3 = 4+3+ 1= 8ohm

Drop across R1=V_{R1} = 12 × (4/8) = 6 volt

DropacrossR2=V_{R2}=12×(3/8)=4.5volt

Drop acrossR3=V_{R3}= 12×(1/8)=1.5volt

Example10:Whataretheoutputvoltagesoftheunloadedvoltagedividershownin figure what is the direction of current Through AB?



Solution:

It mayberememberthat both V1andV2 arewith respect to ground.

R=6+4+2= 12ohm

Therefore

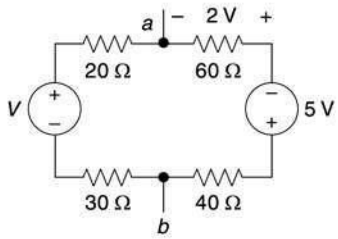
V1= Drop across R2= 24 × (4/12) = 8 volt

V2=DropacrossR3=-24×(2/12)=-4volt

It should be noted that point B is negative potential with respect to the ground. Current flows from A to b i.e. from a point at a higher potential to appoint at a lower potential.

Problem1

Find the values of V, V_{ab} and the power delivered by the 5V source. All values of resistances are in ohm.



Solution

Current, $i = \frac{2}{60} = \frac{1}{30}$ A

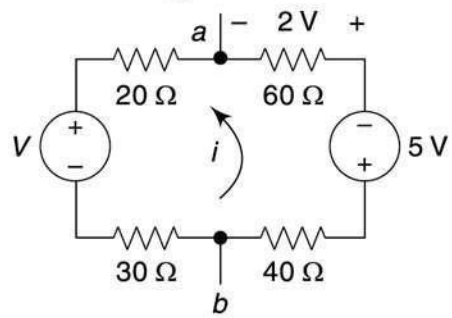
By KVL,

$$20i + 2 + 5 + v + 70i = 0$$

$$v = -7 - 90i = -7 - 90 \times \frac{1}{30} = -10 \text{ V}$$

$$\therefore\ v_{ab} = 20i + v + 30i = 50i - 10$$

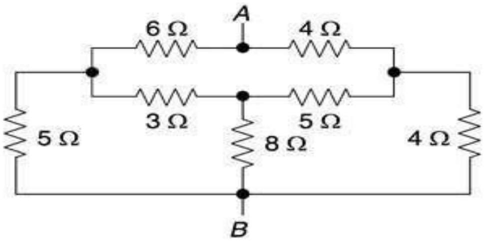
$$= 50 \times \frac{1}{30} - 10 = -8.33 \text{ V}$$



Power drawn by the 5V source = − (Power taken source) = $-5 \times \frac{1}{30}$ = −0.166 W

Problem2

Find the equivalent resistance between the terminals *A* and *B* of the circuit shown below.



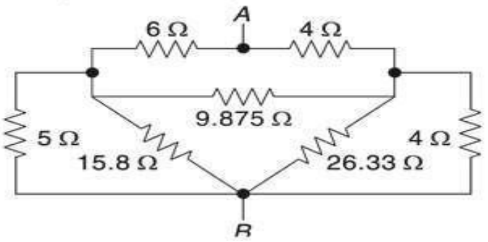
Solution

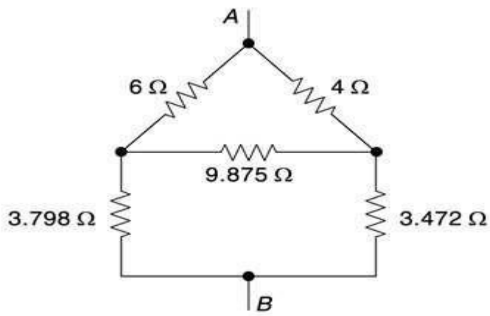
Converting star into delta,

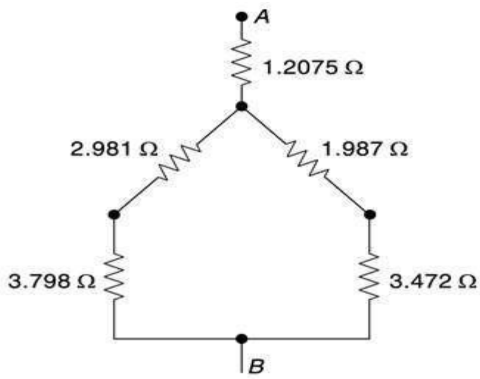
$$r_{12} = \left(r_1 + r_2 + \frac{r_1 r_2}{r_3} \right) = 8 + \frac{15}{8} = 9.875\ \Omega$$

$$r_{23} = \left(r_2 + r_3 + \frac{r_2 r_3}{r_1} \right) = 13 + \frac{40}{3} = 26.33\ \Omega$$

$$r_{31} = \left(r_3 + r_1 + \frac{r_3 r_1}{r_2} \right) = 11 + \frac{24}{5} = 15.8\ \Omega$$







Combining the parallel connections of 5 Ω and 15.8 Ω and 4 Ω and 26.33 Ω, we have the reduced circuit.

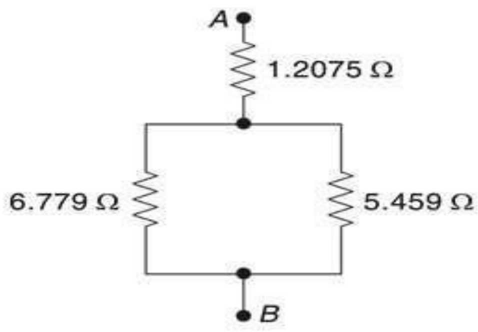
Again, converting the delta made of 6 Ω, 4 Ω and 9.875 Ω into equivalent star,

$$r_1 = \frac{r_{12} r_{31}}{r_1 + r_2 + r_3}$$

$$= \frac{6 \times 4}{19.875} = 1.2075\ \Omega$$

$$r_2 = \frac{4 \times 9.875}{19.875} = 1.987\ \Omega$$

$$r_3 = \frac{6 \times 9.875}{19.875} = 2.981\ \Omega$$

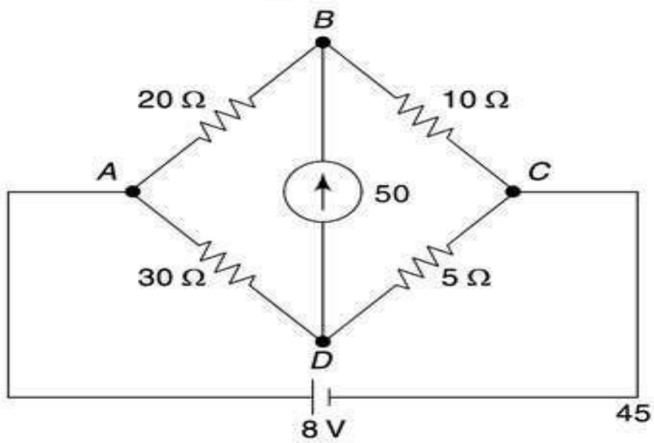


So, the given circuit becomes as shown in figure.

$$\therefore \quad R_{AB} = 1.2075 + \frac{6.779 \times 5.459}{6.779 + 5.459} = 4.23\ \Omega \quad \textit{Ans.}$$

Problem3

Find the current through the galvanometer using delta-star conversion.



Solution

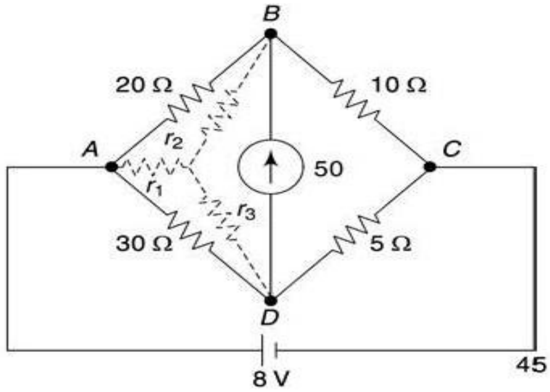
Converting the delta consisting of 20 Ω, 30 Ω and 50 Ω, we get,

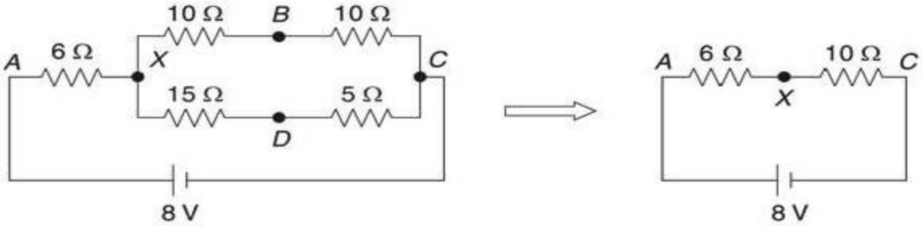
$$r_1 = \frac{20 \times 30}{20 + 30 + 50} = 6\ \Omega$$

$$r_2 = \frac{20 \times 50}{20 + 30 + 50} = 10\ \Omega$$

$$r_3 = \frac{30 \times 50}{20 + 30 + 50} = 15\ \Omega$$

$$\therefore R_{AC} = 16\ \Omega$$





Main current $i = \frac{8}{16} = 0.5$ A

Now, to calculate potential difference between the points *B* and *D* ;

$$V_{XC} = 10 \times 0.5 = 5 \text{ V}$$

$\therefore V_{BD} = (10 \times 0.25 - 5 \times 0.25) = 1.25$ V

$\therefore$ Currant through the galvanometer, (50 (Ω)

$$i_G = \frac{1.25}{50} = 0.025 \text{ A}$$

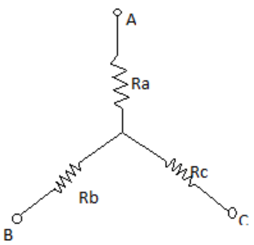
CHAPTER-2
NETWORKTHEOREMS

INTRODUCTION

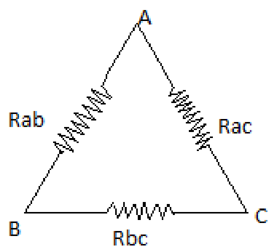
Electriccircuitsonnetworkconsistofanumberofinterconnected single circuit elements. This circuit will generally contain at least one voltage on current source. The arrangement of elements results in a new set of constraintsbetween currents and voltages.These new constraints and their corresponding equations added to thecurrent-voltagerelationshipsoftheindividual elementsprovidethesolutionofthe network. There are different approaches for this but the solution is always unique.

STARDELTATRANSFORMATIONS

FigureshowsaY(staror wage)connectedresistancecircuit.Let the resister value of Y network are R_a, R_b△andR_c.Figureshowsa(delta) connected resistances and Let the resistor values are R_{ab}, $R_{bc}$and R_{ca} .



StarConnected



DeltaConnected

It is possible to substitute a star connected system of resistance for a delta system and vice-versa if proper values are given to the substituted resistances.

DELTATOSTARCONVERSION

The two systems will be exactly equivalent if the resistance between any pair of terminals A, B and C in figure for the star is the same as that between the corresponding pair for the delta connectionwhen the third terminal is isolated.

FortheY-networkresistancebetweentheterminal

Aand BisRab= R_a+R_b --eq.(i)

Forthe△networkresistancebetweentheterminalsABis

$$R_{ab} = R_{ab} \,||\, (R_{ac}+R_{bc})$$

$$= \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{ac}+R_{bc}} \quad \text{------------------------------------ eq.(ii)}$$

Hence $$R_a + R_b = \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{ac}+R_{bc}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(iii)}$$

SimalarlyforY-networkresistancebetweenterminalBandCis R_{bc}=

R_b+R_c

Forthe △ networkresistancebetweenterminalBandCis

$R_{bc} = R_{bc} \,||\, (R_{ab}+R_{ac})$

$$R_b+R_c = \frac{R_{bc}(R_{ab}+R_{ac})}{R_{bc}+R_{ab}+R_{ac}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_eq.(iv)}$$

SimilarlywecanfindR$_{ac}$betweenterminalAandCis

$$R_a+R_c = \frac{R_{ac}(R_{ab}+R_{bc})}{R_{ac}+R_{ab}+R_{ac}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_eq.(v)}$$

Subtractingeq.(v)fromthesumofeq.(iii)andeq.(iv)yields 2

$$R_b = \frac{2\, R_{ab}.\, R_{bc}}{R_{ab}+R_{bc}+R_{ca}}$$

$$R_b = \frac{R_{ab}.\, R_{bc}}{R_{ab}+R_{bc}+R_{ca}}$$

Subtractingeq.(iv)fromthesumofeq.(iii)&eq.(v) yields

$$2\, R_a = \frac{2R_{ab}.R_{ac}}{R_{ab}+R_{bc}+R_{ac}}$$

$$R_a = \frac{R_{ab}.R_{ac}}{R_{ab}+R_{bc}+R_{ac}}$$

Similarlysubtractingeq.(iii)fromthesumofeq.(iv)andeq.(v)yields 2

$$R_c = \frac{2.\, R_{bc}.\, R_{ca}}{R_{ab}+R_{bc}+R_{ca}}$$

$$R_c = \frac{R_{bc}.R_{ca}}{R_{ab}+R_{bc}+R_{ca}}$$

Therefore, the equivalent impedance of each arm of the star is given by the product of the impedance of the two delta sides that meet at its ends divided by the sum of there delta impedance

STARTODELTACONVERSION

Similarlywe canfindconversionformulaforYto △ as

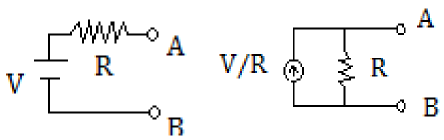
$$R_{ab} = \frac{R_a.R_b + R_b.R_c + R_c.R_a}{R_c}$$

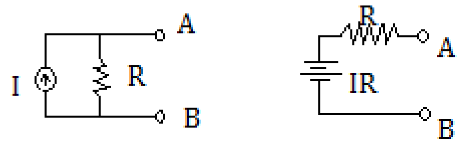
$$R_{bc} = \frac{R_a.R_b + R_b.R_c + R_c.R_a}{R_a}$$

$$R_{ca} = \frac{R_a.R_b + R_b.R_c + R_c.R_a}{R_b}$$

SOURCETRANSFORMATIONS

In the circuit analysis, a circuit with either voltage source or current sources is preferred. Sometimes a circuit may have both i.e. voltage source & current source. In that case it is convenient to transform voltage source to equivalent current source and current source to equivalent voltage source .





(Transformation of Voltage source to an equivalent current source)

(currentsourcetoanequivalent voltage source)

NODEANALYSIS&MESHANALYSIS

Two methods one Node analysis and the other mesh analysis are used to analyse a circuit depending on the arrangement and types of elements in the circuit. Nodal analysis is based on Kirchhoff's Current Law (KCL) and Mesh analysis is based on Kirchhoff's Voltage Law (KVL).

NODALANALYSIS

Letusconsideracircuitshowninfig2.2withfour nodes. A convenient way of defining voltagesfor any network is the set of node voltages.

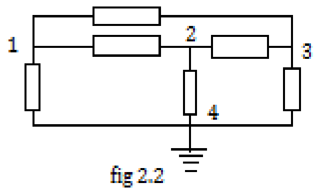


fig 2.2

One node i.e. 4 (generally the node at the bottom)ismarkedasreferencenodewithgroundandother nodes are associated with a voltage. The reference node also can be called as Ground Node. In fig 2.2, the voltages V_1, V_2, V_3 are called Node Voltages because they represent thepotentialdifferencesbetweenthenodes1,2&3andreferencenoderespectively. That is the voltage of each of the non-reference nodes with respect to thereference node is defined as a node voltage.

Considerthecircuitinfigure

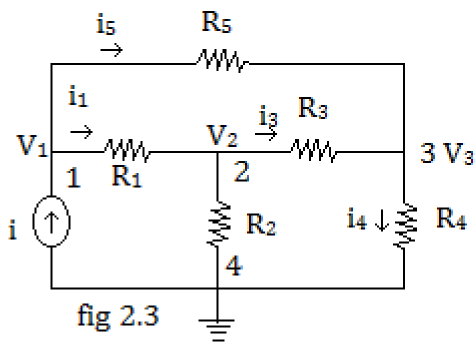


fig 2.3

$$i_1 = \frac{V_1 - V_2}{R_1} \quad , i_5 = \frac{V_1 - V_3}{R_5}$$

Now applying KCL at node 1, the sum of currents leaving is zero.

Therefore$i_1 + i_5 - i = 0$

$$i = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_3}{R_5} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(1)}$$

Similarlyatnode2 $-i_1 = \frac{V_2 - V_1}{R_1}, i_2 = \frac{V_2}{R_2}, i_3 = \frac{V_2 - V_3}{R_3}$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_3} + \frac{V_2}{R_2} = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_ eq.(2)}$$

AtNode3 Sumofcurrentsleavingare

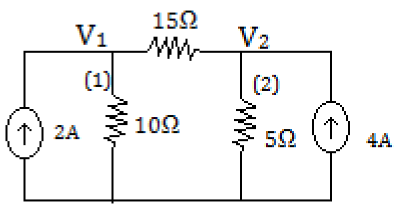
$-i_3 = \frac{V_3 - V_2}{R_3}, i_4 = \frac{V_3}{R_4}, -i_5 = \frac{V_3 - V_1}{R_5}$

$$\frac{V_3 - V_2}{R_3} + \frac{V_3}{R_4} + \frac{V_3 - V_1}{R_5} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(3)}$$

Alltheabovetheseequationcanbesolvedtodeterminetheindividual node voltages V_1, V_2 & V_3.

Example1

Find the node voltages V_1 and V_2 for the circuit at figure.



Solution At node 1 apply KCL sum of all the current leaving the node (1) is zero current leaving node 1 are $\frac{V_1}{10}$, $\frac{V_1 - V_2}{15}$ and -2A (2A is entering)

$$\frac{V_1}{10} + \frac{V_1 - V_2}{15} - 2 = 0$$

$$V_1\left(\frac{1}{10} + \frac{1}{15}\right) - \frac{V_2}{15} = 2$$

$$5V_1 - 2V_2 = 60 \quad \text{- eq.(1)}$$

SimilarlyAtnode2currentleavingare $\frac{V_2}{5}$, $\frac{V_2 - V_1}{15}$ and -4A

$$\frac{V_2}{5} + \frac{V_2 - V_1}{15} - 4 = 0$$

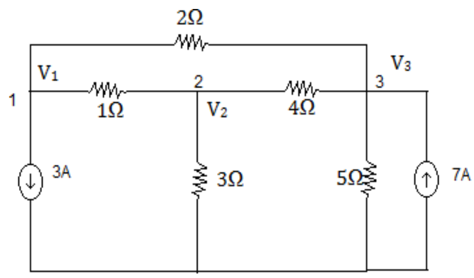
$$4V_2 - V_1 = 60 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(2)}$$

Solvingtheabovetwoequations(1)&(2)

We get $V_1 = 20V$, $V_2 = 20V$

Example2

FindV_1,V_2andV_3forthecircuit infigure.



Solution

Atnode1

$$V_1 - V_2 + \frac{V_1 - V_3}{2} + 3 = 0$$

$$3V_1 - 2V_2 - V_3 = -6 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(1)}$$

Atnode2

$$V_2 - V_1 + \frac{V_2}{3} + \frac{V_2 - V_3}{4} = 0$$

$$-12V_1 + 19V_2 - 3V_3 = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ eq.(2)}$$

Atnode3

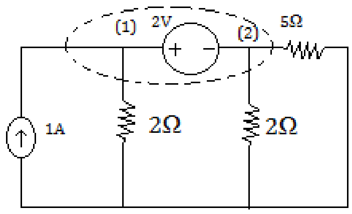
$$\frac{V3-V1}{2}+\frac{V3}{5}+\frac{V3-V2}{4}=7$$

$-10V_1-5V_2+19V_3= 140$ __________________________eq.(3)

BysolvingwegetV$_1$ =5.238V,V$_2$ =5.12V&V$_3$= 11.47V Find the

Example3

node voltage V_1& V_2



Solution

Towritenodeequation treatnode1and2 and the voltage source together as a Sort of Super node and apply KCL to both nodes at the same time.The super node is individual by dotted line.

ApplyingKCL,weget

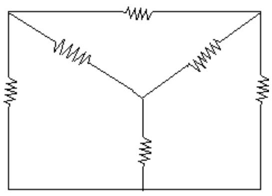
$$-1+\frac{V1}{2}+\frac{V2}{2}+\frac{V2}{5}=0$$ ____________________________eq.(1)

And from voltage source $V_1 - 2 = V_2$ ____________________________eq.(2)

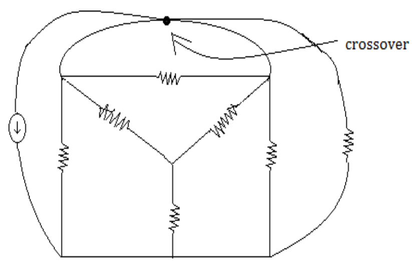
Now we can solve for $V_1$and V_2using both equations.

MESHANALYSIS

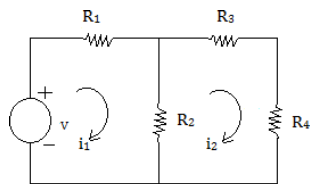
Mesh analysis is restricted to the category called Planar Circuit whereas nodal analysis can applied to any electrical circuits. A planer circuit is a circuit if the diagram of the circuit can be drawn on a plane surface without crossover. Example of planner and non-planar circuit are shown in fig (2.7).



Planer Circuit



Figuredepictsacircuitcomprisingtwomeshes.



They are

Mesh1:$V_5 \rightarrow R_1 \rightarrow R_2 \rightarrow V_3$

Mesh2:$R_3 \rightarrow R_4 \rightarrow R_2 \rightarrow R_3$

The two mesh currents are labeled as i_1 and $i_2$flowing in clockwise direction. Now we will apply KVL around each mesh.

Formesh1

$i_1R_1+(i_1-i_2)R_2=V_5$ ______________________________ eq.1

Formesh2

$i_2R_3+i_2R_4+R_2(i_2-i_1)=0$ ____________________eq.2

Eq.(1)&(2)canberewrittenas

$(R_1+R_2)i_1-R_2i_2=V_5$ _______________________________eq.3

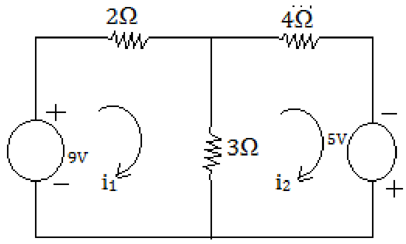
$-R_2i_1+(R_2+R_3+R_4)i_2=0$

Finallythetwoequationscanbeputinmatrixform

$$\begin{bmatrix} R1+R2 & -R2 \\ -R2 & R2+R3+R4 \end{bmatrix}\begin{bmatrix} i1 \\ i2 \end{bmatrix}=\begin{bmatrix} V5 \\ 0 \end{bmatrix}$$

Whichcanbesolvedfor $i_1$andi_2.

<u>Examples 4</u> findthemeshcurrent $i_1$and$i_2$for the circuit shown infigure.



For first mesh $2i_1+3(i_1-i_2) = 9$____________eq.1

$4i_2-5+3(i_2-i_1)=0$____________eq.2

Equationcanberewrittenas

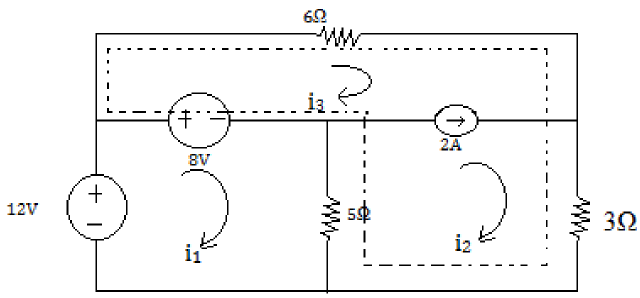
$5i_1-3i_2=9$ ___________eq.3

$-3i_1+7i_2=5$ ___________eq.4

Bysolving $i_1=3A, i_2=2A$

<u>Example5</u>

Determinethevoltagedropacross3Ωresisterusingmeshanalysisin figure.



<u>SUPERMESH</u>

When a current source is common to two meshes we use the concept of super mesh to analysis the circuit using mesh current method. A super mesh is a larger mesh created from two meshes that have a current source as common element. A super mesh encloses more than one mesh for each common current source between two meshes,thenumberofmeshesreducebyone,thusreadingthenumberofmesh

Solution toExample6

The2Acurrentsourceiscommontomesh2&3.Sowecreateasuper mesh as shown in dotted line.

Forsuper mesh

$6i_3+3i_2+5(i_2-i_1)-8=0$

$\Rightarrow -5i_1+8i_2+6i_3=8$ ________________________________ eq.1

Formesh1

$-12+8+5(i_1-i_2)=0$

$\Rightarrow 5i_1-5i_2=4$ ________________________________ eq.2

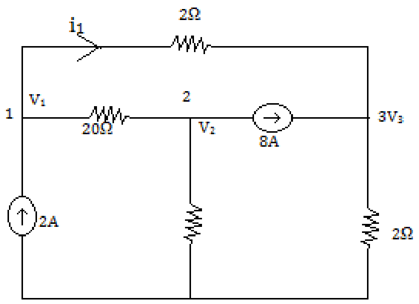
Fromcurrent source $i_2-i_3=2$

By solving we get $i_2=2.664$

Voltageacross3Ωresistor=2.66×3=8v.

Example7

UsenodeanalysistofindV$_1$,V$_2$,V$_3$&i$_1$



Solution

ApplyingKCLatnode1

We get

$\frac{V1-V2}{20}+\frac{V1-V3}{2}=2$ __________________________ eq.1

ApplyingKCLatnode2

$\frac{V2-V1}{20}+\frac{V2}{4}+8=0$ __________________________ eq.2

ApplyingKCLatnode3

$\frac{V3}{2}+\frac{V3-V1}{2}=8$ __________________________ eq.3

Bysolvingall theseequationswecangetV$_1$= 16v,V$_2$=-24v,V$_3$=16v,i$_1$= 0A

Examples8

FindthevoltageV$_2$usingmeshanalysis.

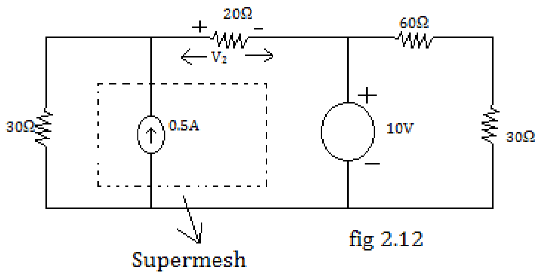


fig 2.12

Solution

ApplyingKVLforsupermesh

$30i_1+20(05+i_1)+10=0$

$\Rightarrow 50i_1=-20$

$\Rightarrow i_1=-\frac{2}{5}=-0.4A, V_2=20(i_1+0.5)$

$=20\times 0.1=2v$

SuperpositionTheorem

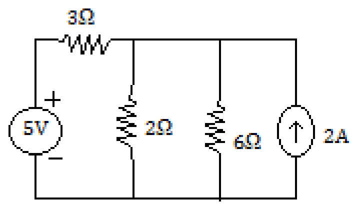
In a linear bilateral network containing two or more independent sources, the voltage across or current in any branch is algebraic sum of individual voltages or currents produced by each independent sources acting separately with all the independent sources set equal to zero.

Proceduretosolvethecircuitusingsuperposition theorem

1. Select only one source and replace all other sources with their internal resistance. If the source is an ideal current source replace it by open circuit. If the source is an ideal voltage source, replace it by short circuit.
2. Findthecurrentanditsdirectionthroughthedesired branch.
3. Addallthebranchcurrentstoobtaintheactualbranch current.

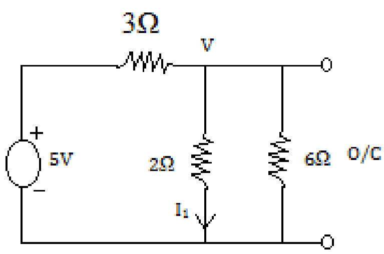
Examples9

Findthecurrentthrough2Ωregisterusing superpositiontheorem.



Solution

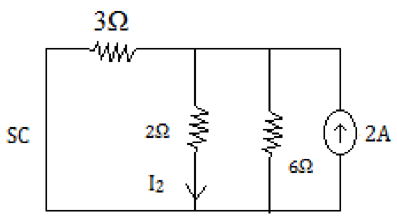
FirstwefindthecontributiontoIdueto5V sourcebyreplacing2Acurrentsourcewithopen-circuit. Applying KCL for the circuit in figure.

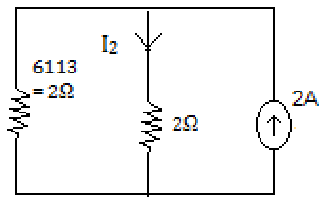


$$\frac{V-5}{3}+\frac{V}{2}+\frac{V}{6}=0$$

$$V=\frac{5}{3}v, I_1=\frac{5}{6}Amp$$

Next we findthe contributions I_2 due to 2A currentsource by replacing the voltage source by short-circuit.





$$I_2=2\times\frac{2}{4}=1Amp$$

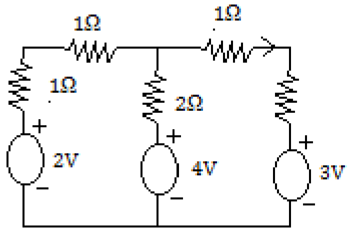
$$\text{Totalcurrentflowingthroughthe2Ωresistor}=I_1+I_2=1+\frac{5}{6}=\frac{11}{6}Amp$$

LimitationofSuper-positionTheorem

1. Notapplicabletothecircuitsconsistingofonlydependent sources.
2. Notapplicabletothecircuitsconsistingofnon-linearelements.
3. Not applicable for calculation of power, since power is potential is propositional to the sequence of current or voltage.
4. Notusefultothecircuitsconsistingoflessthantwoindependent sources.

Example10

FindcurrentIusingSuperpositiontheoremforthecircuitinthefigure.

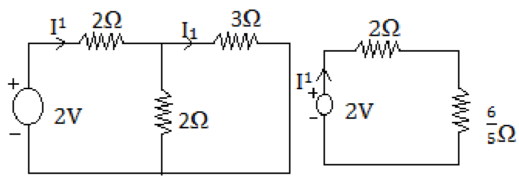


Solution:

Thecircuithasthreevoltagesources.FirstwefindthecontributiontoI_1 dueto2V.Thereforeshort-circuittheremainingtwovoltagesourcesasshowninfigure.

$$I^1 = \frac{2}{2+\frac{6}{5}} = \frac{10}{16} = \frac{5}{8}A$$

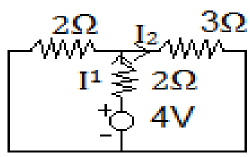
$$I_1 = \frac{5}{8} \times \frac{2}{5} = \frac{1}{4}A$$



When4Vactingasshowninfigure

$$I^1 = \frac{4}{2+\frac{6}{5}} = \frac{5}{4}A$$

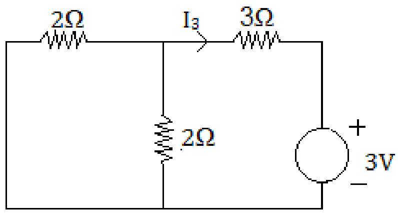
$$I_2 = \frac{5}{4} \times \frac{2}{5} = \frac{1}{2}A$$



When3Visactingaloneasshowninfigure $I_3 = -\frac{3}{4}$ Amp

Whenall thesourcesareacting together total current will be

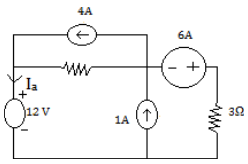
$$I = I_1 + I_2 + I_3 = \frac{1}{4} + \frac{1}{2} - \frac{3}{4} = \frac{1+2-3}{4} = 0 \text{Amp}$$

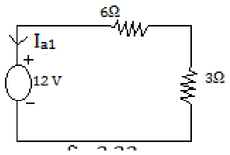


Example11

FindcurrentI_a

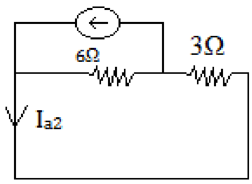
Solution:Letusassumethatonly12Visactingdoneandcurrent through it i_{a1}, open circuit 4A and 1A current source and short-circuit the 6V voltage source as in the figure.





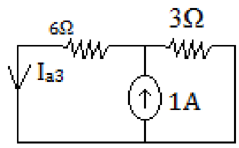
$$I_{a1} = -\frac{12}{9} = -\frac{4}{3} A$$

When4Acurrent sourceisactingaloneasshownin figure.



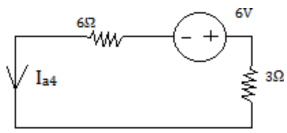
$$I_{a2} = \frac{4\times 6}{9} = \frac{24}{9} = \frac{8}{3} A$$

When 1Aisactingaloneasshown infigure.



$$I_{a3} = 1\times\frac{3}{9} = \frac{1}{3} A$$

When6Visactingaloneasinfigure



$$I_{a4} = \frac{6}{9} = -\frac{2}{3} A$$

Whenallthesourcesareactingtotalcurrentwillbe

$$I_a = I_{a1} + I_{a2} + I_{a3} + I_{a4}$$

$$= -\frac{4}{3} + \frac{8}{3} + \frac{1}{3} - \frac{2}{3} = \frac{-4+8+1-2}{3}$$

$$= \frac{3}{3}$$

= 1amp

I_a =1A

APPLICATIONOFSUPER-POSITIONTHEOREM

The super-position theorem is applicable for any linear circuit having time varying or time invariant elements. It is useful in circuit analysis for finding current & voltage when the circuit has a large number of independent sources.

LIMITATIONOFSUPER-POSITIONTHEOREM

1. Notapplicabletothecircuitsconsistingofdependent sources.
2. Notapplicabletothecircuitsconsistingofnonlinearelementslike diode, transistor etc.
3. Notapplicableforcalculationofpower.

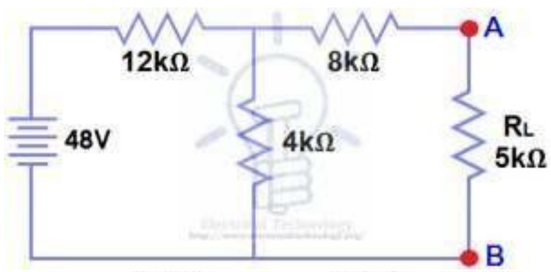
THEVENIN'STHEOREM

Thevenin's theorem states that any linear active two terminal network containing resistance and voltage sources or current sources can be replaced by a single voltage sources V_{th} in series with single resistance R_{th}. The Thevenin equivalent voltage V_{th} is the open circuit voltage at the network terminal and the Thevenin resistance R_{th} is the resistance between the network terminals when all the sources are replaced with their internal resistance.

Fig (a) shows a linear network containing resistance, voltage sources or current sources with output terminal AB using Thevenin's theorem the linear network can be replaced by single voltage source V_{th}in series with a single resistor R_{th}as shown in fig(b). Now any resistor can be corrected between the terminal AB andcurrent through it can be obtained easily.

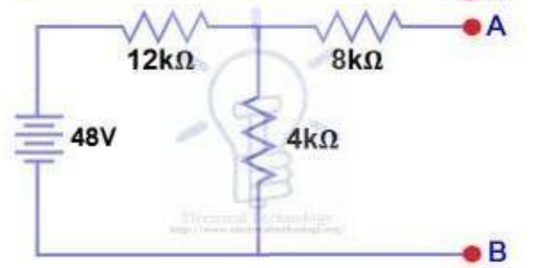
Proceduretofindthecurrentthroughabranchusing Thevenin'sTheorem.

1. Remove the branch through which current is to be found and mark the terminal AB.
2. CalculatetheopencircuitvoltageV$_{th}$betweentheterminalAB.
3. Replace the independent sources with their internal resistance. (if the internal resistances are zero, then voltage source should be short-circuited and current source should be open-circuited)
4. CalculateR$_{th}$betweentheterminalAB.
5. Correct thevenin's voltage sources in series with Thevenin resistance with output terminal AB.
6. CorrecttheremovedresistancebetweenABandfindthecurrentthroughit.



Example

FindV$_{TH}$,R$_{TH}$andtheloadcurrent flowingthroughandloadvoltageacrosstheload resistor in figure by using Thevenin's Theorem.



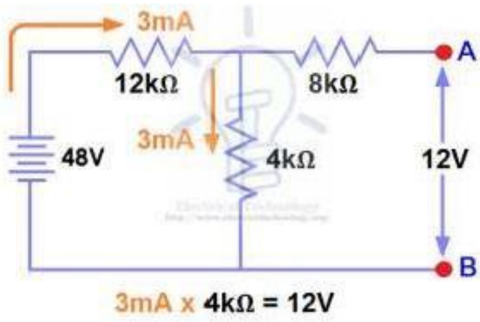
Solution

Step1

Openthe5kΩloadresistorfigure. Step 2

Calculate/measure theOpenCircuit Voltage.ThisistheTheveninVoltage(V$_{TH}$) figure. Wehave alreadyremoved theload resistor from figure1,so thecircuit becamean opencircuitasshowninfig 2.NowwehavetocalculatetheThevein'sVoltage.Since

3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open.

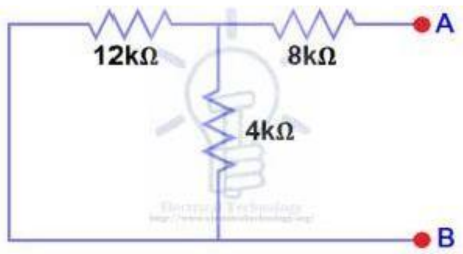


So12V(3mAx4kΩwill appearacrossthe 4kΩresistor.Wealsoknwthat currentisnot flowingthroughthe8kΩ resistorasitisopen circuit,butthe8kΩresistorisinparallelwith4k resistor.Sothesamevoltage(i.e.12V)willappear across the 8kΩ resistor a s 4kΩresistor.Therefore12VwillappearacrosstheAB terminals. So,

V_{TH}=12V

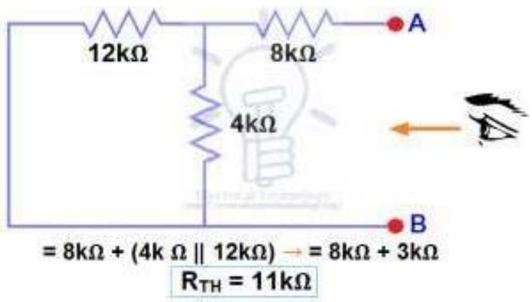
Step3

OpenCurrentSourcesandShort VoltageSourcesfigure.



Step4

Calculate/measuretheOpenCircuit Resistance.ThisistheThevninResistance (R_{TH})



WehaveReducedthe48VDCsource tozeroisequivalenttorepl a ceitwithashortinstep (3), as shown in figure () We can see that 8kΩ resistorisinserieswithap a rallelconnectionof4kΩ resistor and12kΩresistor.i.e.:

8kΩ+(4kΩ||12kΩ).....(||=inparallelwith) R_{TH} =

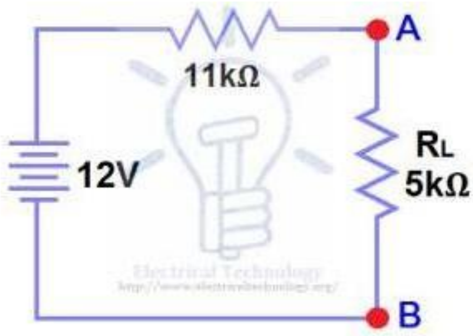
8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)] R_{TH}=

8kΩ + 3kΩ

R_{TH}=11kΩ

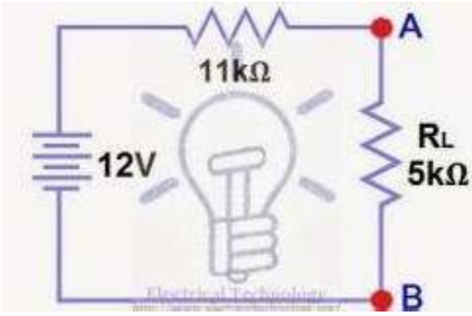
Step5

ConnecttheR$_{TH}$inserieswithVoltage SourceV$_{TH}$ andre-connecttheloadresistor.Thisis shown in figure i.e. Thevenin circuit with loadresistor.



Step6

Nowapplythelaststepi.e.calculatethe totalloadcurrent&loadvoltageasshowninfigure.



I_L $= V_{TH}/(R_{TH}+ R_L)$

$=12V / (11k\Omega+ 5k\Omega)\rightarrow$

$=12/16k\Omega$

I_L $=0.75mA$

And

V_L $= I_L x R_L$

V_L $=0.75mA x 5k\Omega$

V_L $=3.75V$

NORTON'STHEOREM

Norton's theorem states that any linear active two terminalnetwork contains resistance and voltage source or current source can be replaced by single current source or current source can be replaced by single current source $I_N$in parallel with a single resistanceR_N. The Norton's equivalent current $I_N$is the state circuit current through the terminals AB and resistance $R_N$is the resistance between the network terminalswhenallthe sourcesare replacedwithinternal resistances.

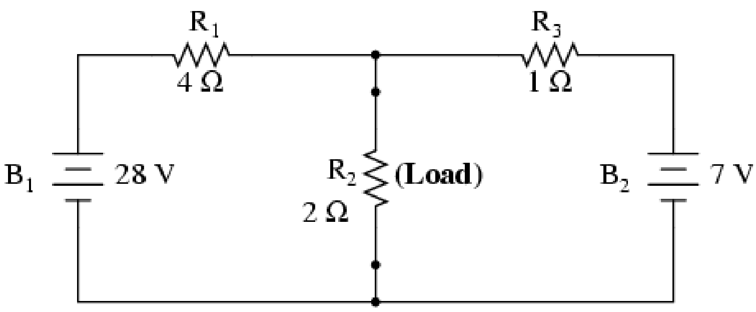
Proceduretofindthecurrent throughabranchusingNorton'stheorem.

1. Removethebranchthroughwhichcurrent istobefoundandmarkterminalAB.
2. Short-circuittheterminalABandfindcurrentthroughitanddenoteitasI_{SC}.
3. Replace the independent sources with their internal resis ances (if internal resistances are zero then voltage source should be short circuited and current sources should be open-circulated).
4. Calculate$R_N$betweentheterminalsAB.
5. Connect the short-circuit current (Norton's) I_n in parallel with R_N with output terminal AB.

CorrecttheremovedbranchbetweenterminalsABandfindcurrent.

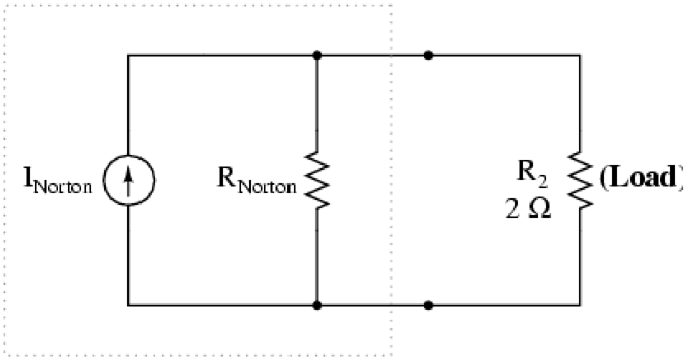
Example

Findthecurrentin*RL*usingNorton's Theorem

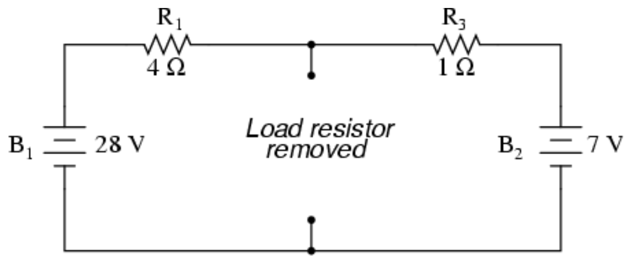


AfterNorton conversion...

Norton Equivalent Circuit

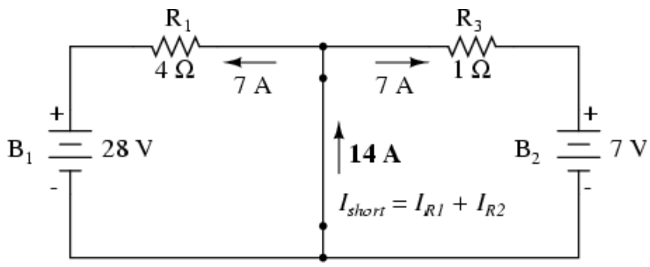


Remember that a *currentsource* isacomponent whose job is to provide a constant amount of current, outputting as much or as little voltage necessary to maintain that constant current.



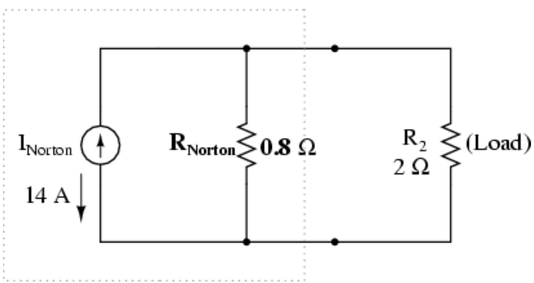
As with Thevenin's Theorem, everything in the original circuit except the load resistance has been reduced to an equivalent circuit thatissimplertoanalyze.Alsosimilar to Thevenin's Theorem are the steps usedinNorton'sTheoremto calculatetheNortonsourcecurrent(I_{Norton})andNortonresistance(R_{Norton}).

Asbefore,thefirststepistoidentifytheloadresistanceandremoveit from the original circuit.



Then, to find the Norton current (for the current source in the Norton equivalent circuit),placeadirectwire (short) connection betweenthe load points and determinetheresultantcurrent. Note that this step is exactly oppositetherespectivestepinThevenin'sTheorem,wherewereplacedtheloadresistor with a break (open circuit).

Norton Equivalent Circuit



With zero voltage dropped between the load resis or connectionpoints, the current through R_1 is strictly a function of B_1's voltage and R_1's resistance: 7 amps (I=E/R). Likewise, the current through R_3 is now strictly a function of B_2's voltageand R_3'sresistance:7 amps(I=E/R).

The total current through the short between the load connection points is the sum of these two currents: 7 amps + 7 amps = 14 amps. This figure of 14 amps becomes the Norton source current (I_{Norton}) in our equivalent circuit.

Currentthroughloadof2Ωresistor =14X .8/2.8 =4 Amp.

<u>MaximumPowerTransferTheorem</u>

In a linear bilateral network containing an independent voltage source in series with resistance R_S delivers maximum power to the load resistance $R_L$when R_L=R_S

Letusconsideracircuitshowninfig(a)

CurrentI=$\frac{Vs}{Rs+Rl}$

Power delivered totheload $P_L=IR_L^2=(\frac{s_2}{Rs+Rl})R_L$

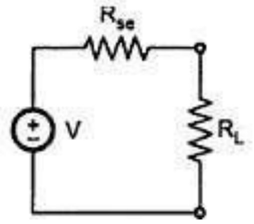
TofindthevalueofR$_L$foroptimumpowertransferdifferentiateP$_L$withrespecttoR$_L$and equal to 2nd

$$\frac{dPl}{dRl}=V^2\left[\frac{(Rs+Rl)2-2Rl(Rs+Rl)}{(Rs+Rl)2}\right]=_S0$$

$\Rightarrow(R_S+R_L)^X = 2\,R_L(R_S/+R_L)$

$\Rightarrow R_S+R_L = 2R_L$

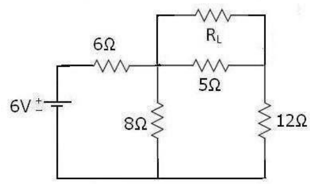
$\Rightarrow \boxed{R_S=R_L}$



Maximum power will be $=(V_S/2R_L)^2\times R_L$

$=V_S^2/4R_L$<u>Example</u>

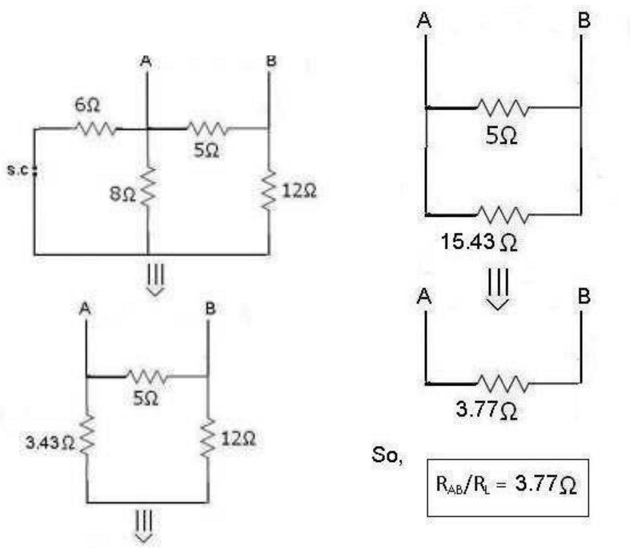
Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



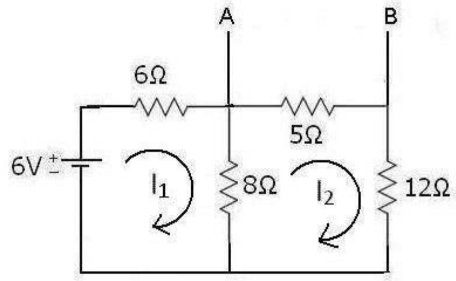
<u>Solution</u>

For the above network,we are going to find-out the value of unknown resistance called "R_L". In previous post, I already show that when power is maximum through load-resistance is equals to the equivalent resistance between two ends of load-resistance after removing.

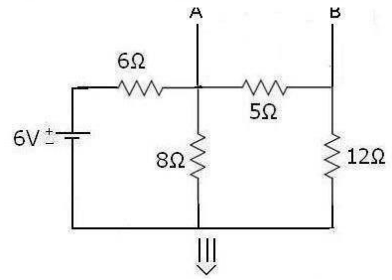
So, for finding load-resistance R_L. We have to find-out the equivalent resistance like that for this circuit.



Now, For finding Maximum Power through load-resistance we have to find-out the value of $V_{o.c}$. Here, $V_{o.c}$isknownasvoltagebetween open circuits.So,stepsare



ForthiscircuitusingMesh-analysis.We get



ApplyingKvlinloop1st:-

$6-6I_1-8I_1+8I_2=0$

$-14I_1+8I_2=-6$ .... (1)

Again, Applying Kvl in loop 2nd:-

$-8I_2-5I_2-12I_2+8I_1=0$

$8I_1-25I_2=0$ .... (2)

On solving,eqn (1) & eqn (2), We get

$I_1=0.524A$

$I_2=0.167A$

Now,FromthecircuitV$_{o.c}$is

$V_A-5I_2-V_B=0$

$V_{o.c}/V_{AB}=5I_2=5X0.167=0.835v$

So,themaximumpowerthroughtheR$_L$isgivenby:-

$$P_{max}=\frac{V_{o.c}^{2}}{4R_L}$$

$$P_{max}=\frac{0.835^{2}}{4X\ 3.77}$$

$$\boxed{P_{max}=0.046\text{ watt}}$$

Milliman'sTheorem

This theorem states that Any number of current sources in parallel may be replaced by a single current source whose current is the algebraic sum of individual source currents and source resistance is the parallel combination of individual source resistance.

The alternative statement of Milliman's theorem is Any number of voltage sourceV_1, V_2, V_3,----------$V_n$havingsourceresistanceR_1, R_2, R_3-- R_n respectively connected in parallel may be replaced y a single voltage source V_n and resistance $R_n$where

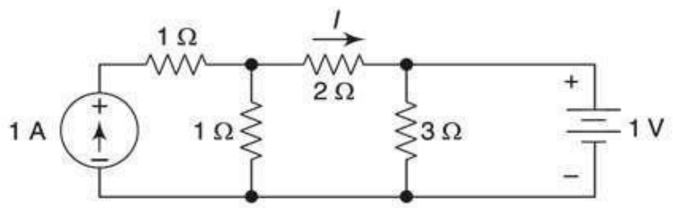
$$V_n = \frac{1}{G1+G2\pm---\pm Gn} \qquad \text{where} G_1 = \frac{1}{R1}, G_2 = \frac{1}{R2} \text{etc.}$$

The above two statements are identical because a voltage source can be connected in to current source and vice-versa.

ReciprocityTheorem

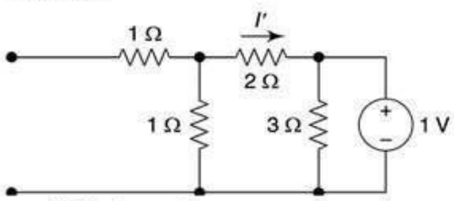
The Reciprocity theorem states that if the source voltage and zero resistance ammeters are integrated, the magnitude of the current through the ammeter will be the same. In lead the principle states that in a linear positive network, supply voltage *V* and current *I* are mutually transferable. The ratio of V and I is called thetransfer resistance.

Problem1

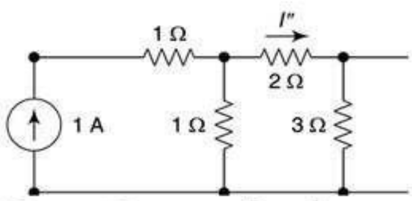


Find the current *I* in the circuit shown in the figure. using superposition theorem.

Solution



(i) Voltage Source acting alone



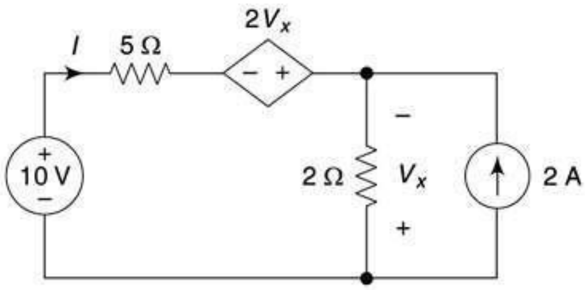
(ii) Current Source acting alone

For Figure (i) $I' = -\frac{1}{3}$ A

For Figure (ii) $I'' = 1 \times \frac{1}{1+2} = \frac{1}{3}$ A

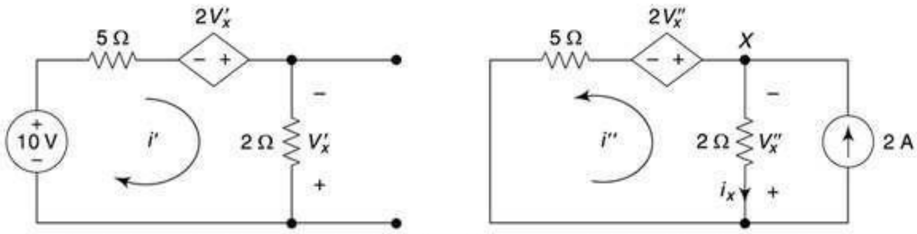
By superposition, $I = (I' + I'') = -\frac{1}{3} + \frac{1}{3} = 0$

Problem2



Use superposition theorem on the circuit shown in figure to find I.

Solution



(i) Voltage source acting alone (ii) Current source acting alone

For Fig. (i), by KVL, $5i' - 2vx' + 2i' = 10$ with $v_x' = -2i'$

$\Rightarrow 7i' + 4i' = 10$

$\Rightarrow i' = 10/11$ A

For Fig (ii), by KCL at node (x)

$2 = i_x + i'' = -\frac{v_x''}{2} + i''$ (i)

But loop analysis in the left loop gives,

$5i'' + 3v_x'' = 0$

or $i'' = -\frac{3}{5}v_x''$

From (i), $2 = -\frac{v_x''}{2} - \frac{3}{5}v_x''$

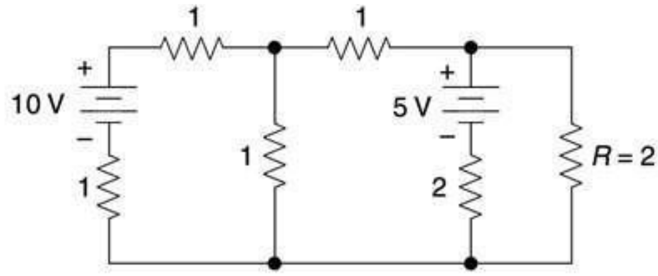
$\Rightarrow v_x'' = -\frac{20}{11}$

$\therefore i'' = -\frac{3}{5} \times \left(-\frac{20}{11}\right) = \frac{12}{11}$ A

So, by superposition theorem total current

$$I = (i' - i'') = \left(\frac{10}{11} - \frac{12}{11}\right) = -\frac{2}{11} \text{ A}$$

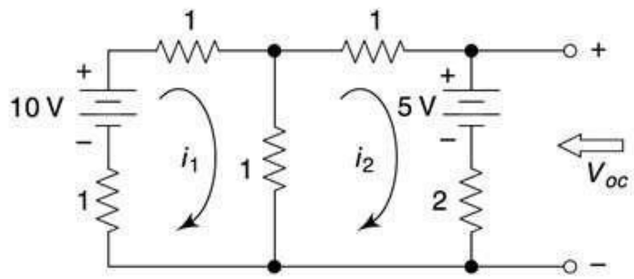
Problem3



Draw the Thevenin's equivalent of the circuit in figure and find the load current, *i*. All values are in ohm.

Solution

Open circuiting the terminals,



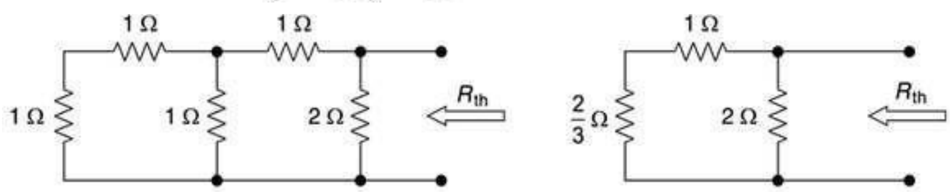
By KVL for two meshes,

$$3i_1 - i_2 = 10$$

and $-i_1 + 4i_2 = -5$

Solving, $i_1 = 5/11$ and $i_2 = -5/11$

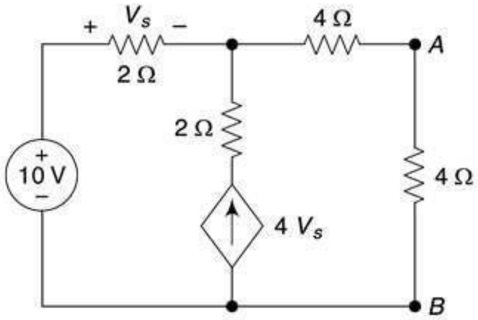
$$\therefore V_{oc} = (5 + 2i_2) = \left(5 - \frac{10}{11}\right) = \frac{45}{11}\text{ V}$$



Equivalent resistance, $R_{th} = \dfrac{\frac{5}{3} \times 2}{5/3 + 2} = \dfrac{10}{11}\ \Omega$

So, the load current is, $i = \dfrac{V_{oc}}{R_{th} + 2} = \dfrac{45/11}{10/11 + 2} = \dfrac{45}{32} = 1.40625\text{ A}$

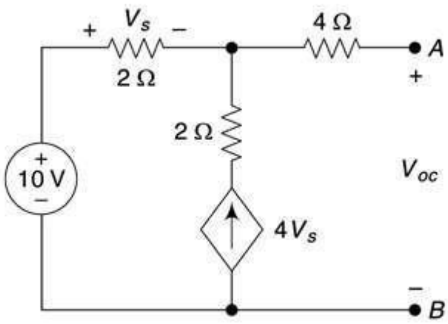
Problem4

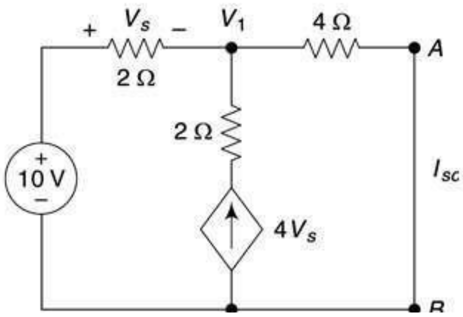


Find Thevenin's equivalent about *AB* for the circuit shown in figure.

Solution

Open-circuiting The 4 Ω resistor, by KCL,





$$\frac{V_{oc}-10}{2} = 4v_s = 4(10 - V_{oc})$$

$$\Rightarrow V_{oc} = 10 \text{ V}$$

Short-circuiting the terminals *AB*, by KCL,

$$\frac{V_1-10}{2} + \frac{V_1}{4} = 4v_s = 4(10 - V_1)$$

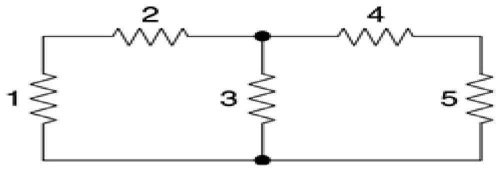
$$V_1 = \frac{180}{19} = 9.47 \text{ V}$$

$$\therefore\ I_{sc} = \frac{9.47}{4} = 2.368 \text{ A}$$

$$\therefore\ R_{th} = \frac{V_{th}}{I_{sc}} = 4.22\ \Omega$$

Problem5

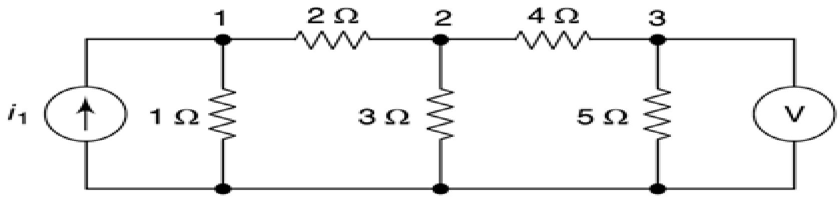
Verify the Reciprocity Theorem for the network shown in the figure using current source and a voltmeter. All the values are in ohm.



Solution

Using a current source and a voltmeter,

Let, e_1, e_2 be node voltages, v_1 be the voltmeter reading.



By KCL,

At node (1) $\Rightarrow$ $3e_1 - e_2 - 2i_1 = 0$ (i)

At node (2) $\Rightarrow$ $-6e_1 + 13e_2 - 3v_1 = 0$ (ii)

At node (3) $9v_1 = 5e_2$ (iii)

From (ii) $\Rightarrow$ $-6e_1 + 13 \times \frac{9}{5} v_1 - 3v_1 = 0$

$\Rightarrow -6e_1 + \left(\frac{117}{5} - 3\right) v_1 = 0$

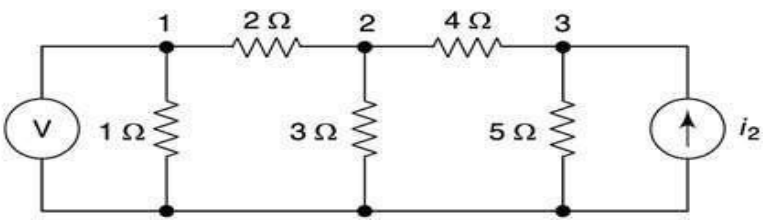
$\Rightarrow 6e_1 + \frac{102}{5} v_1 \Rightarrow e_1 = \frac{17}{5} v_1$

From (i) $\Rightarrow$ $3 \times \frac{17}{5} v_1 - \frac{9}{5} v_1 = 2i$

$\Rightarrow \left(\frac{i_1}{v_1}\right) = \left(\frac{21}{5}\right)$ (A)

Interchanging the positions of the current source and the voltmeter,

Now, let v_2 be the voltmeter reading



By KCL,

At node (1) $\Rightarrow$ $3v_2 = e_2$ (iv)

At node (2) $\Rightarrow$ $-6v_2 + 13e_2 - 3e_3 = 0$

$\Rightarrow$ $-6v_2 + 13 \times 3v_2 - 3e_3 = 0$

$\Rightarrow$ $e_3 = 11v_2$ (v)

At node (3) $\Rightarrow$ $5e_3 - 5e_2 + 4e_3 - 20i_2 = 0$

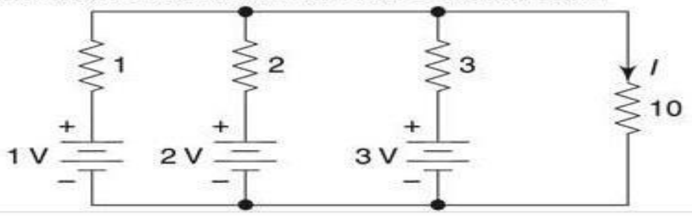
$\Rightarrow$ $20i_2 = 9e_3 - 5e_2 = 9 \times 11v_2 - 5 \times 3v_2 = 84v_2$

$\Rightarrow$ $\left(\frac{i_2}{v_2}\right) = \left(\frac{21}{5}\right)$ (B)

From equations (A) and (B), Reciprocity theorem is proved.

Problem6

Find the load current using Millman's theorem. All values are in ohm.



Solution

Here, $E_1 = 1$ V, $E_2 = 2$ V, $E_3 = 3$ V

$Z_1 = 1\ \Omega$, $Z_2 = 2\ \Omega$, $Z_3 = 3\ \Omega$

$\therefore$ $Y_1 = 1\ \mho$, $Y_2 = 0.5\ \mho$, $Y_3 = \frac{1}{3}\ \mho$

By Millman's theorem, the equivalent circuit is shown.

$$\therefore E = \frac{\sum_{i=1}^{3} E_i Y_i}{\sum_{i=1}^{3} Y_i} = \frac{1 \times 1 + 2 \times 0.5 + 3 \times \frac{1}{3}}{1 + 0.5 + \frac{1}{3}} = \frac{3}{\frac{11}{6}} = \frac{18}{11}\ \text{V}$$

and $$Z = \frac{1}{\sum_{i=1}^{3} Y_i} = \frac{6}{11}\ \Omega$$

$$\therefore I = \frac{E}{Z + 10} = \frac{\frac{18}{11}}{\frac{6}{11} + 10} = \frac{18}{116} = \frac{9}{58}\ \text{A}$$

CHAPTER-3

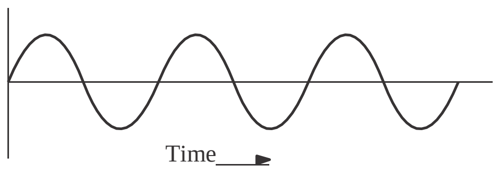
AC FUNDAMENTAL & AC

CIRCUITWHATIS ALTERNATINGCURRENT (A.C.)

Alternating current is the current which constantly changes in amplitude, and reverses direction at regular intervals. We know that direct current flows only in one direction, and that the amplitude of current is determined by the number of electrons flowing past a point in a circuit in one second. If, for example, a coulomb of electrons moves past a point in a wire in one second and all of the electrons are moving in the same direction, the amplitude of direct current in the wire is one ampere. Similarly,ifhalfacoulombofelectronsmovesin onedirectionpastapointinthewirein half a second, then reverses direction and moves past the same point in the opposite direction during the next half-second, a total of one coulomb of electrons passes the point in one second. The amplitude of the alternating current is one ampere.

PROPERTIESOFALTERNATINGCURRENT

An A.C. source of electrical power changes constantly in amplitude and the + changesaresoregularAlternatingvol tageand 0 current have a number of properties associated – with any such waveform. These basic properties include the following list:



Frequency

One of the most important properties of any regular waveform identifies the number of complete cycles it goes through in a fixed period of time. For standard measurements, the period of time is one second, so the frequency of the wave is commonly measured in cycles per second (cycles/sec) and, in normal usage, is expressed in units of Hertz (Hz). It is represented in mathematical equations by the letter 'f'.

Period

Sometimes we need to know the amount of time required to complete one cycle of the waveform, rather than the number of cycles per second of time. This is logically the reciprocal of frequency

Wavelength

Because an A.C. wave moves physically as well as changing in time, sometimes we need to know how far it moves in one cycle of the wave, rather than how long that cycle takes to complete. This of course depends on how fast the wave is moving as well. The Greek letter (lambda) is used to represent wavelength in mathematical expressions. And, $\lambda = c/f$. As shown in the figure to the above, wavelength can be measured from any part of one cycle to the equivalent point in the nextcycle. Wavelength is very similar to period as discussed above, except that wavelength is measured in distance per cycle while period is measured in time per cycle.

Amplitude

Mathematically,theamplitudeofasinewaveisthevalueofthatsinewave at its peak. This is the maximum value, positive or negative, that it can attain. However, when we speak of an A.C. power system, it is more useful to refer to the effective voltage or current.

THESINEWAVE

In discussing alternating current and voltage, you will often find it necessary to express the current and voltage in terms of maximum or peak values, peak-to-peak values, effective values, average values, orinstantaneousvalues.Eachofthesevalues hasadifferentmeaningandisusedtodescribe adifferentamountofcurrentorvoltage.

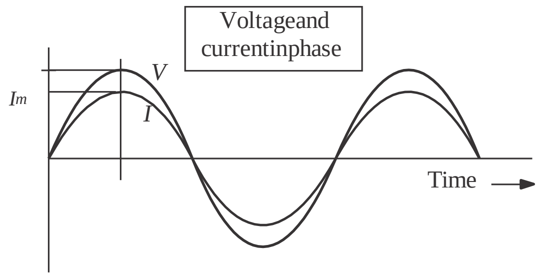


Fig.1.6

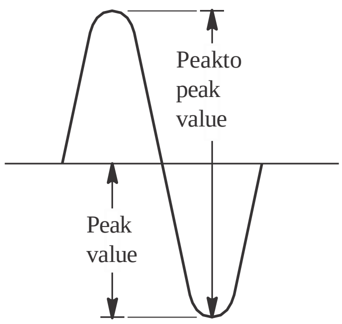
PeakValue[Ip]

Refertofigure,itisthemaximum valueofvoltage[V_p]orCurrent[I_p].Thepeak valueappliestobothpositiveandnegativevalues of the cycle.

Peak-Peakvalue[Ip-p]

During each complete cycle of ac there are always two maximum or peak values, one for the positive half-cycle and the other for the negative half-cycle. The difference between the peak positive value and the peak negative value is called thepeak-to-peak value of the sine wave. This value is twice the maximum or peak value of the sine wave and is sometimes used for measurement of ac voltages.

Note the difference between peak and peak to-peak values in the figure. Usually alternating voltage and current are expressed in effective values rather than in peak-to-peak values.



INSTANTANEOUSVALUE

The instantaneous value of an alternating voltage or current is the value of voltage or current at one particular instant. The value may be zero if the particular instant is the time inthe cycle atwhich thepolarity of the voltage is changing.It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing.

There are actually an infinite number of instantaneous valuesbetween zero and the peak value.

AVERAGEVALUE

The average value of an alternating current or voltage is the average of all the instantaneous values during one alternation. Since the voltage increases from zero to peak value and decreases back to zero during one alternation, the average value must be some value between those two limits.

The average value of A.C. is the average over one complete cycle and is clearly zero, because there are alternately equal positive and negative half cycles.

Averagevoltage= $\frac{2}{\pi} \times peakvalue$

ROOTMEANSQUAREVALUE

Circuit currents and voltage in A.C. circuits are generally stated as root-mean-square or rms values rather than by quoting the maximum values. The root-mean-square for a current is defined as the value of steady state current which when flowing through a resistor for a given time produces the same amount of hit as generated by the alternating current when passed through the same resistor for the same time.

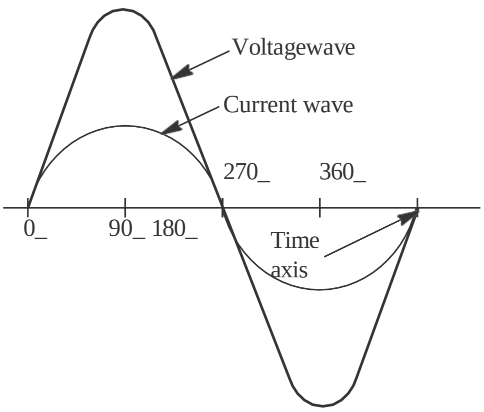
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

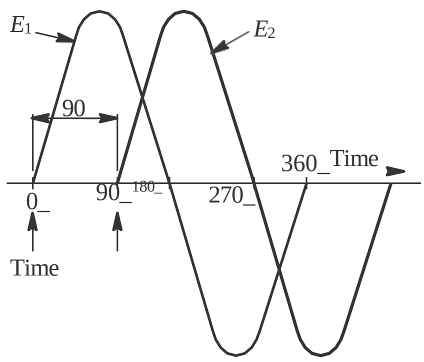
FormFactor= $\frac{V_{rms}}{V_{ave}} = 1.11$

ItistheratioofRMSvaluetoaveragevaleofvoltageorcurrent.

SINEWAVESINPHASE

When a sine wave of voltage is applied to a pure resistance, the resulting current is also a sine wave. This follows Ohm's law which states that current is directly proportional to the applied voltage. To be in phase, the two sine waves must go through their maximum and minimum points at the same time and in the same direction as shown in the figure.





SineWavesOutof Phase

Figure shows voltage wave $E1$ which is considered to start at 0° (time one). As voltage wave $E1$ reaches its positive peak, voltage wave $E2$ starts its rise (time two). Since these voltage waves do not go through their maximum and minimum points at the same instant of time, a phase difference exists between the two waves. The two waves are said to be out of phase. For the two waves in figure, the phase difference is 90°.

PHASORS

In an a.c. circuit, the e.m.f. or current vary sinusoidally wih time and may be mathematically represented as

$$E = E0\sin\omega t$$

and $$I = I0\sin(\omega t \pm \theta)$$

Whereθisthephaseanglebetweenalternating e.m.f.andcurrent.

Displacement of S.H.M. also varies sinusoidally with time *i.e.*

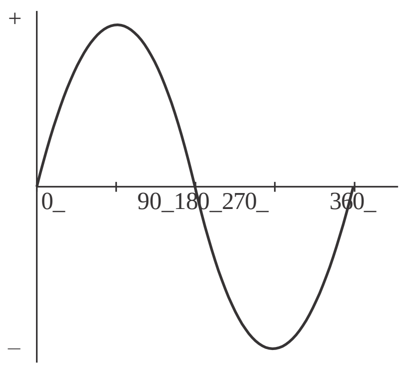
$$Y = A\sin\omega t$$

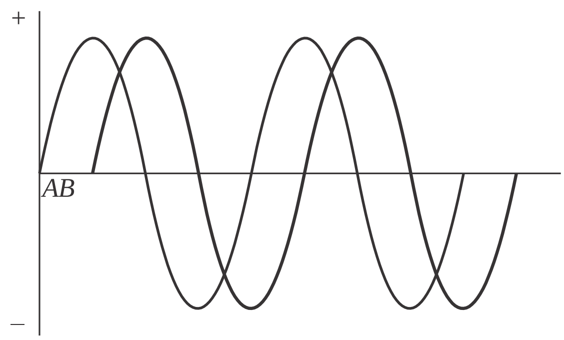
And its instantaneous value is equal to the projection of the amplitude A on$Y$-axis.Therefore,instantaneousvaluesofalternatinge.m.f.(E) andcurrent(I)maybe considered as the projections of e.m.f. amplitude ($E0$) and current amplitude ($I0$) respectively. The quantities, such as alternating e.m.f. and alternating current are called phasor. Thusaphasor isaquantitywhich variessinusoidally with timeand represented as the projection of rotating vector.

PHASORDIAGRAM

Thegenerator at thepowerstation which producesour A.C.mainsrotates through 360 degrees to produce one cycle of the sine wave form which makes up the supply.

Inthenextdiagramthereаretwo sinewaves.
Theyareoutofphasebecausetheydonotstart fromzeroatthesametime. To be in phase they must start at the same time.
Thewaveform*A*startsbefore*B* andisLEADINGby90 degrees.

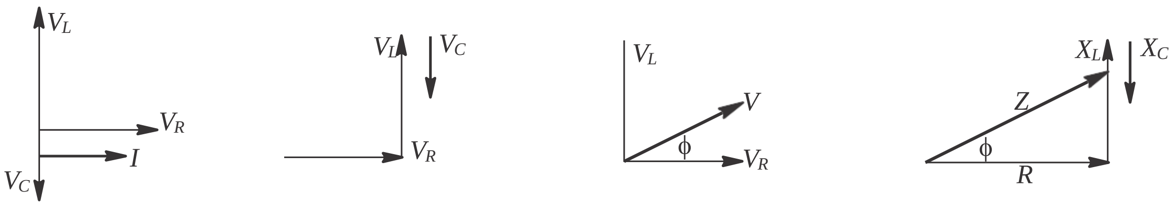




Waveform *B*isLAGGING *A*by90degrees.

Thenextlefthanddiagram,knownasaPHASORDIAGRAM,showsthisinanother way.

It is sometimes helpful to treat the phase as if it defines a vector in aplane. The usual reference for zero phase is taken to be the positive *x*-axis and is associated with the resistor since the voltage and current associated with the resistor are in phase. The length of the phasor is proportional to the magnitude of the quantity represented,and its angle represents its phase relative to that of the current through the resistor. The phasor diagram for the RLC series circuit shows the main features.



Notethatthephaseangle,thedifference inphasebetweenthevoltageand the current in an A.C. circuit, is the phase angle associated with the impedance *Z* of the circuit.

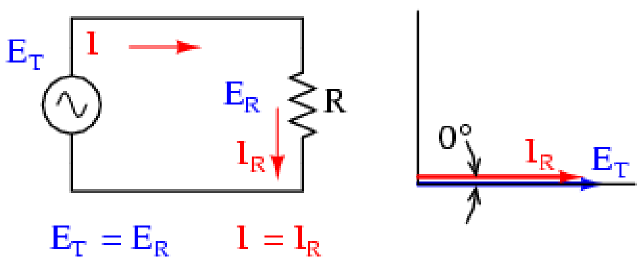
ACSERIESCIRCUIT

RESISTANCEACCIRCUIT

AresistanceRconnectedtoanacsourceisshown.Itsvoltagecanbe writtenas

$$e_t = E_{tm}\sin wt \quad i = I_m \sin wt$$

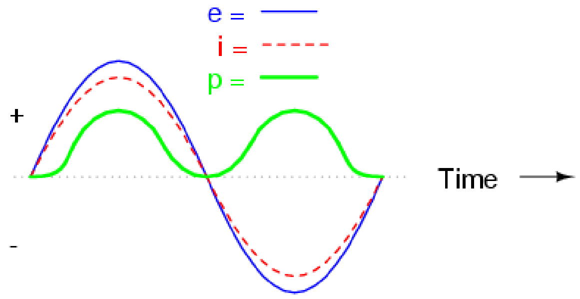
$$i = \frac{E_{tm}}{R}\sin wt = I_m \sin wt$$



The above two equations depict that voltage and current in resistive network are in phase. Figure shows the voltage and current waveform and phasor diagram.

POWERINRESISTIVENETWORK

The instantaneous power curve is plotted in figure it is seen that the power curve is always positive in case of resistive network and equal to

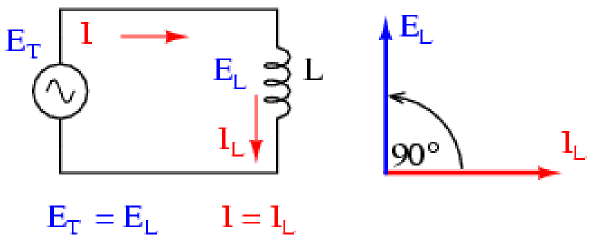


$$p=e\times i=E_tI\sin^2 wt=E_{tm}I_{mm}\left(\frac{1-\cos 2wt}{2}\right)=\frac{E_{tm}I_m}{2}-\frac{E_{tm}I_m}{2}\times\cos 2wt$$

Theabovepowerequationshowsthatthepowerhastwocomponents,oneis constanti.e. $\frac{E_{tm}I_m}{2}$&anaccomponent $\frac{E_{tm}I_m}{2}\cos 2wt$.Theaveragevalueofaccomponent inonecycleiszero.ThereforeAveragepower$p=\frac{E_{tm}I_m}{2}=\frac{E_{tm}}{\sqrt{2}}\times\frac{I_m}{\sqrt{2}}=E_tI$

InductanceACCircuit

Figure shows an inductance Lconnectedtoanacsupplywhichvoltage isgivenby $v=E_{Tm}\sin wt, i=I_m\sin\left(wt-\frac{\pi}{2}\right)$



Theaboveequationshows thatcurrentlagstheappliedvoltageby90°Where$I_m=\frac{E_{Tm}}{wL}$,thequantitywLcontrolsthe currentinductorandthisquantitywLisknownasinductivereactancedenotedX_L .Hence $X_L=wL$

POWERININDUCTIVENETWORK

Theinstantaneouspowerinapurelyinductivenetworkis

$$p=e_T\times i=E_{Tm}\sin wt\times I_m\sin\left(wt-\frac{\pi}{2}\right)$$

$$=-E_{Tm}I_m\sin wt.\cos wt$$

$$=\frac{-E_{Tm}I_m\sin 2wt}{2}$$

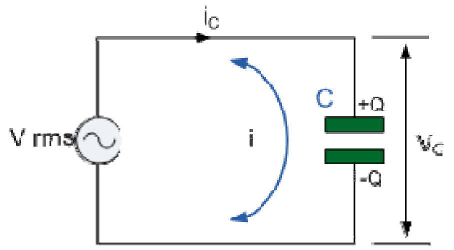
Theaveragepowerinapureinductorduringacycleiszero.

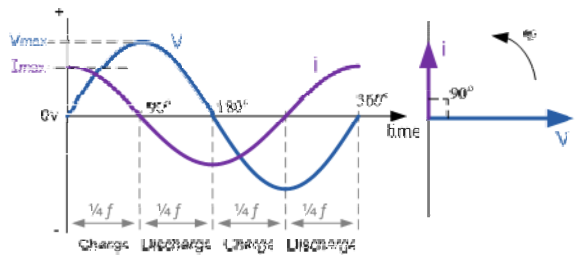
CAPACITANCE IN AC CIRCUIT

Figure shows a capacitor C connected to an ac source equation of voltage & current are given below

$$v = V_m \sin wt, \; I_m \sin\left(wt + \frac{\pi}{2}\right)$$

Equation shows that current leads voltage by 90° and $I_m = \dfrac{V_m}{\frac{1}{wC}}$ Where $\frac{1}{wC}$ is known as capacitive reactance denoted as X_C. Its unit is ohm.





POWER IN CAPACITIVE NETWORK

The instantaneous power in a purely capacitive network is

$$p = v \times i = V_m \sin wt \times I_m \sin\left(wt + \frac{\pi}{2}\right)$$

$$= \frac{V_m I_m \sin 2wt}{2}$$

The average power in a pure capacitive network is zero.

SERIES RL NETWORK

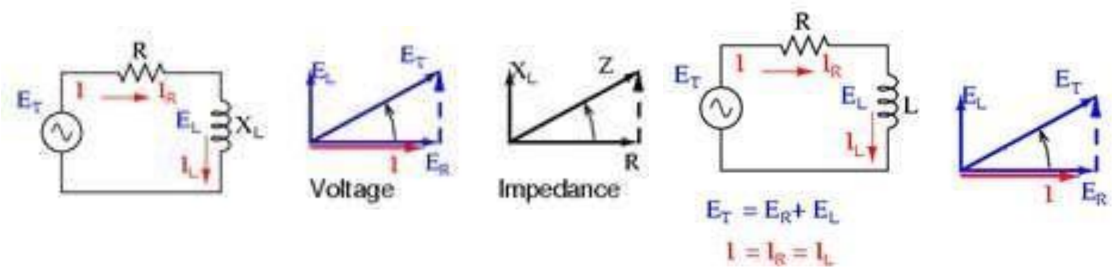
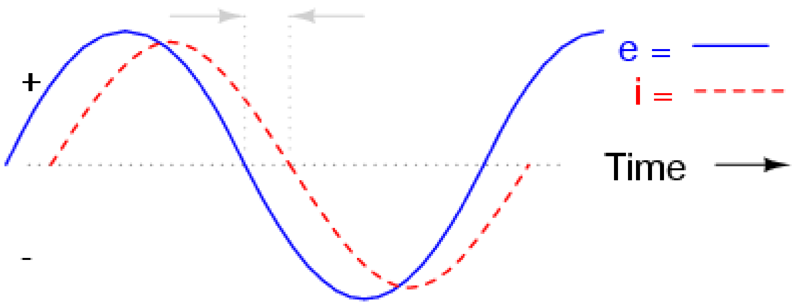


Figure shows a resistor (R) and inductor (L) series network with its phasor diagram and impedance diagram. As discussed earlier E_R is in phase with I and E_L leads I by 90°.



$$E_T = E_L + E_R = I(R + jX_L)$$

$$\text{Hence} \Rightarrow IZ = I(R + jX_L)$$

$$\Rightarrow Z = (R + jX_L) = R + jwL$$

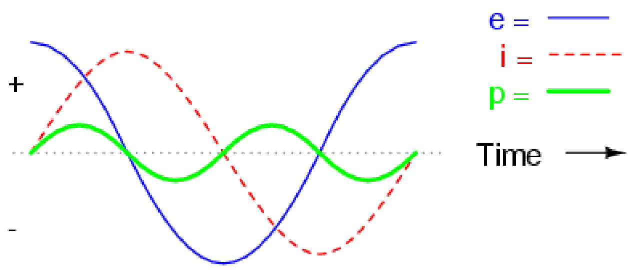
WheremagnitudeofZ= $\sqrt{R^2 + X_L^2}$

Thequantities$R$,X_L,Zareshownintheimpedence diagram.

<u>POWERINSERIESRLNETWORK</u>

The average power in RL series circuit is $p = \frac{E_{Tm} I_m}{2} \cos\theta = \frac{E_{Tm}}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\theta = E_T I \cos\theta$

$E_T I\cos\theta$ isknownasactivepower. $p = E_T I\cos\theta = E_T I \frac{R}{Z} = \frac{E_T}{Z} . IR = I^2 R \because I = \frac{E_T}{Z}$



Thus the active power in ac circuit represents the power dissipated across resistance. It is measured in watt. The product of RMS voltage & current i.e. VI is known as apparent power and measured in volt ampere. The ration of active power to apparent power equalstocosθwhereθisthephase anglebetween V&I.Thetermcosθiscalled

power factor of the circuit. The power factor is zero in case of pure inductiveor capacitive network. The power factor of a circuit may be either leading or lagging. A leading power factor means that the current in the circuit leads the voltage and lagging power factor means the current lags the voltage. The power factor of a circuit is the ratio of resistance to impedence .

Theinstantaneouspoweracrossinductorofcapacitor isknown asreactive power. That is$Q = I^2X = I^2_L wL = I^2Z \sin\theta = EI \sin\theta$

Thereactivepowerdoes notcontributeanything to thenetenergytransfer from source to load. Yet it constitutes a loading of the equipment.

The apparent power VI, active power VIcosθand reactive power VIsinθis also applicable in this case too. Current in RC circuit leads the apply voltage and therefore the power factor is leading.

<u>SERIESRCNETWORK</u>

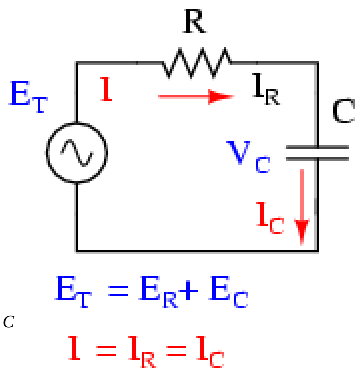
Figure shows a seriesRCnetwork

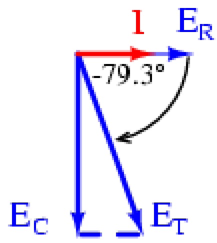
$$E_T = RI - j\frac{I}{wC} = I\left(R - \frac{j}{wC}\right)$$

$$\Rightarrow IZ = I\left(R - \frac{j}{wC}\right) \Rightarrow Z = R - \frac{j}{wC} = R - jX_C$$

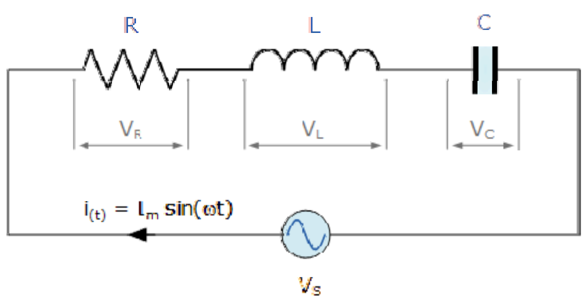
$$X_C = \text{Capacitive Reactance} = \frac{1}{wC}$$

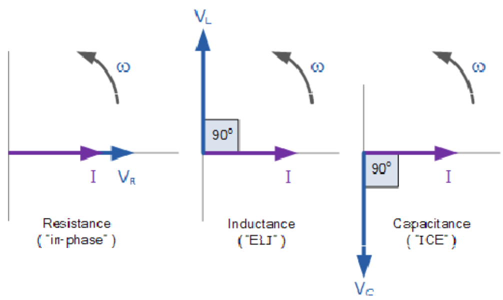
$$\text{magnitude of } Z = \sqrt{R^2 + \frac{1}{w^2C^2}}$$

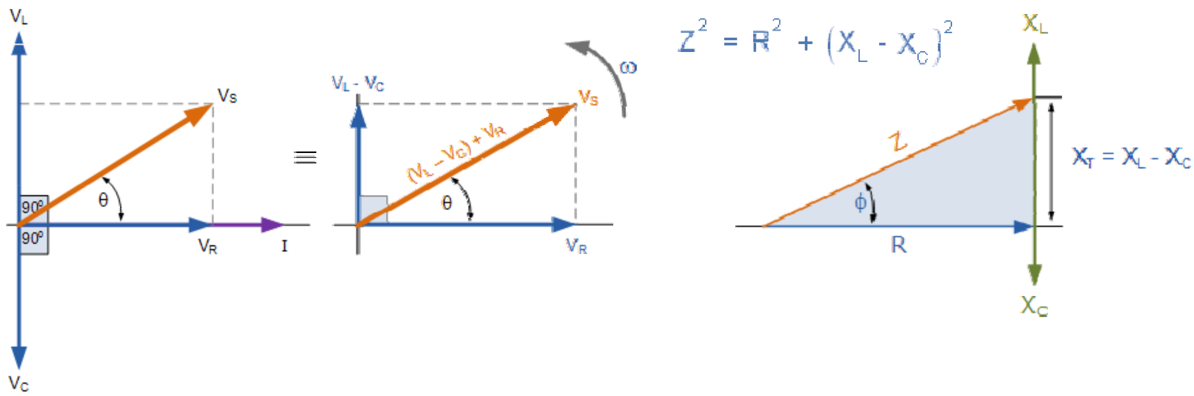




<u>SERIESR-L-CCIRCUIT</u>







Consider a series R-L-C circuit as shown in the figure. The voltage V_R is in phase with the current, the voltage V_L leads the current by 90° and the voltage V_C lags the current by 90°. The total impedence

$$Z = Z_R + Z_L + Z_C$$
$$= R + j(X_L - X_C)$$

We can find that the reactance is positive if $X_L \succ X_C$ and negative if $X_C \succ X_L$. If $X_L \succ X_C$ the circuit behaves like an R-L series circuit and current lags voltage by an angle θ if $X_C \succ X_L$ the circuit behaves as an R-C series circuit and current leads the applied voltage by angle θ. The phasor diagram for both cases are shown.

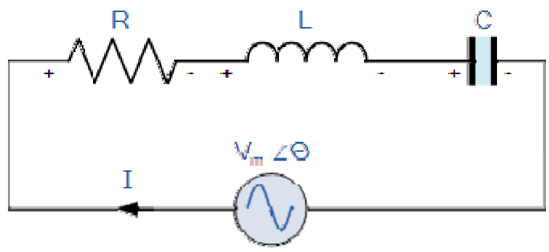
The magnitude of the impedence is given by

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \qquad i = \frac{VS}{Z}$$

$$\theta = \tan^{-1}\frac{wL - \frac{1}{wC}}{R}$$

CHAPTER-4

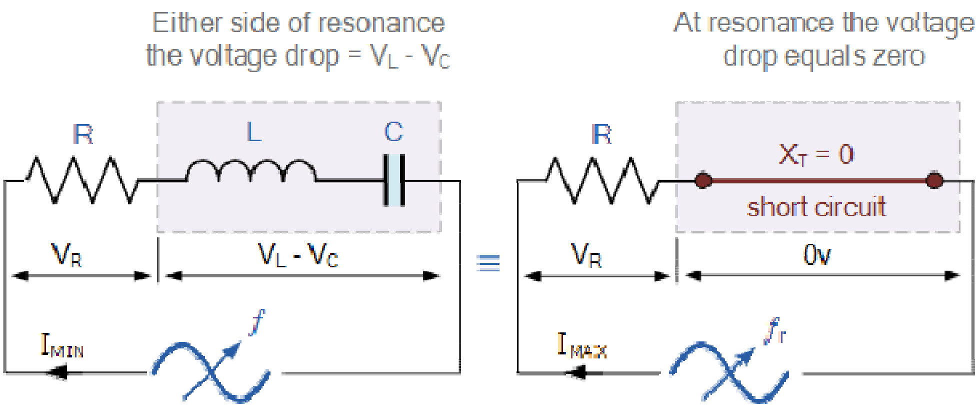
RESONANCE

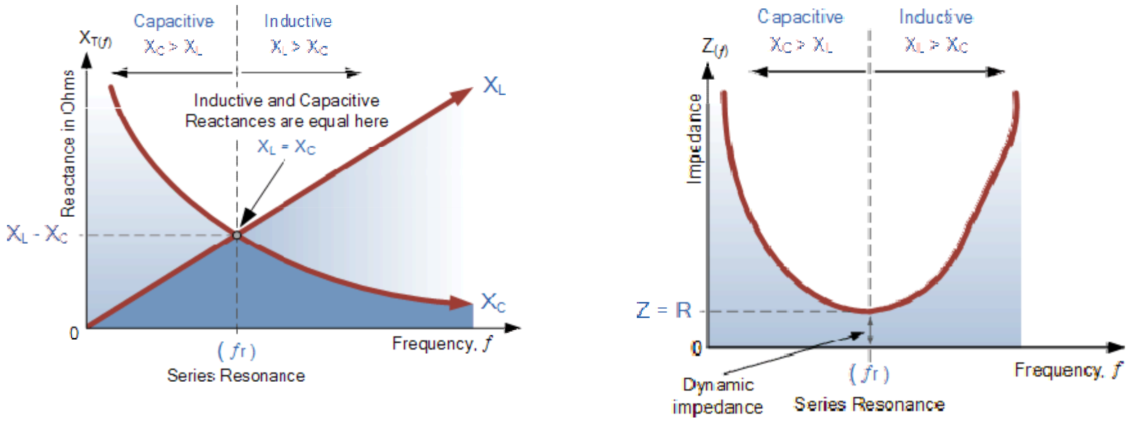


Consider a series R-L-C circuit as shown in the figure. The impedence of the circuit is given by

$$Z = R + j(X_L - X_C)$$

where X_L is inductive reactance $= wL$ and X_C = capacitive reactance $= \frac{1}{wC}$.





As frequency of the supply is increased X_L increases and X_C decreases. At one particular frequency $X_L = X_C$ and the total reactance of the circuit become zero. At this particular frequency the impedence is resitive and voltage & current are in phase. This phenomenon is known as resonance.

$$X_L = X_C$$

$$\Rightarrow 2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$f_0$iscalledasfrequencyofresonance.ImpedenceZoftheR-L-Cseries circuit is equal to R at resonance and current is equal to $\frac{V}{R}$.

QFactor

Theratioofcapacitorvoltageorinductorvoltageatresonantfrequencyto supplyvoltageisameasureofqualityofaresonancecircuit.Thistermisknownas quality factor (Q factor).

Atthefrequencyoftheresonance(f_0)

$$V_L = IX_L = \frac{V}{R}X_L \because I = \frac{V}{R}$$

$$Q = \frac{V_L}{V} = \frac{X_L}{R} = \frac{2\pi f_0 L}{R}$$

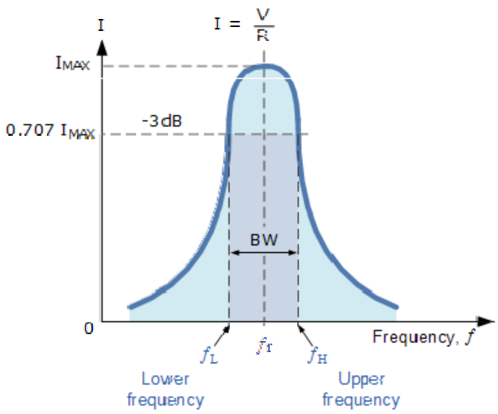
$$= \frac{V_C}{V} = \frac{X_C}{R} = \frac{1}{2\pi f_0 RC}$$

Bandwidth

At resonant frequency current in the R-L-C series circuit is maximum. Let us define two frequenciesw_1&$w_2$atwhichcurrentis707I_{max}.

Thefrequency w_1&w_2 arecalledhalfpower frequency.

Bandwidth=$w_2 - w_1$



Wherew_2=upperhalfpowerfrequency,w_1=lowerhalf powerfrequency.

RelationshipbetweenQandBandwidthofR-L-Cseriescircuit

Bandwidth=$w_2 - w_1$

At$w = w_1$,thereactanceiscapacityas$X_C \succ X_L$

Hence $\frac{1}{w_1 C} - w_1 L = R$ eq.1

At$w = w_2$thereactanceisinductiveas$X_L \succ X_C$

Hence$w_2 L - \frac{1}{w_2 C} = R$..........................eq.2

Fromequation1weget$w_1^2 LC + w_1 RC - 1 = 0$

Dividing by LC we get $w_1^2 + w_1\frac{R}{L} - \frac{1}{LC} = 0$

$$w_1 = \frac{-R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

Similarly from equation 2

$$w_2^2 - \frac{R}{L}w_2 - \frac{1}{LC} = 0$$

$$w_2 = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

Hence bandwidth

$$w_2 - w_1 = \frac{R}{2L} + \frac{R}{2L} = \frac{R}{L}$$

$$\Rightarrow 2\pi(f_2 - f_1) = \frac{R}{L}$$

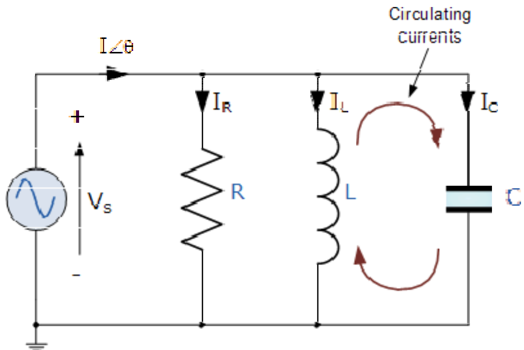
$$\Rightarrow f_2 - f_1 = \frac{R}{2\pi L} = \frac{f_0}{Q}$$

$$\Rightarrow Q = \frac{f_0}{f_2 - f_1} = \frac{w_0}{BW}$$

The Parallel Resonance Circuit

In many ways a parallel resonance circuit is exactly the same as the series resonance circuit we looked at in the previous tutorial. Both are 3-element networks that contain two reactive components making them a second-order circuit, both are influenced by variations in the supply frequency and both have a frequency point where their two reactive components cancel each other out influencing the characteristics of the circuit. Both circuits have a resonant frequency point.

The difference this time however, is that a parallel resonance circuit is influenced by the currents flowing through each parallel branch within the parallel LC tank circuit. A tank circuit is a parallel combination of L and C that is used in filter networks to either select or reject AC frequencies. Consider the parallel RLC circuit below.



Letusdefinewhatwealreadyknowaboutparallel RLC circuits.

$$\text{Admittance, } Y = \frac{1}{Z} = \sqrt{G^2 - B^2}$$

$$\text{Conductance, } G = \frac{1}{R}$$

$$\text{Inductive Susceptance, } B_L = \frac{1}{2\pi f L}$$

$$\text{Capacitive Susceptance, } B_C = 2\pi f C$$

A parallel circuit containing a resistance, R, an inductance, L and a capacitance, C will produce parallel resonance (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations, then parallel circuits produce current resonance.

A *parallel resonant circuit* stores the circuitenergy in the magnetic fieldof the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between theinductor and thecapacitorwhichresultsin zero current and energy being drawn from the supply. This is because the corresponding instantaneous values of I_L and I_C will always be equal and opposite and therefore the current drawn from the supply is the vector addition of these two currents and the current flowing in I_R.

In the solution of AC parallel resonance circuits we know that the supply voltage is common for all branches, so this can be taken as our reference vector. Each parallelbranchmustbetreatedseparately as withseriescircuits sothatthetotalsupply current taken by the parallel circuit is the vector addition of the individual branch currents. Then there are two methods available to us in the analysis of parallel resonance circuits. We can calculate the current in each branch and then add togetheror calculate the admittance of each branch to find the total current.

We know from the previous series resonance tutorial thatresonance takesplacewhenV_L=-$V_C$andthissituationoccurswhenthetworeactanceare equal, X_L= X_C. The admittance of a parallel circuit is given as:

$$Y = G + B_L + B_C$$

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

or

$$Y = \frac{1}{R} + \frac{1}{2\pi f L} + 2\pi f C$$

Resonanceoccurswhen $X_L = X_C$andtheimaginarypartsof$Y$becomezero.

Then:

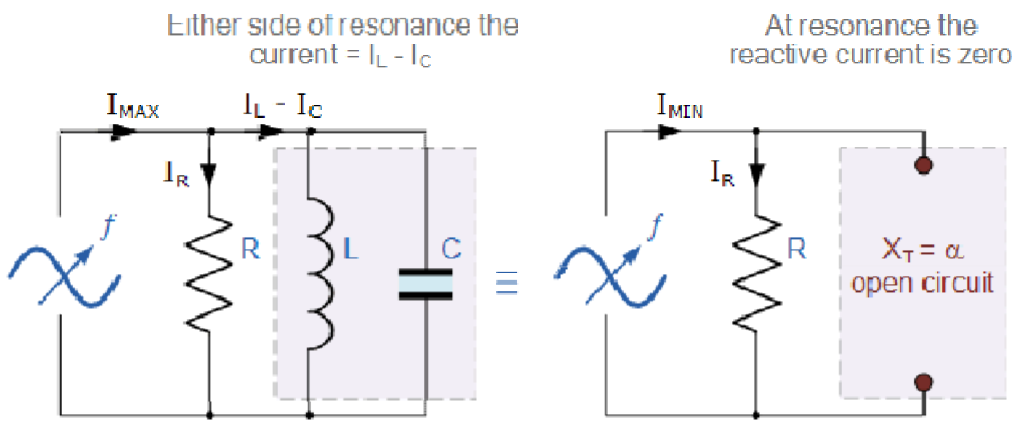
$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 - \frac{1}{2\pi L \times 2\pi C} - \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

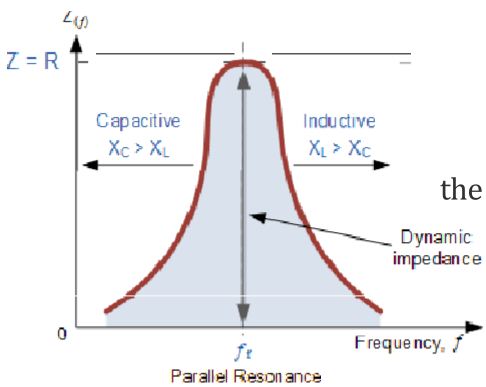
$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)} \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}} \text{ (rads)}$$

Notice that at resonance the parallel circuit produces the same equationas for the series resonance circuit. Therefore, it makes no difference if the inductor or capacitor isconnected in parallelor series. Also at resonancetheparallel LC tank circuit actslikeanopencircuitwiththecircuitcurrentbeingdeterminedbytheresistor, R only. So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and Z = R as shown.



ImpedanceinaParallelResonanceCircuit

Notethatiftheparallelcircuit's impedanceisatitsmaximumatresonancethen consequently,thecircuit'sadmittancemustbeatits minimumandoneofthecharacteristicsofaparallel resonance circuitisthatadmittanceisverylowlimiting the circuitscurrent.Unliketheseriesresonancecircuit,the resistorinaparallelresonancecircuithasadamping effectonthecircuit'sbandwidthmakingthecircuit lessselective.



Also, since the circuit current is constant for any value of impedance, Z, the voltage across a parallel resonance circuit will have the same shape as the total impedance and for a parallel circuit the voltage waveform is generally taken from across the capacitor.

We now know that at the resonant frequency, f_r the admittance of the circuit is at its minimum and is equal to the conductance, G given by 1/R because in a parallel resonance circuit the imaginary part of admittance, i.e. the susceptance, B is zero because $B_L = B_C$ as shown

Bandwidth & Selectivity of a Parallel Resonance Circuit

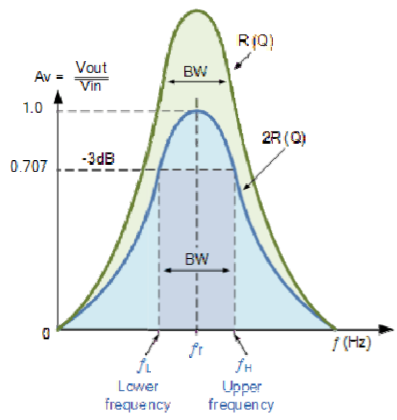
The bandwidth of a parallel resonance circuit is defined in exactly the same way as for the series resonance circuit. The upper and lower cut-off frequencies given as: f_{upper} and f_{lower} respectively denote the half-power frequencies where the power dissipated in the circuit is half of the full power dissipated at the resonant frequency $0.5(I^2 R)$ which gives us the same -3dB points at a current value that is equal to 70.7% of its maximum resonant value, $(0.707 \times I)^2 R$.

As with the series circuit, if the resonant frequency remains constant, an increase in the quality factor, Q will cause a decrease in the bandwidth and likewise, a decrease in the quality factor will cause an increase in the bandwidth as defined by: $BW = f_r/Q$ or $BW = f_2 - f_2$.

Also changing the ratio between the inductor, L and the capacitor, C, or the value of the resistance, R the bandwidth and therefore the frequency response of the circuit will be changed for a fixed resonant frequency. This technique is used extensively in tuning circuits for radio and television transmitters and receivers.

The selectivity or Q-factor for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as:

$$\text{Quality Factor, } Q = \frac{R}{2\pi f L} = 2\pi f C R = R\sqrt{\frac{C}{L}}$$



Note that the Q-factor of a parallel resonance circuit is the inverse of the expression for the Q-factor of the series circuit. Also in series resonance circuits the Q-factor gives the voltage magnification of the circuit, whereas in a parallel circuit it gives the current magnification.

CHAPTER-5

TransientResponseofSimpleCircuit(DC)

Circuits that contain capacitors and inductors can be represented by differential equation. If a circuit contains one resistor and one Inductor (or one capacitor), it can be represented by first order differential equation. On the other hand if a circuit contains a resistor, inductor and Capacitor it can be represented by a second order differential equation. The solution of the differential equation represents the response of the circuit. The response consists of two parts (1) Transient response (2) SteadyStateresponse.Thetransientresponsedependsonthecircuitelementsand initial energy stored init. Toobtain the transient response of thenetworkit is necessary to find the initial state of the network.

InitialCondition

Initial condition of a circuit is important to be calculated when a change of state occurs and the change of state of the network occurs when the switch change its position at time t=0. The value of voltage, current derivatives of both at $t=0^-$ and $t=0^+$,that is immediately before and after change of switch position. Initial conditions in a circuit depends on the past history of the network prior to $t= 0^-$. We will assume that the switch in the network has been in a position for along time and at t=0, the switch changes its position. That is we say the circuit is in steady state at the time of switching.

Initialconditionincircuitelements.

1. Resistor:- By Ohm's Law we have V= IR , if there is a change involtage,thecurrent through resistance will change simultaneously. Similarly if the current change, voltage across resistance changes simultaneously.
2. Inductor:- Current through inductor cannot change instantaneously,ifthecurrent through an inductor before switching is zero, then the current through inductor after switching is also zero.

   i.e.$i_L(0^+)= i_L(0^-)=0$

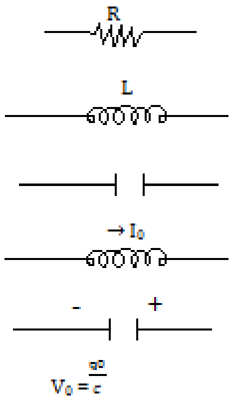
   In the same way if the current through inductor before switching is I_0, then the current through inductor after switching is also I_0. i.e.$i_L(0^+)=i_L(0^-)= I_0$.
3. Capacitor:- Voltage across capacitor cannot change instantaneously. If the voltage across capacitor before switching is zero, then the voltage across capacitor after switching is also zero.

   $V_C(0^+)=V_C(0^-)=0$

   If the voltage across capacitor prior to switching is V_0 then the voltage across capacitor immediately after switching is

   $V_C(0^+)=V_C(0^-)=V_0$

The equivalent from of the elements in terms of the initial condition of the elements is shown below.

<table>
<tr><th>ELEMENT</th><th>EQUIVALENTFORMAT t=0</th><th>t=0+/EQUATIONCIRCUITAT t=∞</th></tr>
<tr>
<td>
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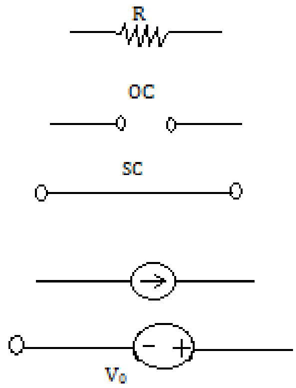
<td>
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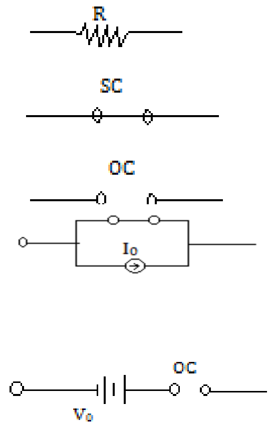
<td>
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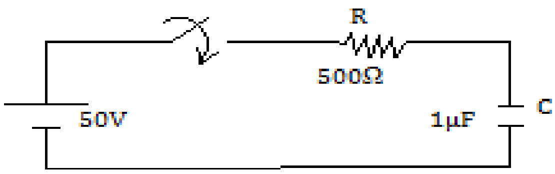
To solve the initial condition of an element it is necessary to study the steady statebehaviorofthiselement.The steadystatebehaviorcanbeobtainedfromthe basic relations.

$$V_L=L\frac{di}{dt} \quad i_c=C\frac{dvc}{dt}$$

At t=∞, V_L= 0 hence the inductor acts as short-circuit

Similarlyatt=∞,i_L=0hencethecapacitoractsasopen-circuit.

Example:Inthenetworkshown infig.1,theswitchKis calledatt=0withthecapacitor uncharged.Findthevalue of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ att= 0^+.



Solution:

ApplyKVLtothecircuit

$Ri+\frac{1}{C}\int idt=V$ ---eq. (i)

$\Rightarrow \quad 500i+\frac{1}{1\times10^{-6}}\int idt=50$ --eq.(ii)

$V_C(0^+)= V_C(0^-)=0$

Att=0^+ $\quad$ $500i\,(0^+)+0=50$

$i(0^+)=\frac{50}{500}= 0.1A$

Differentiating eq.(ii)

$$500\frac{di}{dt}+i\frac{1}{1\times10^{-6}}=0 \quad \text{-- eq.(iii)}$$

At $t=0^+$ $\quad 500\frac{di}{dt}(0^+)=-i(0^+)\frac{1}{1\times10^{-6}}=-\frac{1}{1\times10^{-6}}\times0.1$

$$\Rightarrow \frac{di}{dt}(0^+)=-\frac{10^5}{500}=-2000\text{Amp/sec.}$$

Differentiating eq.(iii)

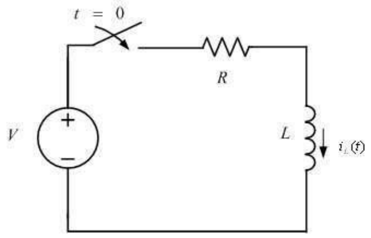
$$500\frac{d^2i}{dt^2}+\frac{1}{1\times10^{-6}}\frac{di}{dt}=0$$

$$\Rightarrow \quad 500\frac{d^2i}{dt^2}(0^+)=-\frac{1}{1\times10^{-6}}\frac{di}{dt}(0^+)=-\frac{1}{1\times10^{-6}}(-2000)$$

$$\Rightarrow \frac{d^2i}{dt^2}(0^+)=\frac{2000\times10^6}{500}=4\times10^6\text{A/sec}^2$$

Transient Response of series R-L circuit having DC Excitation.

Consider a R-L series circuit as shown in figure. The switch is closed at time t=0 Applying KVL



$$L\frac{di(t)}{dt}+Ri(t)=V$$

$$\Rightarrow \frac{di(t)}{dt}+\frac{R}{L}i(t)=\frac{V}{L}$$

General solution of the differential equation

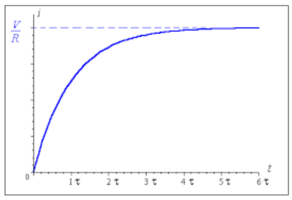
$$i(t)=\frac{V}{R}+ke^{\frac{-R}{L}t}$$

Since inductor behaves as an open circuit as switching

$i(0^+)=0$

$0=\frac{V}{R}+K$ $\quad$ or $\quad$ $K=-\frac{V}{R}$

Therefore $i(t)=\frac{V}{R}-\frac{V}{R}e^{-R/L(t)}=\frac{V}{R}[1-e^{(-R/L)t}]$



Voltage across inductor $V_L(t)=L\frac{di(f)}{dt}=Ve^{(-R/L)t}$

Voltage across resistor $V_R(t)=V[1-e^{(-R/L)t}]$

At t=0, $i(t)=0$, $V_L(t)=V$ $\quad$ $V_R(t)=0$

At $t=\infty$, $i(t)=\frac{V}{R}$, $V_L(t)=0$, $\quad$ $V_R(t)=V$

At $t=\frac{L}{R}$ $\quad i(t)=\frac{V}{R}(1-e)^{-1}=0.632\frac{V}{R}$, $V_L(t)=0.368V$

i(t)&V(t) are plotted in figure.

$\tau=\frac{L}{R}$ is known as the time content and is defined as the interval after which current or voltage charges 63.2% of its total change.

Letusanalysesthetransientconditionofthe R-Lcircuitasthecircuit reaches steady state charging switch to S^1

$$L\frac{di^1(t)}{dt}+Ri^1(t)=0$$

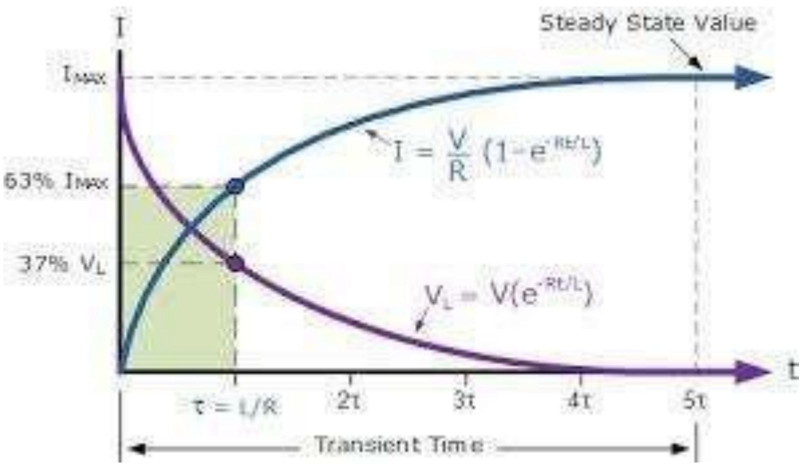
Solutionof $i^1(t)=K^1e^{(-R/L)t}$
Steadystatecurrent $i(0^+)= i(\infty)= \frac{V}{R}$

$\frac{V}{R}=K^1e^0$

$\Rightarrow \quad K^1=\frac{V}{R}$

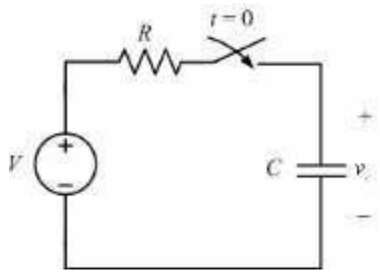
Therefore $i^1(t)= \frac{V}{R}e^{(-R/L)t}, V^1R(t)=Ve(-R/L)t, V_L^1(t)=L\frac{di^1(t)}{dt} =-Ve^{(-R/L)t}$

$i^1(t)$ and $V^1R(t), V^1L(t)$ areplottedbelow.



<u>TransientresponseofseriesR-CcircuithavingDCexcitation.</u>

Consider a series R-C circuit as shown in figure. The switch S is closed attime t=0. Applying KVL



$$Ri(t)+\frac{1}{C}\int i(t)dt=V$$

Differentiating,weget

$$R\frac{di(t)}{dt}+\frac{1}{C}i(t)=0$$

Generalsolutionofthisdifferentialequationis

$$i(t)=Ke^{-t/RC}$$

at $t=0^+, i(0^+)=\frac{V}{R}$ $\because$ capacitoractsasashort-circuitat switching.

$\frac{V}{R}=Ke^0 \Rightarrow K=\frac{V}{R}$

Therefore $i(t)=\frac{V}{R}e^{-t/RC}$

Voltageacrosstheresistorandcapacitorare

$V_R(t)$=i(t).R=Ve$^{-t/RC}$

$V_C(t)$=– = – – $^{-t/RC}$dt

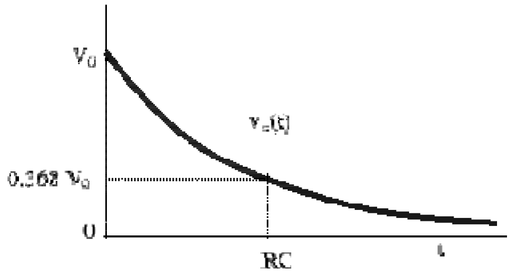
= — (-RC)e$^{-t/RC}$=V(1-e$^{-t/RC}$)

Att=0,i(t)=–$V_C(t)$=0 ,$V_R(t)$=V

Att=∞,i(t)=0,$V_c(t)$=V,$V_R(t)$=0 At t=

RCi(t) = –e^{-1}= 0.368 –,

$V_i(t)$=V(1-e^{-1})=0.632V



Let usanalyze another transient conditionofR-Ccircuitasthecircuitreaches atsteadystate(att=∞)byclosingswitchatpoint2

Ri^{l}(t) + – ^{1}(t)=0

Differentiating we get

R —— +–$i^1(t)$=0

Itssolutionisi^{1}(t)=Ke$^{-t/RC}$

However at t=0^{+}, capacitor keeps the steady state voltage $V_C(0^+)$ = V anddirection of i^{1}(t) during discharge is negative

i(0^{+})= -–

- –= Ke^{0}⇒K= - -–

$\boxed{i(t) = - – e^{-t/RC}}$ $V^1_R(t)$=i^{1}(t).R=-Ve$^{-t/RC}$

$V_C^1(t)$ = – ^{1}(t)dt=Ve$^{-t/RC}$

CHAPTER6

LAPLACETRANSFORM

TheLaplace domain or the "Complex s Domain" is the domain into which the Laplace transform transforms a time-domain equation. s is a complex variable, composed of real and imaginary parts:

$$s = \sigma + j\omega$$

The Laplace domain graphs the real part (σ) as the horizontal axis, andthe imaginary part (ω) as the vertical axis. The real and imaginary parts of s can be considered as independent quantities. The similarity of this notation with the notation used in Fourier transform teory is no coincidence; for $\sigma = 0$,theLaplacetransformis the same as the Fourier transform if the signal is causal.

ThemathematicaldefinitionoftheLaplacetransformisasfollows:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt$$

Thetransfor m ,byvirtueofthedefiniteintegral,remvesalltfromthe resultingequation,leavinginsteadthenewvariables,acomplexnum erthatisnormally writtenas$s=\sigma+j\omega$. In essence, this transform takes the functionf(t),and "transforms it" into a function in terms of s, F(s). As a general rule the transform of a function f(t) is written as F(s). Time-domain functions are written in lower-case, and the resultant s-domain functions are written in upper-case.

Wewillusethefollowingnotationtoshowthetransfor n of a function:

$$f(t) \Leftrightarrow F(s)$$

Weusethisnotation,becausewecanconvertF(s)backintof(t)using the inverse Laplace transform.

TheInverseTransfor m

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

InitialValueTheorem

$$f(0) \Leftrightarrow \lim_{s\to\infty} sF(s)$$

This is useful for finding the initial conditions of a function needed when we perform the transform of a differentiation operation.

FinalValueTheorem

Similar to the Initial Value Theorem, the Final Value Theorem states that we can find the value of a function f, as t approaches infinity, in the laplace domain, as such:

$$\lim_{t\to\infty} f(t) \Leftrightarrow \lim_{s\to 0} sF(s)$$

This is useful for finding the steady state response ofcircuit. The final value theorem may only beapplied to stable systems.

LaplaceTransformatinofSignalWaveform

Laplacetransformofunitstepfunction is $\frac{1}{S}$

Laplacetransformoframp functionis $\frac{1}{S^2}$

Laplacetransformofunit impulsefunction isunity.

The laplace transform can be used independently r elements, and then the circuit can be solved entirely in the S Domain (Which is much easier). Let's take a look at some of the circuit elements:

Resistor

Resistors are time and frequency invariant. Therefore, the transform of a resistor is the same as the resistance of the resistor:

$$R(s) = r$$

Comparethisresulttothephasorimpedancevalueforaresistancer:

$$Z_r = r\angle 0$$

You can see very quickly that resistance values are very similar between phasors and laplace transforms.

Ohm'sLaw

IfwetransformOhm'slaw,weget thefollowingequation:

$$V(s) = I(s)R$$

Now, following ohms law, the resistance of the circuit element is a ratio of the voltage to the current. So, we will solve for the quantity $\frac{V(s)}{I(s)}$, anthe result will be theresistanceofourcircuit element.

$$R = \frac{V(s)}{I(s)}$$

This ratio, the input/output ratio of our resistor is animportant quantity, and we will find this quatity for all of our circuit elements. We can say that the transform of a resistor with resistance r is given by:

$$\mathcal{L}\{resistor\} = R = r$$

Capacitors

Let us look at the relationship between voltage, current, and capacitance, in the time domain:

$$i(t) = C\frac{dv(t)}{dt}$$

Solvingforvoltage,wegetthefollowingintegral:

$$v(t) = \frac{1}{C}\int_{t_0}^{\infty} i(t)dt$$

Then,transformingthisequationintothelaplacedomainassumingthe zero initial condition, we get the following:

$$V(s) = \frac{1}{C}\frac{1}{s}I(s)$$

Again,ifwesolvefor theratio $\frac{V(s)}{I(s)}$,wegetthe following:

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

Therefore,thetransformforacapacitorwithcapacitanceCisgivenby:

$$\mathcal{L}\{\text{capacitor}\} = \frac{1}{sC}$$

Inductors

Letuslookatourequationfor inductance:

$$v(t)=L\frac{di(t)}{dt}$$

Puttingthisi ɪ tothelaplacedomainassumingthezer initialcondition, we get the formula:

$$V(s)=sLI(s)$$

Andsolvingforour ratio $\frac{V(s)}{I(s)}$,wegetthefollowing:

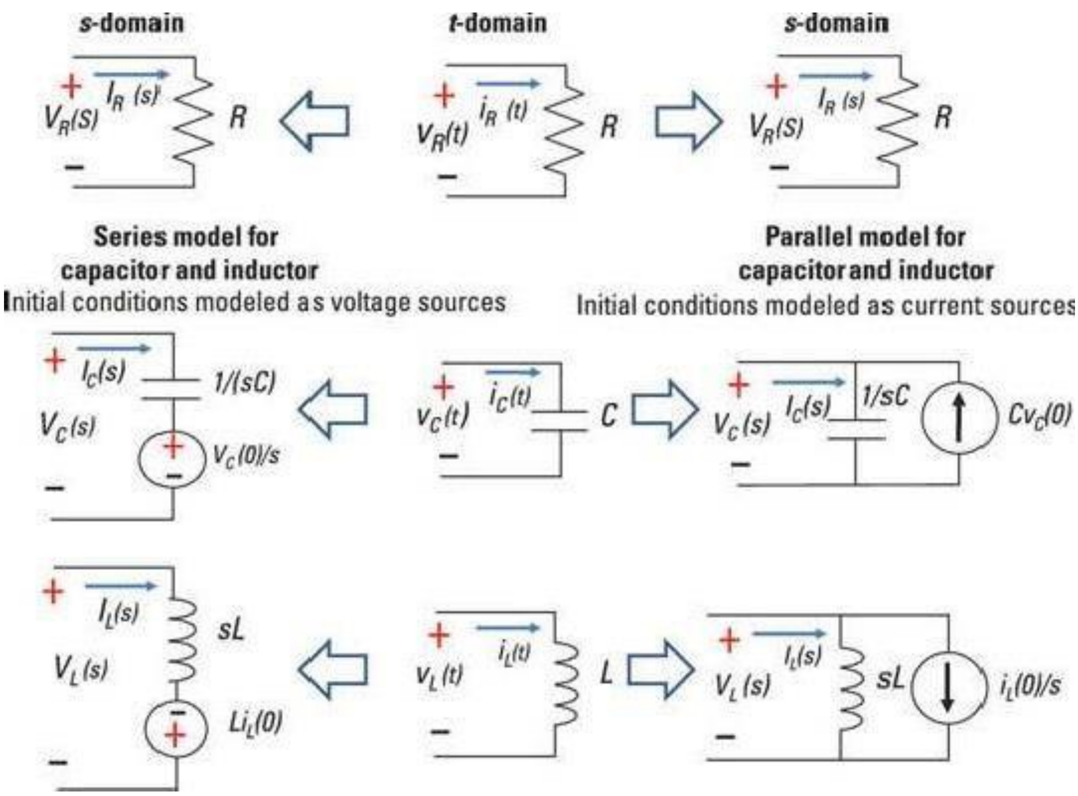
$$\frac{V(s)}{I(s)} = sL$$

Therefore,thetransformofaninductorwithinductanceLisgivenby:

Impedance

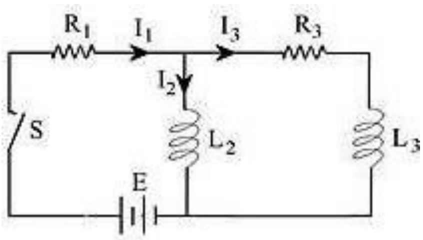
$$\mathcal{L}\{Inductor\} = sL$$

Impedanceofalltheloadelementscanbecombinedintoasingleformat dependentons,wecallthe effectofallloadelementsimpedance,thesameaswecallitin phasorrepresentation.We denoteimpedancevalueswithacapital Z(butnotaphasor$\mathbb{Z}$).



Determiningelectriccurrentincircuits

In the network shown, determine the character of the currents $I_1(t)$, $I_2(t)$,and $I_3(t)$ assuming thateachcurrent iszerowhentheswitch isclosed.



Solution:

Currentflowatajointincircuit

Sincethealgebraicsmuofthecurrentsatanyjunctioniszero,then

$$I_1(t) - I_2(t) - I_3(t) = 0$$

Voltagebalanceonacircuit

Applyingthevoltagelawtothecircuitontheleftweget

$$I_1(t)R_1 + L_2\frac{dI_2(t)}{dt} = E(t)$$

Applyingagainthevoltagelawtotheoutsidecircuit,giventhat Eiscostant,weget

$$I_1(t)R_1 + I_3(t)R_3 + L_3\frac{dI_3(t)}{dt} = E(t)$$

Laplace transformsof currentandvoltageequations

Transforming the abo veequations,weget

$$i_1(s) - i_2(s) - i_3(s) = 0$$

$$i_1(s)R_1 + sL_2 i_2(s) = \frac{E}{s}$$

$$i_1(s)R_1 + (R_3 + sL_3)i_3(s) = \frac{E}{s}$$

The above three Laplace transformed equationsshow thebenefits of integral transformation in converting differential equationsinto linear algebraic equations that could be solved for the dependent variables (the three currents in this case), then inverse transformed to yield the required solution

$$\text{Resistor:}\quad V_R(s) = Ri_R(t) \rightarrow I_R(s) = \left(\frac{1}{R}\right)V_R(s)$$

$$\text{Capacitor:}\quad V_c(s) = \frac{1}{sC}I_c(s) + \frac{v_c(0)}{s} \rightarrow I_c(s) = (sC)V_c(s) - Cv_c(0)$$

$$\text{Inductor:}\quad V_L(s) = sLI_L(s) - Li_L(0) \rightarrow I_L(s) = \left(\frac{1}{sL}\right)V_L(s) + \frac{i_L(0)}{s}$$

Example:Findthecapacitorvoltage.

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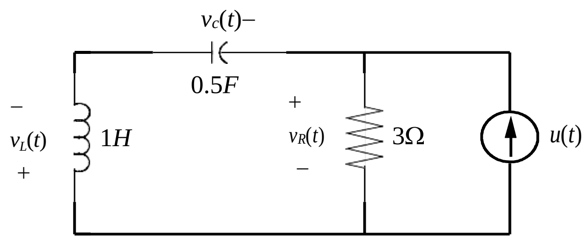

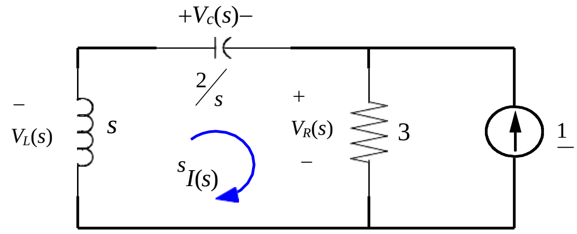

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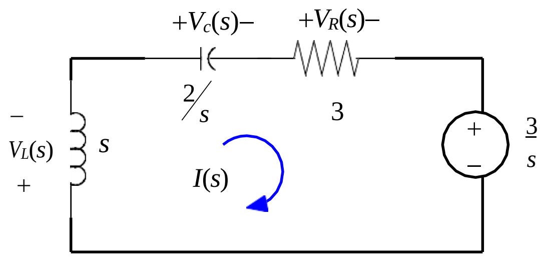


$$\left(s+\frac{2}{s}+3\right)I(s)+\frac{3}{s}=0$$

$$\Rightarrow I(s)=\frac{-3}{s^2+3s+2}$$

Thecapacitor’svoltage

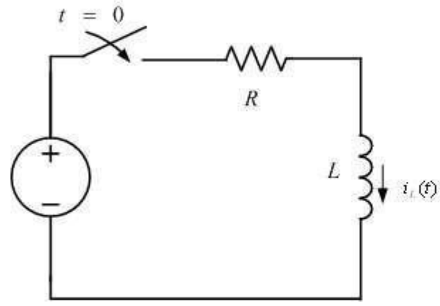
$$V_c(s)=\frac{2}{s}\cdot I(s)=\frac{-6}{s(s^2+3s+2)}$$

ExpandingV$_c$(s)bypartial fraction

$$V_c(s)=\frac{-6}{s(s+1)(s+2)}=\frac{K_1}{s}+\frac{K_2}{s+1}+\frac{K_3}{s+2}$$

$$v_c(t)=\left(-3+6e^{-t}-3e^{-2t}\right)u(t)$$

StepresponseofanR-Lcircuit



Consider the RL circuit as shown in the figure assuming the initial current to be zero. At t = 0the switch is closed and the voltage E is impressed on the circuit. The differential equation on application of KVL is

$$Ri(t)+L\frac{di(t)}{dt}=E$$

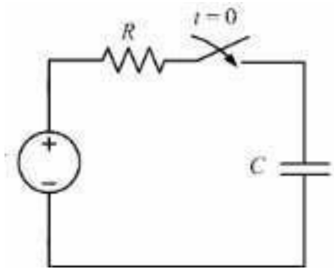
$$\Rightarrow RI(s)+L\left[sI(s)-i_L(0)\right]=\frac{E}{s}$$

$$\Rightarrow I(s)\left[sL+R\right]=\frac{E}{s}\quad \because i_L(0)=0$$

$$\Rightarrow I(s)=\frac{\frac{E}{s}}{sL+R}=\frac{\frac{E}{L}}{s\left(s+\frac{R}{L}\right)}=\frac{E}{R}\left[\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right]$$

$$taking\,inverse\,LT \Rightarrow i(t)=\frac{E}{R}\left[1-e^{-\frac{R}{L}t}\right]$$

StepresponseofanR-Ccircuit



Consider the R-C circuit as shown in the figure assuming the initial current to be zero. At t = 0 the switch is closed and the voltage E is impressed on the circuit. The differential equation on application of KVL is

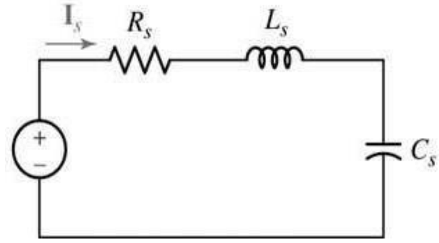
$$Ri(t)+\frac{1}{C}\int_0^t i(t)\,dt=E$$

$$\Rightarrow RI(s)+\frac{1}{C}\left[\frac{I(s)}{s}\right]=\frac{E}{s}$$

$$\Rightarrow I(s)\left[R+\frac{1}{Cs}\right]=\frac{E}{s}$$

$$\Rightarrow I(s)=\frac{\frac{E}{R}}{s+\frac{1}{RC}}$$

$$taking\,inverse\,LT\; i(t)=\frac{E}{R}e^{-t\frac{1}{RC}}$$



StepresponseofanR-L-Ccircuit

Consider the R-L-C series circuit as shown in the figure assuming the initial current to be zero. At t = 0 the switch is closed and the voltage E is impressedon thecircuit.Thedifferential equation on application of KVL is

$$Ri(t)+L\frac{di(t)}{dt}+\frac{1}{C}\int_0^t i(t)dt=E$$

$$\Rightarrow RI(s)+L\left[sI(s)\right]+\frac{1}{C}\left[\frac{I(s)}{s}\right]=\frac{E}{s}$$

$$\Rightarrow I(s)=\frac{\frac{E}{s}}{sL+R+\frac{1}{Cs}}=\frac{\frac{E}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

$$\Rightarrow I(s)=\frac{\frac{E}{L}}{(s-s_1)(s-s_2)}=\frac{K_1}{(s-s_1)}+\frac{K_2}{(s-s_2)}$$

$$\Rightarrow i(t)=K_1e^{s_1t}+K_2e^{s_2t}$$

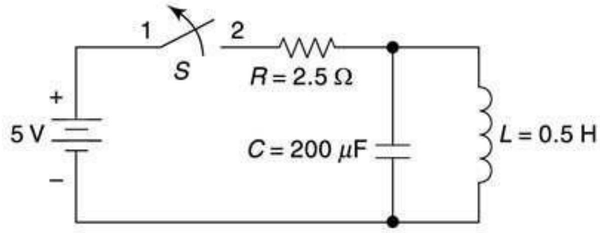
Wheres_1&$s_2$arethe rootsofthecharacteristicequation $s^2+\frac{R}{L}s+\frac{1}{LC}$ andK_1&$K_2$are constant.

Valueofs_1&$s_2$canbedeterminedas

$$s_1,s_2=-\frac{R}{2L}\pm\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}$$

Problem1

In the network shown in the figure, the switch S is closed and a steady state is attained. At t = 0, the switch is opened. Determine the current through the inductor for t > 0.



Solution

When the switch S is closed and the steady-state exists, the current through the inductor is,

$$i(0-) = \frac{V}{R} = \frac{5}{2.5} = 2\text{ A}$$

The voltage across the capacitor, $V_C(t) = 0$ as it is shorted.

For $t > 0$, the switch is opened. By KVL,

$$L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt = 0$$

Taking Laplace transform,

$$L[sI(s) - i(0-)] + \frac{I(s)}{Cs} = 0$$

or $I(s)\left[sL + \frac{1}{Cs}\right] = Li(0-)$

Putting the values,

$$I(s) = 2\frac{s}{s^2 + 10^4}$$

Taking inverse Laplace transform,

$$i(t) = 2\cos 100t \quad (A);\ t \geq 0$$

Problem2

A series *R-L-C* circuit with $R = 3\Omega$, $L = 1H$ and $C = 0.5$ F is excited with a unit step voltage. Obtain an expression for the current, using Laplace transform. Assume that the circuit is relaxed initially.

Solution

By KVL,

$$RI(s) + sLI(s) - Li(0-) + \frac{1}{sC}I(s) + \frac{Q(0-)}{sC} = \frac{1}{s}$$

Since the circuit is initially relaxed,

$\therefore\ i(0-) = 0 \quad \text{and} \quad Q(0-) = 0$

Putting the values,

$$I(s)\left[3 + s + \frac{2}{s}\right] = \frac{1}{s}$$

or $I(s) = \dfrac{1}{s^2 + 3s + 2} = \dfrac{1}{(s+1)\,(s+2)} = \dfrac{A_1}{s+1} + \dfrac{A_2}{s+2}$

where, $A_1 = \left.\dfrac{1}{s+2}\right|_{s=-1} = 1 \quad \text{and} \quad A_2 = \left.\dfrac{1}{s+1}\right|_{s=-2} = -1$

$\therefore\ I(s) = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

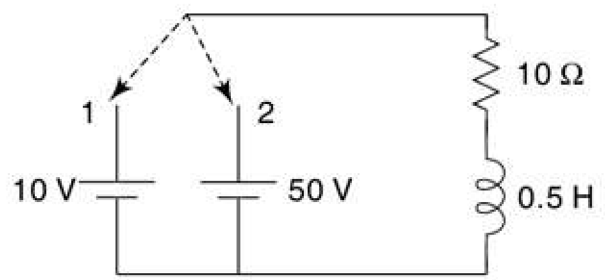
Taking inverse Laplace transform,

$$i(t) = e^{-t} + e^{-2t} \text{ (A)}$$

$$= 2e^{3t/2} \sinh\left(\frac{t}{2}\right) \text{(A)}$$

Problem3

The circuit was in steady state with the switch in position 1. Find the current i(t) for $t > 0$ if the switch is moved from position 1 to 2 at $t = 0$.



Solution

When the switch is in position 1, steady-state exists and the initial current through the inductor is,

$$i(0-) = \frac{10}{10} = 1A$$

After the switch is moved to position 2, the KVL gives, in Laplace transform,

$$10I(s) + 0.5sI(s) - 0.5 \times 1 = \frac{50}{s}$$

or, $I(s) = \dfrac{100}{s(s+20)} + \dfrac{1}{s+20} = 5\left[\dfrac{1}{s} - \dfrac{1}{s+20}\right] + \dfrac{1}{s+20}$

Taking inverse Laplace transform,

$$i(t) = 5 - 4e^{-20t} \quad (A);\, t > 0;$$

Problem4

Find the response current of a series *RL* circuit consisting of a resistor $R = 3\Omega$ and an inductor $L = 1$ H when each of the following driving force voltage is applied:

a. unit ramp voltage $r(t-2)$,

b. unit impulse voltage $\delta(t-2)$,

c. unit step voltage $u(t-2)$,

Solution

a. Unit ramp voltage $r(t-2)$

Applying KVL to *RL* series circuit,

$$Ri + L\frac{di}{dt} = v(t) = r(t-2)$$

Taking Laplace transform,

$$(R+sL)I(s) = \frac{1}{s^2}e^{-2s}$$

$$I(s) = \frac{e^{-2s}}{s^2(sL+R)}$$

Substituting the values,

$$I(s) = \frac{e^{-2s}}{s^2(s+3)} = e^{-2s}\left[\frac{K_1}{s^2} + \frac{K_2}{s} + \frac{K_3}{s+3}\right]$$

$$\therefore\ K_1 = \left.\frac{1}{s+3}\right|_{s=0} = \frac{1}{3}$$

$\therefore$ K2

$$= \left.\frac{d}{ds}\left[\frac{1}{s+3}\right]\right|_{s=0} = \left.-\frac{1}{(s+3)^2}\right|_{s=0} = -\frac{1}{9}$$

$$\therefore\ K_3 = \left.\frac{1}{s^2}\right|_{s=-3} = \frac{1}{9}$$

$$\therefore\ I(s) = e^{-2s}\left[\frac{1/3}{s^2} + \frac{-1/9}{s} + \frac{1/9}{s+3}\right]$$

Taking inverse Laplace transform,

$$i(t) = -\frac{1}{9}u(t-2) + \frac{1}{3}r(t-2) + \frac{1}{9}e^{-3(t-2)}u(t-2) \qquad Ans.$$

b. Unit impulse voltage $\delta(t-2)$:

In this case,

$$Ri + L\frac{di}{dt} = v(t) = \delta(t-2)$$

Taking Laplace transform,

$$(R+sL)I(s) = e^{-2s}$$

$$I(s) = \frac{e^{-2s}}{(sL+R)} = \frac{e^{-2s}}{(s+3)}$$

Taking inverse Laplace transform,

$$i(t) = e^{-3(t-2)}u(t-2) \qquad Ans.$$

---

c. Unit step voltage $u(t-2)$

In this case,

$$Ri + L\frac{di}{dt} = v(t) = u(t-2)$$

Taking Laplace transform,

$$(R+sL)I(s) = \frac{e^{-2s}}{s}$$

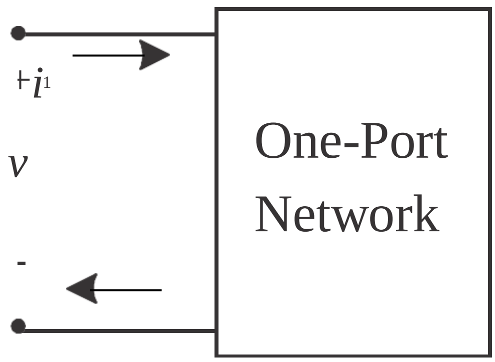
$$I(s) = \frac{e^{-2s}}{(sL+R)} = \frac{e^{-2s}}{s(s+3)} = \frac{1}{3}e^{-2s}\left[\frac{1}{s} - \frac{1}{(s+3)}\right]$$

Taking inverse Laplace transform,

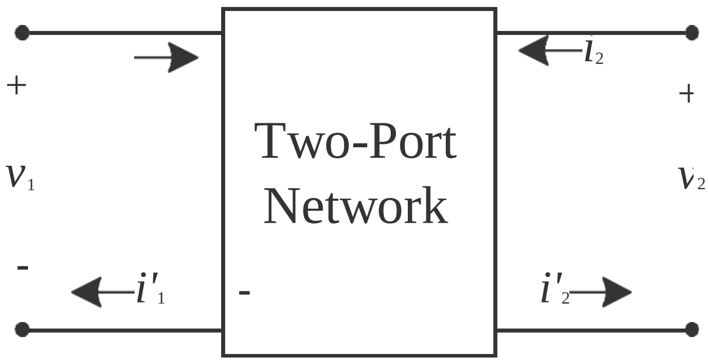
$$i(t) = \frac{1}{3}u(t-2) - \frac{1}{3}e^{-3(t-2)}u(t-2) \qquad Ans.$$

CHAPTER7

TWO-PORTNETWORKS



a) A pair of terminals at which a signal (voltage orcurrent) may enter or leave is called a port.
b) Anetworkhavingonlyonesuchpairofterminalsiscalledaone-port network.
c) Noconnectionsmaybemadetoanyothernodesinternaltothenetwork.
d) ByKCL,wetherefore have $i_1=i_1$



- Two-port networksare usedto describe therelationship between apairof terminals
- The analysis methods we will discuss require the following conditions be met
  1. Linearity
  2. Noindependent sourcesinsidethenetwork
  3. Nostoredenergyinsidethenetwork(zeroinitial conditions)
  4. $i_1=i_1$ and $i_2=i_2$

ImpedanceParameters

- Supposethecurrentsandvoltagescanbemeasured.
- Alternatively,ifthecircuitintheboxisknown,V_1andV_2canbecalculated based on circuit analysis.
- Relationshipcanbewrittenintermsoftheimpedanceparameters.
- Wecanalsocalculatetheimpedanceparametersaftermakingtwosetsof measurements.

$$V_1=z_{11}I_1+z_{12}I_2$$

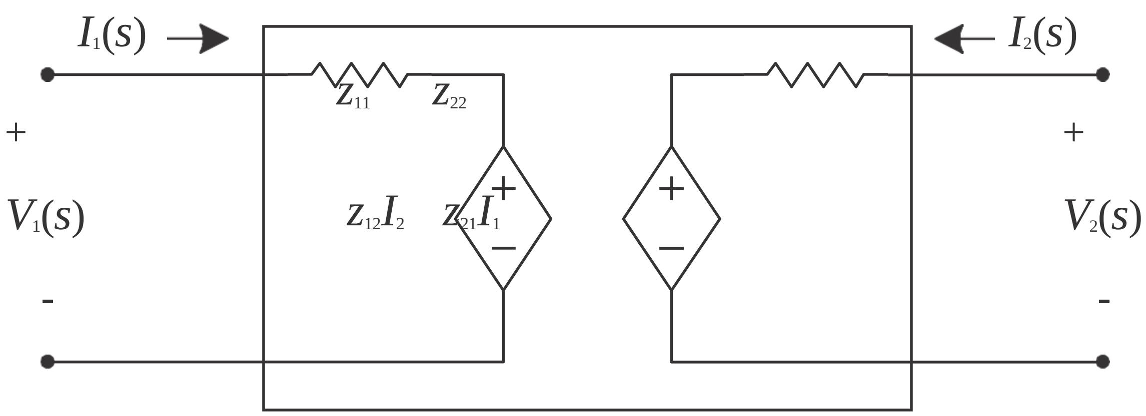
$$V_2=z_{21}I_1+z_{22}I_2$$

Iftherightportisan opencircuit (I_2=0),thenwecan easilysolvefor two of the impedance parameters: Similarly by open circuiting left hand port (I_1=0) we can solve for the other two parameters.

$$Z_{11}=inputimpedence=\frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{21}=forwardtransferimpedence=\frac{V_2}{I_1}\Big|_{I_2=0}$$

$$Z_{12}=reversetransferimpedence=\frac{V_1}{I_2}\Big|_{I_1=0} \qquad Z_{22}=outputimpedence=\frac{V_2}{I_2}\Big|_{I_1=0}$$

ImpedanceParameterEquivalent

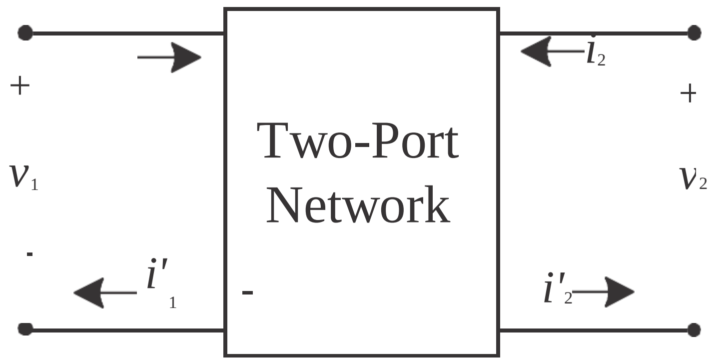


$$V_1=z_{11}I_1+z_{12}I_2$$

$$V_2=z_{21}I_1+z_{22}I_2$$

- Onceweknowwhattheimpedanceparametersare,wecanmodel the behavior of the two-port with an equivalent circuit.
- NoticethesimilaritytoTh´eveninandNorton equivalents

AdmittanceParameters



$$I_1=y_{11}V_1+y_{12}V_2$$

$$I_2=y_{21}V_1+y_{22}V_2$$

$$Y_{11}=\text{inputadmittance}=\left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21}=\text{forwardtransfer admittance}=\left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$Y_{22}=\text{outputadmittance}=\left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{12}=\text{reversetransferadmittance}=\left.\frac{I_1}{V_2}\right|_{V_1=0}$$

HybridParameters

$$V_1=h_{11}I_1+h_{12}V_2$$

$$I_2=h_{21}I_1+h_{22}V_2$$

$$h_{11}=\text{inputimpedance}=\left.\frac{V_1}{I_1}\right|_{V_2=0}$$

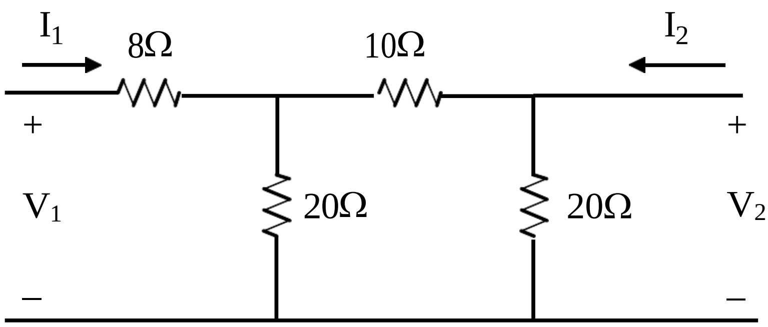
$$h_{21}=\text{forwardcurrentratio }=\left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12}=\text{reversevoltageratio}=\left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{22}=\text{outputadmittance}=\left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Example:

Giventhefollowingcircuit.DeterminetheZparameters.



$Z_{11} = 8 + 20 \| 30 = 20\Omega$

$Z_{22} = 20 \| 30 = 12\Omega$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$V_1 = \frac{20 x I_2 x 20}{20+30} = 8 x I_2 \quad \text{Therefore } z_{12} = \frac{8 x I_2}{I_2} = 8\Omega = z_{21}$$

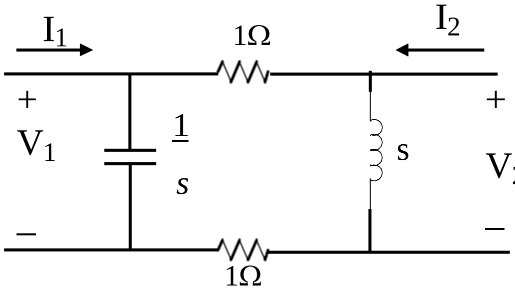
TheZparameterequationscanbeexpressedinmatrixformasfollows.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 8 \\ 8 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

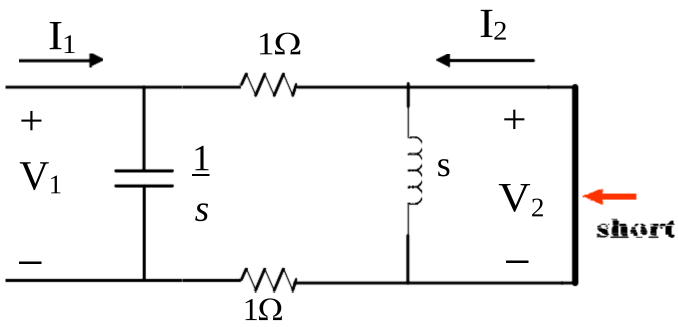
Example:

Giventhefollowingcircuit.DeterminetheYparameters.



$I_1 = y_{11}V_1 + y_{12}V_2$ I_2

$= y_{21}V_1 + y_{22}V_2$



To find y_{11}

$$V_1 = I_1\left(\frac{2/s}{2+1/s}\right) = I_1\left[\frac{2}{2s+1}\right]$$

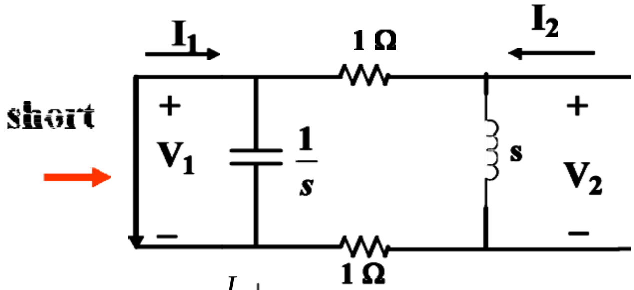
So $y_{11} = \frac{I_1}{V_1} y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = s+0.5$

To find y_{12} and y_{21} we reverse things and short V_1

$$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$$V_1 = -2I_2$$

$$y_{21} = \frac{I_2}{V_1} = 0.5\text{S}$$



$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$V_2 = -2I_1$ $\qquad y_{12} = \frac{I_1}{V_2} = 0.5s$

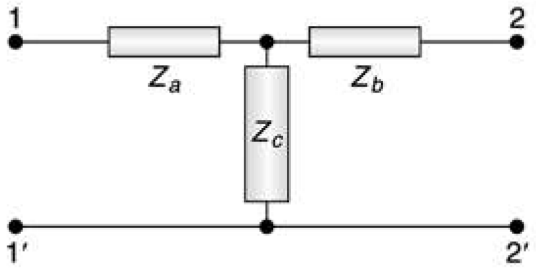
$$y_{22} = 0.5 + \frac{1}{s}$$

$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ $\qquad V_2 = I_2\frac{2s}{(s+2)}$ $\qquad y_{22} = 0.5 + \frac{1}{s}$

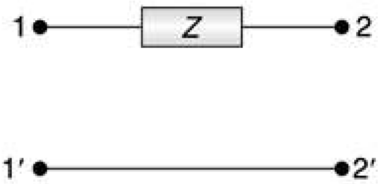
Problem1

Find the Z and Y parameter for the networks shown in figure.

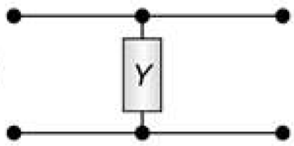
a.



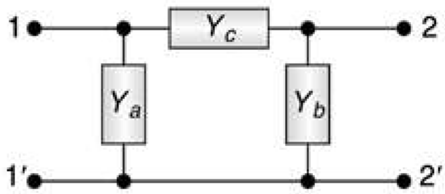
b.



c. (c)



d.



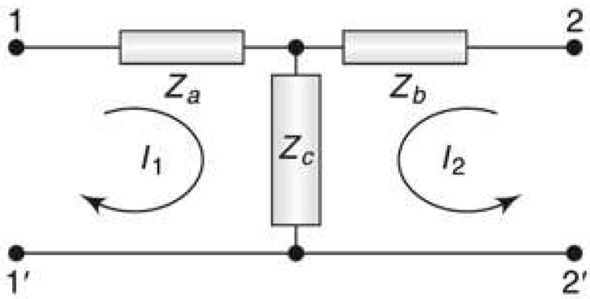
Solution

a. By KVL, $(Z_a + Z_c)I_1 + Z_c I_2 = V_1$

and $Z_c I_1 + (Z_b + Z_c)I_2 = V_2$

Thus, the Z-parameters are:

$$z_{11} = (Z_a + Z_c),\ z_{12} = z_{21} = Z_c, \quad z_{22} = (Z_b + Z_c)$$



b. By KCL,

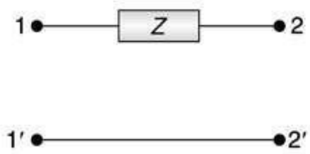
$$I_1 = \frac{V_1 - V_2}{Z} = \frac{1}{Z}V_1 - \frac{1}{Z}V_2$$

and $$I_2 = \frac{V_2 - V_1}{Z} = -\frac{1}{Z}V_1 + \frac{1}{Z}V_2$$

Thus, the y-parameters are,

$$y_{11} = \frac{1}{Z} = y_{22} \quad y_{12} = y_{21} = -\frac{1}{Z}$$

Since, $\Delta y = y_{11}y_{22} - y_{12}y_{21} = 0$, the z-parameters do not exist for this network.



c. By KVL,

$V_1 =$

$$\frac{I_1 + I_2}{Y} = V_2 \quad \text{or, } V_1 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2 \quad \text{and} \quad V_2 = \left(\frac{1}{Y}\right)I_1 + \left(\frac{1}{Y}\right)I_2$$

Thus, the z-parameters are,

$$z_{11} = z_{22} = \frac{1}{Y} = z_{12} = z_{21}$$

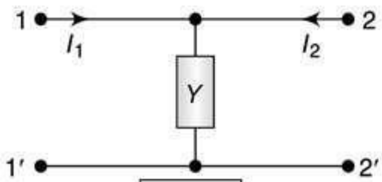
Since, $\Delta z = z_{11}z_{22} - z_{12}z_{21} = 0$, the y-parameters do not exist for this network.

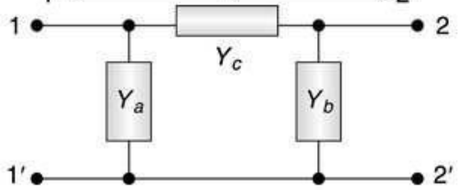
d. By KCL,

$$I_1 = Y_aV_1 + (V_1 - V_2)Y_c = V_1(Y_a + Y_c) - V_2Y_c$$
$$I_2 = Y_bV_2 + (V_2 - V_1)Y_c = -V_1Y_c + V_2(Y_b + Y_c)$$

Thus, the y-parameters are:

$$y_{11} = Y_a + Y_c;\ y_{12} = y_{21} = -Y_c;\ y_{22} = Y_b + Y_c$$





Problem2

a. The following equations give the voltages V_1 and V_2 at the two ports of a two port network, $V_1 = 5I_1+2I_2$, $V_2 = 2I_1+I_2$;

   A load resistance of 3 Ω is connected across port-2. Calculate the input impedance.

b. The *z*-parameters of a two port network are $z_{11} = 5\ \Omega$, $z_{22} = 2\ \Omega$, $z_{12} = z_{21} = 3\ \Omega$. Load resistance of 4 Ω is connected across the output port. Calculate the input impedance.

Solution

a. From the given equations,

$V_1 = 5I_1 + 2I_2$ (i)

$V_2 = 2I_1 + I_2$ (ii)

At the output, $V_2 = -I_2R_L = -3I_2$

Putting this value in (ii),

$-3I_2 = 2I_1 + I_2$ **fi** $I_2 = -I_1/2$

Putting in (i), $V_1 = 5I_1 + \left(\frac{-I_1}{2}\right) = 4I_1$

$\therefore$ Input impedance, $Z_{in} = \frac{V_1}{I_1} = 4\Omega$

b. [Same as Prob. (a)] $Z_{in} = \frac{V_1}{I_1} = 3.5\Omega$

Problem3

Determine the h-parameter with the following data:

i. with the output terminals short circuited, $V_1 = 25$ V, $I_1 = 1$ A, $I_2 = 2$ A

ii. with the input terminals open circuited, $V_1 = 10$ V, $V_2 = 50$ V, $I_2 = 2$ A

Solution

The h-parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

a. With output short-circuited, $V_2 = 0$, given: $V_1 = 25$ V, $I_1 = 1$ A and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 25 &= h_{11} \times 1 \\ \text{and} \quad 2 &= h_{21} \times 1 \end{aligned}\right\} \Rightarrow h_{11} = 25\ \Omega, \text{ and } h_{21} = 2$$

b. With input open-circuited, $I_1 = 0$, given: $V_1 = 10$ V, $V_2 = 50$ V and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 10 &= h_{12} \times 50 \\ \text{and} \quad 2 &= h_{22} \times 50 \end{aligned}\right\} \Rightarrow h_{12} = \frac{1}{5} = 0.2 \text{ and } h_{23} = \frac{1}{25}\ \mho = 0.04\ \mho$$

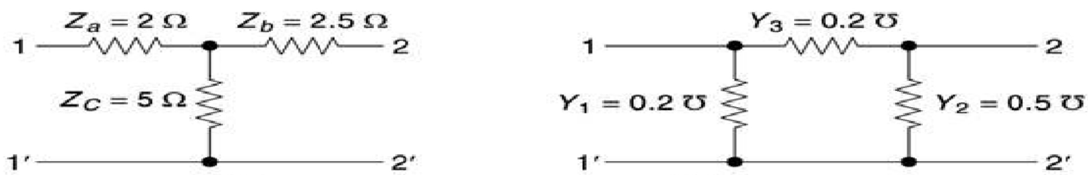
Thus, the h-parameters are:

$$[h] = \begin{bmatrix} 25\ \Omega & 0.2 \\ 2 & 0.04\ \Omega^{-1} \end{bmatrix}$$

Problem4

a. Find the equivalent π-network for the T-network shown in the Fig. (a).

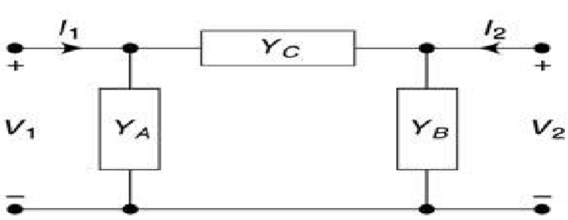
b. Find the equivalent T-network for the π-network shown in the Fig. (b).



(a) (b)

Solution

a. Let the equivalent π-network have Y_C as the series admittance and Y_A and Y_B as the shunt admittances at port-1 and port-2, respectively.



Now, the z-parameters are given as:

$$z_{11} = (Z_A + Z_C) = 7\ \Omega,\ z_{12} = z_{21} = Z_C = 5\ \Omega,\ z_{22} = (Z_B + Z_C) = 7.5\ \Omega$$

$$\therefore \Delta z = (7 \times 7.5 - 5 \times 5) = 27.5\ \Omega^2$$

$$\therefore y_{11} = \frac{z_{22}}{\Delta z} = \frac{7.5}{27.5}\ \mho$$

$$y_{12} = y_{21} = -\frac{z_C}{\Delta z} = -\frac{5}{27.5}\ \mho$$

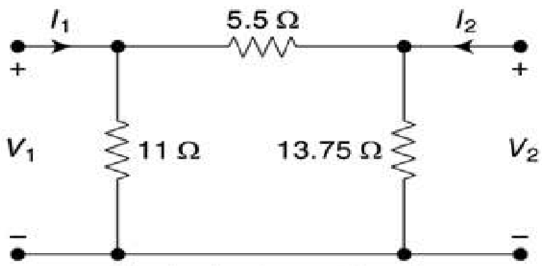
$$y_{22} = \frac{z_{11}}{\Delta z} = \frac{7}{27.5}\ \mho$$

$$\therefore Y_A = (y_{11} + y_{12}) = \frac{2.5}{27.5} = \frac{1}{11}\ \mho$$

$$\therefore Y_B = (y_{22} + y_{12}) = \frac{2}{27.5}\ \mho$$

$$\text{and } Y_C = -y_{21} = \frac{5}{27.5} = \frac{2}{11}\ \mho$$

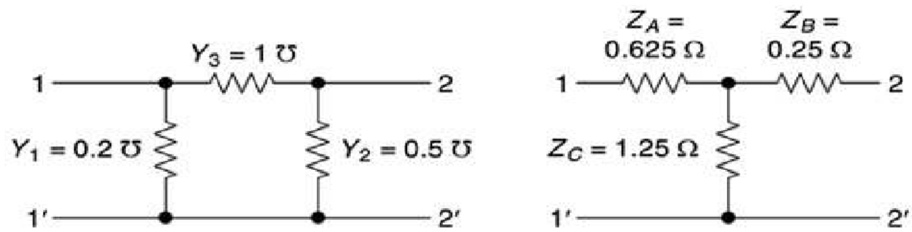
Thus, the impedances of the equivalent π-networks are:



Equivalent π-network

$$\left.\begin{aligned} Z_A &= \frac{1}{Y_A} = 11\ \Omega, \\ Z_B &= \frac{1}{Y_B} = 13.75\ \Omega, \\ Z_C &= \frac{1}{Y_C} = 5.5\ \Omega \end{aligned}\right\}$$

b.



π-network **Equivalent T-network**

The y-parameters,

$$y_{11} = 1.2\ \mho,\ y_{12} = y_{21} = -1\ \mho, \text{ and } y_{22} = 1.5\ \mho$$

$$\therefore \Delta y = (1.2 \times 1.5 - 1) = 0.8$$

$$\therefore z_{11} = \frac{y_{22}}{\Delta y} = \frac{1.5}{0.8}\ \Omega,\ z_{12} = z_{21} = -\frac{y_{12}}{\Delta y} = \frac{1}{0.8}\ \Omega,\ z_{22} = \frac{y_{11}}{\Delta y} = \frac{1.2}{0.8}\ \Omega$$

$$\therefore \left.\begin{aligned} Z_A &= (z_{11} - z_{12}) = \frac{0.5}{0.8} = 0.625\ \Omega \\ Z_B &= (z_{22} - z_{12}) = \frac{0.2}{0.8} = 0.25\ \Omega \\ Z_C &= z_{12} = \frac{1}{0.8} = 1.25\ \Omega \end{aligned}\right\}$$

CHAPTER8

LOWPASSFILTERINTRODUCTION

Basically, an electrical filter is a circuit that can be designed to modify, reshapeorrejectallunwantedfrequenciesofanelectricalsignalandacceptor passonly thosesignalswanted bythecircuit'sdesigner.Inother wordsthey"filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency.

In lowfrequency applications(up to100kHz),passivefiltersaregenerally constructed using simple RC(Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive Filters are made up of passive components such as resistors, capacitorsand inductorsandhaveno amplifying elements(transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

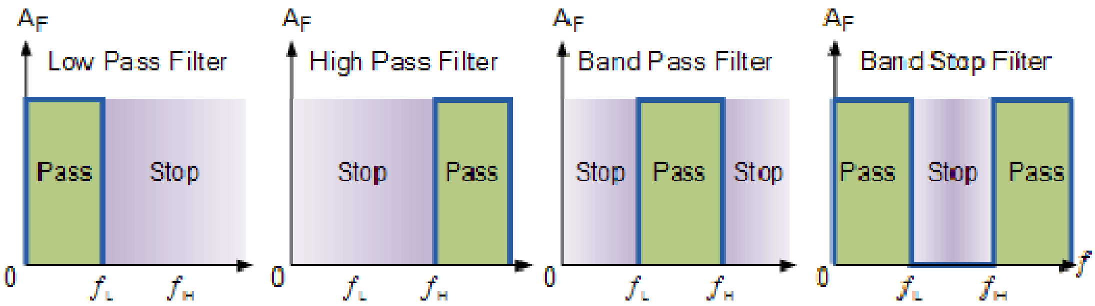
Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or "attenuating" the rest. The most commonly used filter designs are the:

- 1. The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, ƒc point to pass while blocking those any higher.
- 2. The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, ƒc point and higher to infinity to pass through while blocking those any lower.
- 3. The Band Pass Filter – the band pass filter allows signals falling within a certain frequencybandsetupbetweentwopointsto passthroughwhileblockingboth the lower and higher frequencies either side of this frequency band.
- 4 Band Stop Filter - It is so called *band-elimination*, *band-reject*, or *notch* filters; this kind of filter passes all frequencies above and below a particular range set by the component values.

SimpleFirst-orderpassivefilters(1storder)canbemadebyconnecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output of the filter, (Vout) taken from the junction of these two components. Depending onwhich wayaround weconnecttheresistor and thecapacitor with regards to the output signal determines the type of filter construction resulting in either a Low Pass Filter or a High Pass Filter.

As the function of any filter is to allow signals of a given band of frequenciestopassunalteredwhileattenuatingorweakeningallothers thosearenot wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

IDEALFILTERRESPONSECURVES

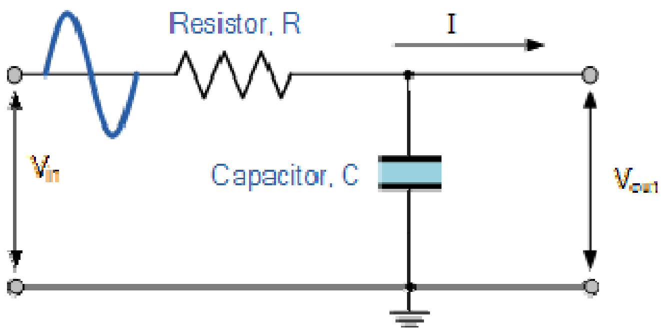


A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the "cutoff" frequency. The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

THELOWPASSFILTER

A simple passive RC Low Pass Filter or LPF, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below.In this type of filter arrangement the input signal (Vin) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (Vout) is taken across the capacitor only. This type of filter is known generally as a "first-order filter" or "one-pole filter", why first-order or single-pole?, because it has only "one" reactive component, the capacitor, in the circuit.

RCLOWPASSFILTERCIRCUIT



As mentioned previously in theCapacitive Reactance tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At lowfrequencies the capacitive reactance, (Xc)ofthecapacitorwillbevery large compared to the resistive value of the resistor, R and as a result the voltage across the capacitor, Vc will also be large while the voltage drop across the resistor, Vr will be much lower. At high frequencies the reverse is true with Vc being small and Vr being large.

While the circuit above is that of anRC Low Pass Filtercircuit, it can also beclassedasafrequencyvariablepotentialdividercircuitsimilartotheonewelooked at in theResistorstutorial. In that tutorial we used the following equation to calculatethe output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

WealsoknowthatthecapacitivereactanceofacapacitorinanACcircuit isgiven as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

OppositiontocurrentflowinanACcircuitiscalledimpedance,symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

RCPOTENTIALDIVIDEREQUATION

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

LOWPASSFILTEREXAMPLE

A Low Pass Filter circuit consisting of a resistor of 4k7Ω in series with a capacitor of 47nF is connected across a 10v sinusoidal supply. Calculate the output voltage (Vout) at a frequency of 100Hz and again at frequency of 10,000Hz or 10kHz.

VoltageOutputata Frequencyof100Hz.

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{Xc}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

Voltage Outputata Frequencyof10,000Hz(10kHz).

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\Omega$$

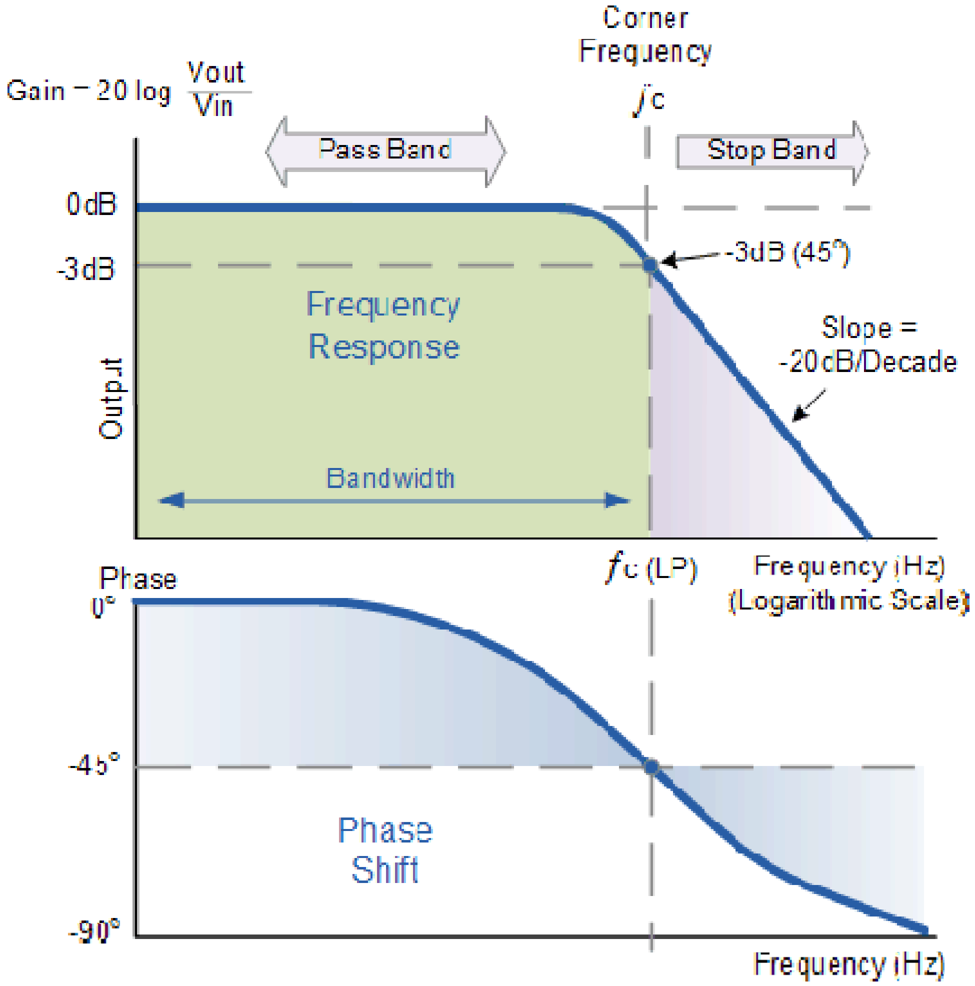
$$V_{OUT} = V_{IN} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

FREQUENCYRESPONSE

We can see from the results above that as the frequency applied to the RC network increasesfrom 100Hz to 10 kHz,thevoltagedropped acrossthe capacitorand therefore the output voltage (Vout) from the circuit decreases from 9.9v to 0.718v.

Byplottingthenetworksoutputvoltageagainstdifferentvaluesofinputfrequency, the Frequency Response Curve or Bode Plot function of the low pass filter circuit can be found, as shown below.

<u>Frequency Response of a 1st-order Low Pass Filter</u>



The Bode Plotshows the Frequency Response of the filter to be nearly flat for low frequencies and the entire input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its Cut-off Frequency point (ƒc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases to zero at a slope of -20dB/ Decade or (-6dB/Octave) “roll-off”. Note that the angle of the slope, this -20dB/ Decade roll-off will always be the same for any RC combination.

Any high frequency signals applied to the low pass filter circuit above this cut-off frequency point will become greatly attenuated, that is they rapidly decrease. This happens because at very high frequencies the reactance of the capacitor becomesso low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

Then by carefully selecting the correct resistor-capacitor combination, we can create a RC circuit that allows a range of frequencies below a certain value to pass through the circuit unaffected while any frequencies applied to the circuit above this cut-off point to be attenuated, creating what is commonly called a Low Pass Filter.

For this type of "Low Pass Filter" circuit, all the frequencies below thiscut-off, ƒc point that are unaltered with little or no attenuation and are said to be in the filters Pass band zone. This pass band zone also represents the Bandwidth of the filter. Any signal frequencies above this point cut-off point are generally said to be inthe filters Stop band zone and they will be greatly attenuated.

This "Cut-off", "Corner" or "Breakpoint" frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input. Although R = Xc, the output is not half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle(Φ)oftheoutputsignal LAGS behindthatoftheinputandatthe-3dBcut-offfrequency(ƒc)andis- 45^{o} outofphase.Thisis dueto thetimetaken to chargetheplates ofthecapacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) "lagging" behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags andthe circuitbecomes more andmore"out ofphase".

The cut-off frequency point and phase shift angle can be found by using the following equation:

CUT-OFFFREQUENCYANDPHASESHIFT

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

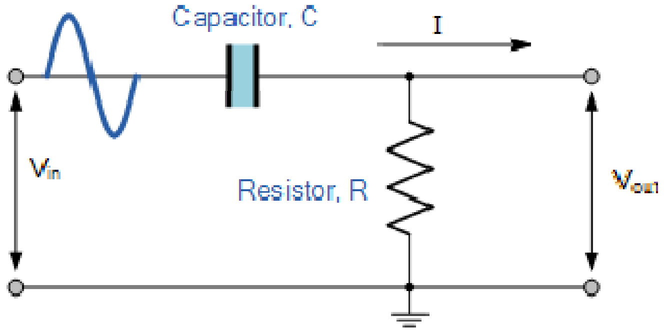
Then for our simple example of a "Low Pass Filter" circuit above, the cut-off frequency (ƒc) is given as720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45^{o}.

HIGHPASSFILTERS

A High Pass Filter or HPF, is the exact opposite to that of the previously seen Low Pass filter circuit, as now the two components have been interchanged with the output signal (Vout) being taken from across the resistor as shown.

Where as the low pass filter only allowed signals to pass below its cut-off frequency point, ƒc, the passive high pass filter circuit as its name implies, only passes signals above the selected cut-off point, ƒc eliminating any low frequency signals from the waveform. Consider the circuit below.

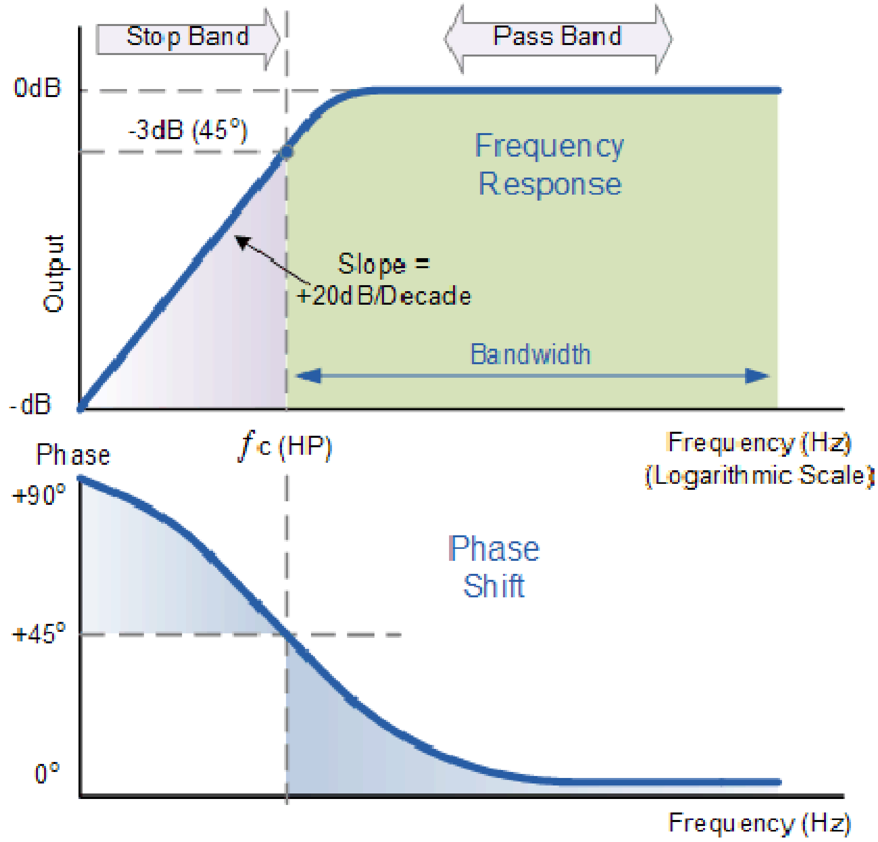
THEHIGHPASSFILTERCIRCUIT



In this circuit arrangement, the reactance of the capacitor is very high atlowfrequenciessothecapacitoractslikeanopencircuitandblocksanyinputsignals at Vin until the cut-off frequency point (ƒc) is reached. Above thiscut-offfrequency point the reactance of the capacitor has reduced sufficiently as to now act more like a short circuit allowing the entire input signal to pass directly to the output as shown below in the High Pass Frequency Response Curve.

FREQUENCYRESPONSEOFA1STORDERHIGHPASSFILTER.

$$\text{Gain (dB)} = 20 \log \frac{Vout}{Vin}$$



TheBodePlotor FrequencyResponse Curve abovefor a High Pass filter is the exact opposite to that of a low pass filter. Here the signal is attenuated or damped at low frequencies with the output increasing at +20dB/Decade (6dB/Octave) until the frequency reaches thecut-off point (ƒc) where again R = Xc. It has a response curve that extends down from infinity to the cut-off frequency, where the output voltage amplitude is 1/√2 = 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input value.

Also we can see that the phase angle (Φ) of the output signal LEADS thatof the input and is equal to+45°at frequency ƒc. Thefrequency responsecurvefor a high pass filter implies that the filter can pass all signals out to infinity. However in practice, the high pass filter response does not extend to infinity but is limited by the electrical characteristics of the components used.

The cut-off frequency point for a first order high pass filter can be found using the same equation as that of the low pass filter, but the equation for the phaseshift is modified slightly to account for the positive phase angle as shown below.

CUT-OFFFREQUENCYANDPHASESHIFT

$$fc = \frac{1}{2\pi RC}$$

$$Phase\ Shift\ \phi = arctan\ \frac{1}{2\pi fRC}$$

Thecircuitgain,AvwhichisgivenasVout/Vin(magnitude)andiscalculated as:

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + XC^2}} = \frac{R}{Z}$$

$$\text{at low } f:\ Xc \to \infty,\ \ Vout = 0$$
$$\text{at high } f:\ Xc \to 0,\ \ Vout = Vin$$

HIGHPASSFILTEREXAMPLE.

Calculate the cut-off or “breakpoint” frequency (ƒc) for asimple highpass filter consisting of an82pF capacitor connected in series with a 240kΩ resistor.

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi\ x\ 240{,}000 x 82 x 10^{-12}} = 8{,}087\ Hz\ or\ 8kHz$$

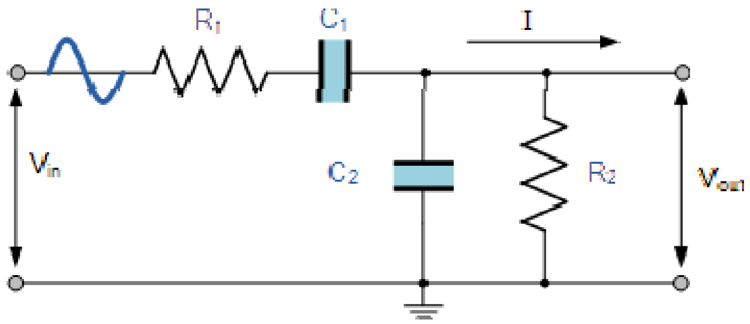
BANDPASSFILTERS

The cut-off frequency or ƒc point in a simple RC passive filter can be accurately controlled using just a single resistor in series with anon-polarized capacitor, and depending upon which way around they are connected either a low pass or a high pass filter is obtained.

One simple use for these types of Passive Filters is in audio amplifier applications or circuits such as in loudspeaker crossover filters or pre-amplifier tone controls. Sometimes it is necessary to only pass a certain range of frequencies that do not begin at 0Hz, (DC) or end at some high frequency point but are within a certain frequency band, either narrow or wide.

By connectingor “cascading” togethera singleLow Pass Filter circuitwith a High Pass Filter circuit, we can produce another type of passive RC filter that passes a selected range or “band” of frequencies that can be either narrow or wide while attenuating all those outside of this range. This new type of passive filter arrangement produces a frequency selective filter known commonly as a Band Pass Filter or BPF for short.

BANDPASSFILTERCIRCUIT



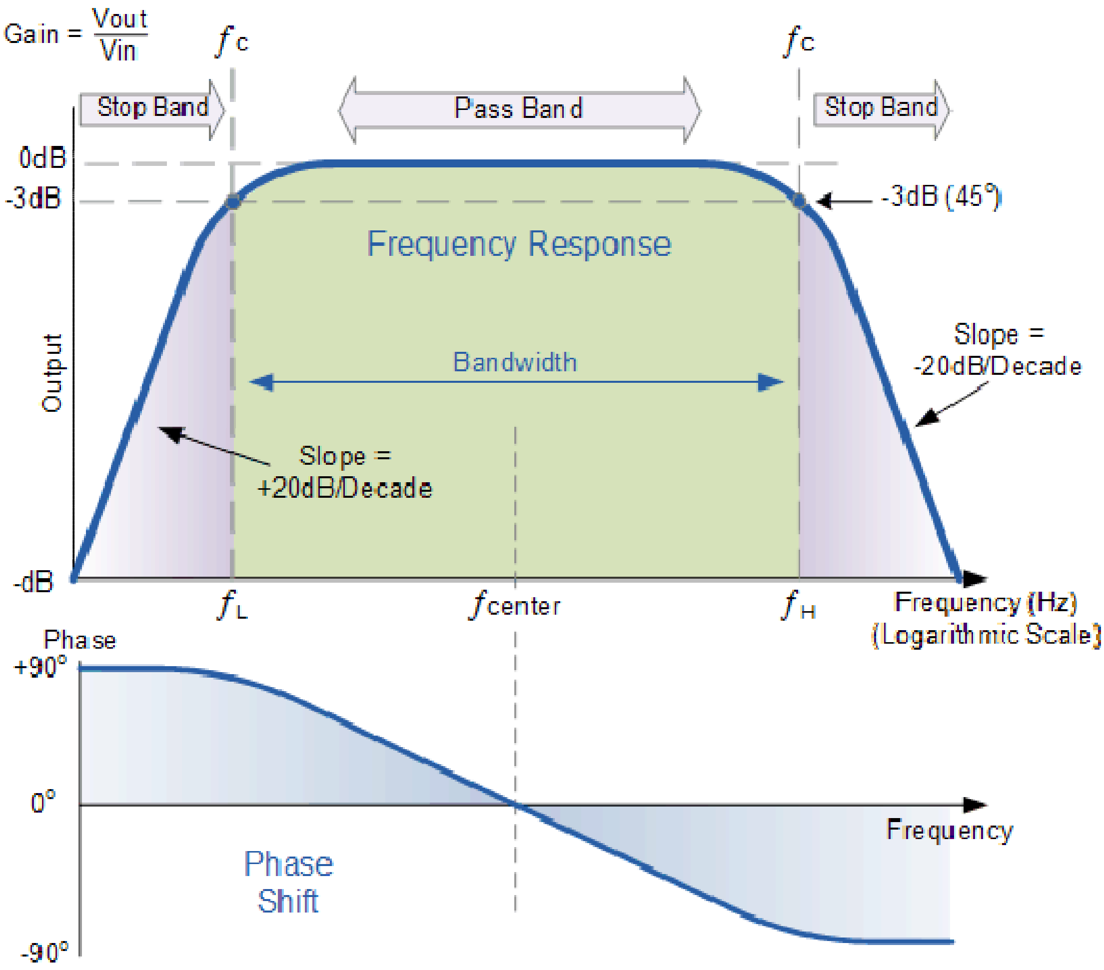
Unlike alowpass filterthat only passsignals of a low frequency range orahighpassfilterwhichpasssignalsofahigherfrequencyrange,aBandPass Filters passes signals within a certain “band” or “spread” of frequencies without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is commonly known as the filters Bandwidth.

Bandwidth is commonly defined as the frequency range that exists between two specified frequency cut-off points (ƒc), that are 3dB below the maximum centre or resonant peak while attenuating or weakening the others outside of these two points.

Then for widely spread frequencies, we can simply define the term “bandwidth”, BW as being the difference between the lower cut-off frequency ($ƒc_{LOWER}$) and the higher cut-off frequency ($ƒc_{HIGHER}$) points. In other words, BW = $ƒ_H - ƒ_L$. Clearly for a pass band filter to function correctly, the cut-off frequency of the low pass filter must be higher than the cut-off frequency for the high pass filter.

The“ideal”Band PassFilter can also beused toisolateor filter out certain frequenciesthatliewithinaparticularbandoffrequencies,forexample,noise cancellation. Band pass filters are known generally as second-order filters, (two-pole) because they have "two" reactive component, the capacitors, within their circuit design. One capacitor in the low pass circuit and another capacitor in the high pass circuit.

FrequencyResponseofa2ndOrderBandPassFilter.



The Bode Plot or frequency response curve above shows the characteristics of the band pass filter. Here the signal is attenuated at low frequencies with the output increasing at a slope of +20dB/Decade (6dB/Octave) untilthe frequency reaches the "lower cut-off" point f_L. At this frequency the output voltage is again $1/\sqrt{2}$ = 70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input.

The output continues at maximum gainuntil itreaches the "upper cut-off" point $f_H$where the output decreases at a rate of -20dB/Decade (6dB/Octave)attenuating any high frequency signals. The point of maximum output gain is generally the geometric mean of the two -3dB value between the lower and upper cut-off points and is called the "Centre Frequency" or "Resonant Peak" value f_r. This geometric mean value is calculated as being $f_r^2 = f_{(UPPER)} \times f_{(LOWER)}$.

A band pass filter is regarded as a second-order (two-pole) type filter because it has "two" reactive components within its circuit structure, then the phase anglewillbetwicethatofthepreviouslyseenfirst-orderfilters,i.e.,180°.Thephase angle of the output signal LEADSthat of the input by+90^{o} up to the centre or resonant frequency,$f_r$point were itecbmes “zero” degrees (0^{o}) or “in-phase” and then changes to LAG the input by -90^{o}as the output frequency increases.

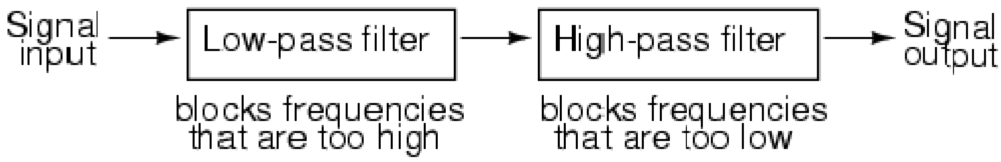
Theupperandlower cut-offfrequencypointsfor abandpassfilter can be foundusingthesameformulaasthatfor boththelowandhighpassfilters,For example.

$$fc = \frac{1}{2\pi RC}\, Hz$$

Thenclearly,thewidthofthepassbandofthefiltercanbecontrolledbythe positioning of the two cut-off frequency points of the two filters.

<u>BandPassFilterExample</u>

Asecond-order bandpassfilter istobeconstructedusingRC components that will onlyllow a rangeε of frequencies to pass above 1kHz (1,000Hz) andbelow30kHz(30,000Hz).Assumingthatboththe resistors havevalues of10kΩ´s, calculatethevaluesofthet wo capacitorsrequired.



<u>TheHighPassFilterStage</u>

The value of thecapacitor C1 required to give a cut-off frequency f_L of 1kHz with a resistor value of10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi x 1{,}000 x 10{,}000} = 15.8\, nF$$

Then, the values ofR1 andC1required for the high pass stage to give a cut-off frequency of 1.0kHz ε re:R1= 10kΩ´sand C1 =15nF.

<u>TheLowPassFilterStage</u>

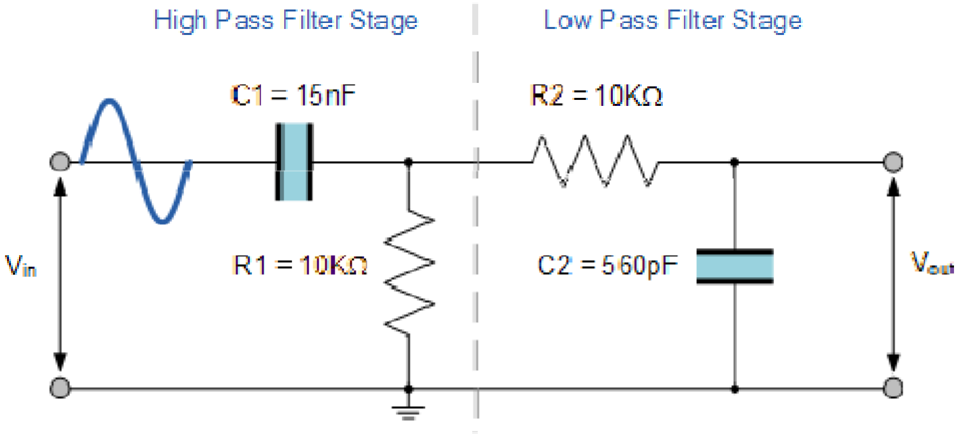
Thevalueofthecapacitor C2 requiredtogiveacut-offfrequency $f_H$of30kHzwitha resistor value of10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi x 30{,}000 x 10{,}000} = 510\, pF$$

Then,thevaluesofR2and C2requiredforthelowpassstage to givea cut- off frequency of 30kHz are, R = 10kΩ´s and C = 510pF. However, the nearest preferred value of the calculated capacitor value of 510pF is 560pF so this is used instead.

With the values of both the resistances R1 and R2 given as 10kΩ, and the twovaluesofthecapacitorsC1 and C2 foundforthehighpassandlowpassfilters as 15nF and 560pF respectively, then the circuit for oursimplepassive BandPassFilter is given as.

CompletedBandPassFilterCircuit



BandPassFilterResonantFrequency

We can also calculate the "Resonant" or "Centre Frequency" (ƒr) point of the band pass filter were the output gain is at its maximum or peak value. This peak value is not the arithmetic averageof theupperand lower -3dBcut-off pointsasyoumight expect but is in factthe "geometric" or mean value. This geometricmeanvalueiscalculatedas being $fr^2 = fc_{(UPPER)} x\ fc_{(LOWER)}$ for example:

CentreFrequencyEquation

$$fr = \sqrt{f_L\ x\ f_H}$$

- Where,$f_r$istheresonantorcentrefrequency
- $f_L$isthelower-3dBcut-offfrequencypoint
- $f_H$istheupper-3dbcut-offfrequencypoint

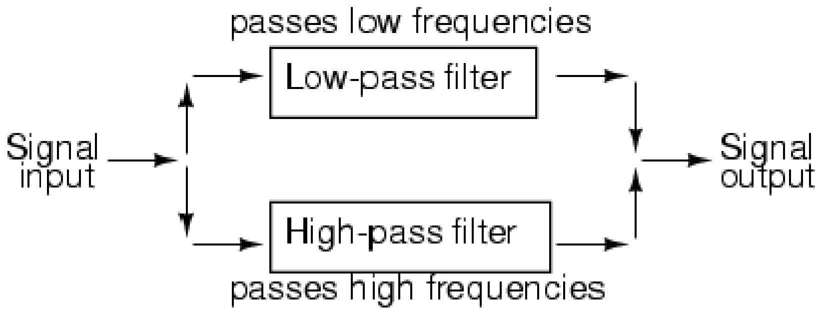
And in our simple example above, the calculated cut-off frequencies were found tobe f_L=1,060Hzandf_H=28,420Hzusingthe filtervalues.

Then by substituting these values into the above equation gives a central resonant frequency of:

$$fr = \sqrt{f_L\ x\ f_H} = \sqrt{1{,}060\ x\ 28{,}420} = 5{,}48\ kHz$$

Band-stop filters

It is so called*band-elimination*,*band-reject*, or*notch* filters; this kind of filter passes all frequencies above and below a particular range sety the component values. Not surprisingly, it can be made out of a low-pass and a high-pass filter, just like the band-pass design, except that this time we connect the two filter sections in parallel with each other instead of in series. (Figure below)



Systemlevelblockdiagramofaband-stopfilter.

Constructed using two capacitive filter sections, it looks something like (Figure below).