KINEMATICS

3. ~~Curvillinear Motion & Kinematics~~

Curvillinear Motion :- Motion of an object in curved path with variable direction of velocity is called as curvillinear motion.

Projectile :- Any object projected into the space & is moving under the influence of gravity only after projection is called as projectile.

Trajectory :- The path of the projectile is called as trajectory & the motion of the projectile is called as projectile motion.

Angle of Projection :- The angle at which the projectile being projected is called as angle of projection.

Maximum Height (H) :- It is the maximum displacement travelled by the projectile in vertical direction.

Horizontal Range (R) :- It is the maximum displacement travelled by the projectile along horizontal dir^n.

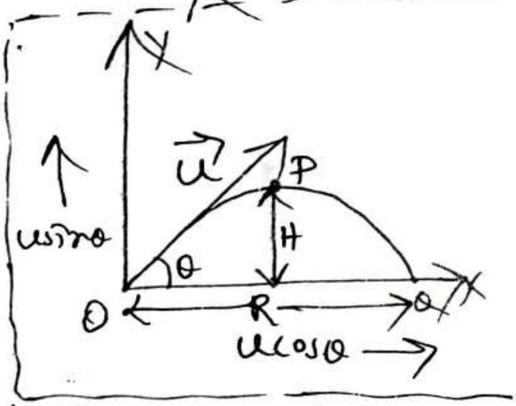
Time of Flight (T) :- It is the total time taken by the projectile to come back to the same level from which it is projected.

Time of ascent :- It is time taken to reach maximum height in vertical dir^n.

Time of descent :- Time taken to come back from max height to the level of projection

Projectile fired at an angle 'θ' with the horizontal :-

Let a projectile is projected with initial velocity '$\vec{u}$' at an angle 'θ' with the horizontal. Suppose the projectile rises to a height 'p' & then fall to the pt. 'Q' on the same level ~~projection~~ from which it was projected.



As the projectile projected in free space i-e in 2-dspace, hence initial velocity '$\vec{u}$' will be resolve into two components. such as :-

(i) $\vec{u}\cos\theta$ along horizontal dirn which is uniform as in this dirn accln due to gravity has no effect &

(ii) $\vec{u}\sin\theta$ along vertical dirn which is non-uniform as in this dirn accln due to gravity 'g' acts exactly opposite to it.

① Equation of Trajectory :-

According to the kinematic equn, the distance travelled by the projectile after time 't' is given by,

$$S = ut + \frac{1}{2}at^2 \text{ —— } ①$$

For horizontal direction suppose the distance travelled in 't' time is ~~given by~~ 'x' then it is given by,

$$x = ut + \frac{1}{2}at^2 \text{ —— } ②$$

But in horizontal dirⁿ,

$\vec{u} = \vec{u}\cos\theta$, accⁿ $a = 0$ as in this dirⁿ velocity is uniform.

Hence from ② we get, $x = u\cos\theta \cdot t + \frac{1}{2} \cdot 0$.

$$\left.\begin{aligned} &\Rightarrow x = u\cos\theta \cdot t \\ &\Rightarrow t = \frac{x}{u\cos\theta} \end{aligned}\right\} \text{— (3)}$$

Now for vertical dirⁿ suppose the distance travelled in 't' is y & is given by,

$$y = ut + \frac{1}{2}at^2 \text{ — (4)}$$

But in this vertical dirⁿ, $\vec{u} = \vec{u}\sin\theta$, accⁿ $a = -g$.

Hence from (4) we get, $\Rightarrow y = u\sin\theta \cdot t + \frac{1}{2}(-g)t^2$

$$\Rightarrow y = u\sin\theta \cdot t - \frac{1}{2}gt^2 \text{ — (5)}.$$

Now putting value of 't' from (3) in (5), we get,

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \boxed{y = \tan\theta \cdot x - \frac{g x^2}{2u^2\cos^2\theta}} \text{ — (6)}.$$

This eqⁿ is a form of parabola, hence the path of the projectile is parabolic.

(2) Time of flight :— The total time taken by the projectile to come back to the level of projection. It can be calculated at pt. 'Q' i.e at final point.

Now at 'Q' the vertical displacement covered by the projectile, $y = 0$.

Using this from eqn (5) we can write

$$0 = u\sin\theta \cdot t - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{1}{2}gt^2 = u\sin\theta \cdot t \Rightarrow \frac{1}{2}gt = u\sin\theta$$

$$\Rightarrow \boxed{t = \frac{2u\sin\theta}{g}} \quad \text{——— (7)}$$

(3) Time of ascent:- It is the time taken by the projectile at a heigh H i.e at point 'P'.

At this point, initial (vertical) velocity, $\vec{u} = \vec{u}\sin\theta$

final (vertical) velocity, $\vec{v} = 0$

distance covered, $s = H$

accl^n, $a = -g$

time (time of ascent), $= t_1$

Now Using the kinematic equation,

$$v = u + at$$

we get,

$$\Rightarrow 0 = u\sin\theta + (-g)t_1$$

$$\Rightarrow u\sin\theta - gt_1 = 0 \Rightarrow u\sin\theta = gt_1$$

$$\Rightarrow \boxed{t_1 = \frac{u\sin\theta}{g}} \quad \text{——— (8)}$$

~~Time of descent:- Here, initial (vertical) velocity $\vec{u} = \vec{u}\sin\theta$ final (vertical) velocity, $\vec{v} =$~~ .

As total time taken by the projectile is the sum of time of ascent (t_1) & time of descent (t_2)

Hence time of flight, $t = t_1 + t_2$

$$\Rightarrow \frac{2u\sin\theta}{g} = \frac{u\sin\theta}{g} + t_2$$ [using eqⁿ (7) & (8)]

$$\Rightarrow t_2 = \frac{2u\sin\theta}{g} - \frac{u\sin\theta}{g} = \frac{u\sin\theta}{g} \quad \text{--- (9)}$$

Hence time of ascent is equal to time of descent.

(4) Maximum Height (H) :-

The ~~time~~ distance ~~taken~~ taken by the projectile after commencing the maximum height 'H' and to reach at point 'P' as shown in fig B. given by, using the kinematic eqⁿ.

$$v^2 = u^2 + 2as.$$

Here at point P,
$v \rightarrow 0$
$u \rightarrow u\sin\theta$
$a \rightarrow -g$
$S \rightarrow H$

Hence we get, $0 = u^2\sin^2\theta + 2(-g)H$

$$\Rightarrow 2gH = u^2\sin^2\theta$$

$$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}} \quad \text{--- (10)}$$

This is the expression for maximum height of a projectile projected with an angle 'θ' with the horizontal.

(i) **Horizontal Range (R)** :- The total horizontal distance covered by the projectile to reach at final p is given by, from eqn (3) ~~we have~~,

$$X = u\cos\theta \cdot t$$

Now using the value of 't' from eqn (7) we get,

$$X = u\cos\theta \cdot \frac{2u\sin\theta}{g}$$

$$\Rightarrow X = \frac{u^2(2\sin\theta \cdot \cos\theta)}{g}$$

$$\Rightarrow \boxed{X = \frac{u^2\sin 2\theta}{g}} \text{ or } \boxed{R = \frac{u^2\sin 2\theta}{g}}$$

~~it is also denoted~~

This is the required expression for the ~~max~~ horizontal range covered by the projectile projected with an angle θ with the horizontal & it is also denoted by 'R'.

NOTE :- Condition for maxm range covered by the projectile :-

maxm range i.e R → maxm

But ~~for~~ for R → maxm ⇒ $\sin 2\theta = 1$

$$\Rightarrow \sin 2\theta = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

Hence the projectile will cover a maxm range when it is projected by an angle ~~θ~~ 45° with the horizontal.

Friction :-

CHAPTER-4

Defn :- Whenever a body in contact or in motion with a surface, then an opposing force comes to play tangentially at the point of contact, This force is called as friction.

Ex :- → Walking of a person on a floor by friction betn feet & floor.

→ Friction enable us to drive & stop the vehicle

Friction are of 4 types, such as
→ Static friction.
→ Kinetic friction. (Dynamic friction)
→ Sliding friction → Fluid friction

① Static Friction :-

→ Defn :- It is the force of friction comes to play when a body is forced to move along a surface but movement ~~does~~ doesn't start.

→ The magnitude of static friction remains equal to the applied force & the dirn is always opposite to dirn of motion.

→ Maximum value of static friction is limiting friction after which only the body ~~once~~ starts to move.

② Dynamic Friction

→ Defn :- It is the force of friction comes to play when a body just starts moving along a surface.

→ If the magnitude of dynamic friction is lesser ~~greater~~ than the external applied force, then only the body will move.

→ Maximum value of dynamic friction may be the sliding friction.

Ex - Pushing on a wall | Ex :- pushing a box across a floor.

Rolling friction :-

Laws of Limiting friction :-

(1) The dirⁿ of force of friction is always opposite to the dirⁿ of motion.

(2) The force of limiting friction depends upon the nature & state of polish of the surfaces in contact & it acts tangentially to the interface betⁿ the two surfaces.

(3) The magnitude of limiting friction (F) is directly proportional to magnitude of normal reaction R betⁿ the two surface in contact.

i.e $F_l \propto R \Rightarrow F_l = \mu_l R \Rightarrow \mu_l = \frac{F_l}{R}$

Here $\mu_l \rightarrow$ co-efficient of limiting friction

(4) Magnitude of limiting friction is independent of the area & shape of the surf in contact, so long as normal reaction remains same.

Methods of Reducing friction :-

→ By rubbing & polishing, friction force can be red

→ By using lubricants on the surfaces in contact, friction may reduce

→ If we convert sliding friction into rolling friction

→ By streamlining the shape of the body, the fluid friction can be reduce.

Unit 5,6 Gravitation, Planetary Motion & S.H.M

Gravitation :- Whenever an object is released in free space, it falls towards the earth. It appears that earth attracts everything towards it which is called Gravitation.

Kepler's Laws of Planetary Motion :-

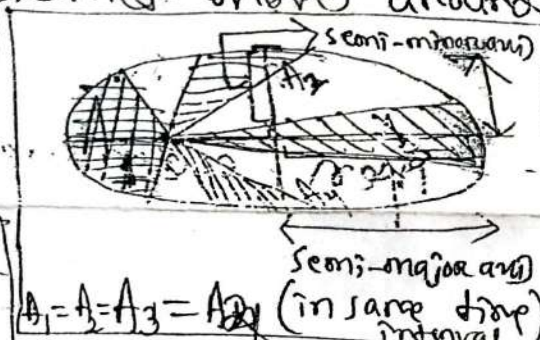
1st Law :- (Law of elliptical orbits)

It states that " a planet moves around the sun in an elliptical orbit with sun situated at one of its foci".

2nd Law :- (Law of areal velocity)

It states that " a planet moves around the sun in such a way that its areal velocity ~~remains constant~~

i.e $\boxed{\text{Areal velocity} = \text{constant}}$



$A_1 = A_2 = A_3 = A_4$ (in same time interval)

3rd law :- (The Harmonic law) → Law of Timeperiod

It states that " a planet moves around the sun in such a way that the square of its time period is directly proportional to the cube of the semi-major axis of its elliptical orbit".

i.e $\boxed{T^2 \propto R^3}$

where, $T \to$ Time period

& $R \to$ Semi-major axis.

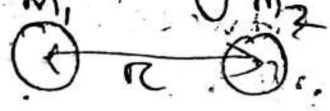
i.e if the distance between the planet & sun is more, hence more time it will take to complete its rotation.

Newton's law of Gravitation:-

Statement:- It states that "every particle of the matter in the universe attracts every other particle with a force which directly proportional to the product of the masses of the two particles & inversely proportional to the square of the distance between them".

⇒ This force of attraction between any two bodies in the universe is known as force of gravitation.

Let m_1 & m_2 → masses of two bodies



& F → Force of attraction betn them

& r → distance betn the two bodies.

Then according to Newton's law,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \boxed{F = G\frac{m_1 m_2}{r^2}}$$

where G → a constant of proportionality called as Universal gravitational constant. ~~& its value is same~~

If $m_1 = m_2 = 1$ unit & $r = 1$ unit

Then, $\boxed{F = G}$

Hence, the gravitational constant 'G' is defined as the magnitude of the force of attraction between two ~~masses~~ bodies each of unit mass & separated by a unit distance from each other.

Unit of G :-

S.I

We know, $F = G\frac{m_1 m_2}{r^2}$

$\Rightarrow G = \frac{FR^2}{m^2}$

Hence in S.I, $= \frac{Newton \times m^2}{kg^2}$

or $N \cdot m^2 kg^{-2}$

& in CGS Dyne. $cm^2 \cdot g^{-2}$

Dimension (G.) :-

$G = \frac{FR^2}{m^2}$

$\Rightarrow [G] = \frac{[M^1 L^1 T^{-2}][L^2]}{[M^2]}$

$= [M^{-1} L^3 T^{-2}]$

Acceleration due to gravity 'g' :-

The acc^n produced by weight (gravity) of a body is called as acc^n due to gravity & is denoted by 'g'.

i.e Gravity (weight) = mg

If m & M → mass of a particle & earth respectively

R → distance bet^n the particle placed on the earth surface & centre of earth

Acc. to, Newton's law,

$$F = G\frac{Mm}{R^2} \Rightarrow mg = G\frac{Mm}{R^2}$$

$$\Rightarrow \boxed{g = G\frac{M}{R^2}}$$

Unit of g :-

In S.I → ~~N/kg~~ m/s^2

In CGS → cm/s^2

Dimension of g

$[g] = [M^0 L^1 T^{-2}]$

Difference betn G & g:-

G	g
→ It is called as universal gravitational constant.	→ It is called as accln due to gravity.
→ It's value remains const everywhere, so it is called as universal const. of gravitation.	→ It's value changes at diff. place of earth as it depends upon mass & radius of the planet
→ It's units are $N m^2 kg^{-2}$ (SI) & $dyne\,cm^2 g^{-2}$ (cgs)	→ It's unit is same as accln i.e m/s^2 or cm/s^2
→ It's dimension is $[M^{-1}L^3T^{-2}]$	→ It's dimn is $[M^0L^1T^{-2}]$
→ $G = \frac{Fr^2}{m^2}$	→ $g = G\frac{M}{R^2}$
→ value of $G = 6.67 \times 10^{-11} Nm^2/kg^2$	→ value of $g = 9.8\ m/s^2$

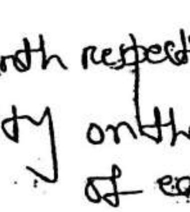
Variation of 'g' with Altitude:-

Consider a body of mass 'm' placed on the surface of the earth.

Let M & R ⇒ Mass & radius of earth respectively.

g ⇒ accln due to gravity on the surface of earth.

Then, $g = \frac{GM}{R^2}$, where, G ⇒ Gravitational constant.



If the body is taken to a height 'h' above the surface of earth & the accln due to gravity at this height is 'g''.

Then, $g' = \frac{GM}{(R+h)^2}$

Now, $\frac{g'}{g} = \left[\frac{GM}{(R+h)^2}\right] \Big/ \frac{GM}{R^2}$

$\Rightarrow \frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow \frac{g'}{g} = 1 - \frac{2h}{R} \Rightarrow g' = g\left(1-\frac{2h}{R}\right) = g - \frac{2gh}{R}$

$\Rightarrow g' - g = -\frac{2gh}{R} \Rightarrow \boxed{g - g' = \frac{2gh}{R}}$

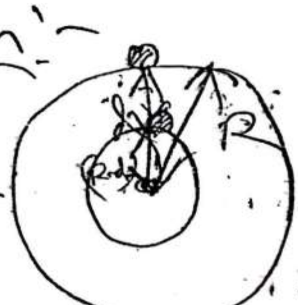
As g & $R \rightarrow$ are constants at a given place on earth

hence, $g - g' \propto h$

It is concluded that if h increases, g' will decrease to increase (g-g') value as g is constant.

Thus the value of acc^n due to gravity 'g' decreases with [redacted] in height above the earth surface.

Variation of 'g' with depth :-



Lets consider the ~~body of mass~~ earth as a homogeneous sphere of radius 'R', mass 'M' density 'ρ'.

Lets consider a body lying on the surface of earth where the acc^n due to gravity is 'g' & it is given by,

$$g = \frac{GM}{R^2}$$

We know, Mass = vol^m × density

$$= \frac{4}{3}\pi R^3 \times \rho$$

Now,
$$g = \frac{G\left(\frac{4}{3}\pi R^3 \rho\right)}{R^2} = \frac{4}{3}\pi G R \rho$$

Let the body taken to a depth 'd' bellow the earth surface, where the accl^n due to gravity is 'g'' & is given by,

$$g' = \frac{4}{3}\pi G (R-d)\rho$$

Now,
$$g'/g = \frac{(4/3)\pi \cancel{G}(R-d)\cancel{\rho}}{(4/3)\pi \cancel{G} R \cancel{\rho}} = \frac{R-d}{R}$$

$$\Rightarrow \frac{g'}{g} = 1 - \frac{d}{R} \Rightarrow g' = g - \frac{dg}{R}$$

$$\Rightarrow \boxed{g - g' = \frac{d}{R} g}$$

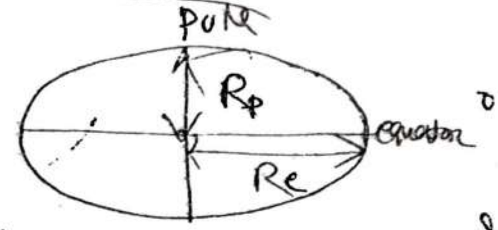
i-e $(g-g') \propto d$

i-e if depth 'd' increase, $(g-g')$ value will increas. But 'g' is constant on the surface, hence to increase $(g-g')$ value with increase of h, 'g'' value will decrease.

Thus the value of accl^n due to gravity 'g' decreases with increase in depth.

Variation of 'g' with latitude:-

The value of acclⁿ due to gravity changes with ~~altitude~~ latitude due to shape of the earth.



As the shape of the earth is not a perfect sphere. i.e flattened at poles & bulges out at the equator. Thus $R_e > R_p$.

As $g = \frac{GM}{R^2}$

& G, M ~~are~~ constants,

Hence, $g \propto \frac{1}{R^2}$

→ As at pole $R \to$ minⁿ, hence g is greater at pole

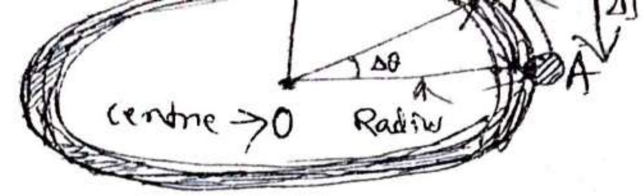
& as at equator $R \to$ maxⁿ, hence g is least at equator.

Circular Motion :-

Defⁿ :- The motion of a body is said to be circular if it moves in such a way that its distance from a certain fixed point always remains the same.

Uniform Circular Motion:-

Defⁿ:- Circular motion is said to be uniform if the speed of the particle along the circular path remains the same.



Angular displacement :- (θ)

Defn :- It is defined as the angle turned by the radius vector of the particle (undergoing rotational) motion.

If $\Delta S \rightarrow$ linear disp.

$r \rightarrow$ radius of the circular path.

$\Delta\theta \rightarrow$ angular disp.

Then, $\boxed{\Delta S = r \times \Delta\theta}$ or $\boxed{\Delta\vec{S} = \vec{r} \times \Delta\theta}$

Angular velocity :- (ω)

Defn :- It is defined as the rate of change of angular disp. with time.

i.e Angular velocity, $\boxed{\omega = \frac{d\theta}{dt}}$.

& the relation betn angular velocity & linear velocity is, $\boxed{V = r \times \omega}$ or $\boxed{\vec{V} = \vec{r} \times \vec{\omega}}$

where, $\omega \rightarrow$ Angular velocity.

Angular Acceleration :- (α)

Defn :- It is defined as the rate of change of angular velocity with time.

i.e angular accln, $\boxed{\alpha = \frac{d\omega}{dt}}$

& reln betn angular accln, linear accln is, $\boxed{a = r \times \alpha}$ or $\boxed{\vec{a} = \vec{r} \times \vec{\alpha}}$

Simple Harmonic Motion (S.H.M) :-

Defn :- The motion of a particle is said to be S.H.M if its acceleration is directly proportional to the disp. & is always directed towards the mean position. (Ex :- Vibration of simple pendulum, ...)

The eqn for S.H.M is given by,

$$\boxed{y = r \sin(\omega t + \phi)}$$

Where,
$y \rightarrow$ displacement.
$r \rightarrow$ amplitude of S.H.M.
$\omega \rightarrow$ angular velocity.
$\phi \rightarrow$ phase angle.

S.H.M parameters :-

(1) **Amplitude** :- Amplitude of a particle executing in S.H.M is defined as its maximum displacement on either side of its mean position.

(2) **Frequency** :- The no. of oscillations made in unit time by the oscillating particle is called its frequency.

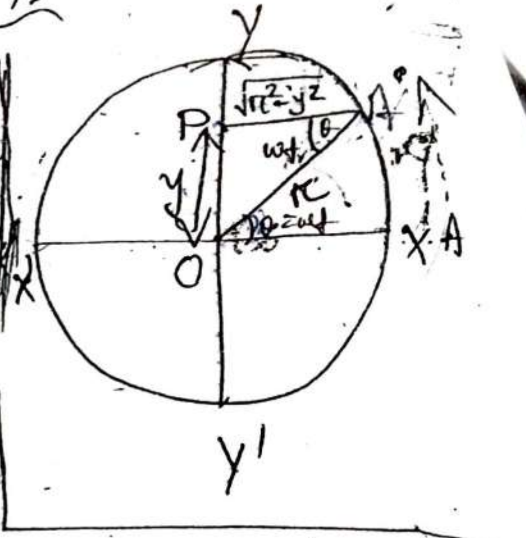
It is given by,

$$n = \frac{1}{T} = \frac{1}{\frac{2\pi}{\omega}} = \frac{\omega}{2\pi}$$

$$= \frac{1}{2\pi}\sqrt{\frac{\text{accl}^n}{\text{disp.}}}$$

S.H.M as a projection of a uniform Circular motion (on any diameter):-

Consider a particle 'A' moving in uniform circular motion in a circular path having XOX' & YOY' as its horizontal & vertical diameter, as shown in fig.



Let 'O' be the projection (foot of the $\perp^{ar}$ drawn from A) of A while it is at 'X'. Let A moves towards Y and after some time it reaches at a point it projection is at 'P' i.e on moving from X to Y along the vertical diameter Projection moves from O to Y & again A moves from to Y to X' & projection moves from Y to O.

Thus A completes its journey along the circumference of the circle, its projection move from O to Y, Y to O, O to Y' & Y' to O.

Hence the motion along YOY' is called S.H.M. So S.H.M is defined as the projection of uniform circular motion on the diameter of circle of reference.

Derivation of disp./velocity & Accl^n:-

(a) Displacement:- (y)

Disp. of a particle vibrating in S.H.M at any point on the circular path is defined as its distance from the mean position at that instant.

In the above fig, 'P' is the projection of particle 'A' at some instant after time 't'

In Δ OAP, $\sin\theta = \frac{OP}{OA} = \frac{y}{r}$

$\Rightarrow y = r\sin\theta$ —— (1)

As, angular velocity, $\omega = \frac{\theta \text{ (angular disp)}}{t \text{ (time)}}$

$\Rightarrow \theta = \omega t$ —— (2)

Using (2) in (1) we get,

$\boxed{y = r\sin\omega t}$ —— (3) $\Rightarrow \boxed{\frac{y}{r} = \sin\omega t}$ (3a)

Special case :-

Disp. y will be max^n when $\sin\omega t$ is max^n & min^n when $\sin\omega t$ is min^n.

Maximum value: y $\rightarrow$ max^n if $\sin\omega t = 1$

$\Rightarrow y = r$ (radius of circular path / amplitude of vibration)

Min^n value:- $y \rightarrow$ min^n if $\sin\omega t = -1$

$\Rightarrow y = -r$

(ii) Velocity :- (V)

From the defination of velocity we know, velocity is the rate of change of disp.

i.e $V = \frac{dy}{dt}$

But from (3), $y = r \sin \omega t$

Hence, $$V = \frac{dy}{dt} = \frac{d}{dt}(r \sin \omega t)$$

$$\Rightarrow V = r \frac{d}{dt} \sin \omega t$$

$$\Rightarrow V = r \cos \omega t \frac{d}{dt}(\omega t)$$

$$\Rightarrow V = r \cdot \cos \omega t \cdot \omega$$

$$\Rightarrow \boxed{V = (r\omega) \cos \omega t} \quad \text{--- (4)}$$

~~But, linear velocity = radius × angular velocity~~

~~i.e $v = r\omega$~~

~~Thus $V = v \cos \omega t$ --- (4)~~

In Δ OAP, $$\cos \omega t = \frac{AP}{OA} = \frac{\sqrt{r^2 - y^2}}{r}$$

Using this in (4) we get

$$V = r\omega \frac{\sqrt{r^2 - y^2}}{r} \Rightarrow \boxed{V = \omega \sqrt{r^2 - y^2}}$$

Spl cases:-

(i) At O, $y = 0 \Rightarrow V = \omega\sqrt{r^2} = \omega r = v$

(ii) At Y/Y', $y = \pm r \Rightarrow V = \omega\sqrt{r^2 - r^2} \Rightarrow V = 0$.

Thus a particle executing in S.H.M. passes with maxm velocity through the mean position and is at rest at the extreme position.

Acceleration :-

As accl^n is the rate of change of velocity,

$$\Rightarrow accl^n = \frac{dv}{dt}$$

$$\Rightarrow a = \frac{d}{dt}(r\omega \cos \omega t) \quad [\text{using } (4)]$$

$$= r\omega \frac{d}{dt} \cos \omega t$$

$$= r\omega(-\sin \omega t) \frac{d}{dt}(\omega t)$$

$$= -r\omega^2 \sin \omega t \quad \text{——— } (5)$$

In the $\triangle^{le}$ OAP, $\sin \omega t = \frac{y}{r}$

Using this in (5) we get,

$$a = -r\omega^2 \cdot \frac{y}{r}$$

$$\Rightarrow \boxed{a = -\omega^2 y}$$

Special case :-

(i) At O, $y = 0 \Rightarrow a = 0$

(ii) At Y/Y', $y = \pm r \Rightarrow a = \pm \omega^2 r$

Thus a particle vibrating in S.H.M has zero accl^n while passing through mean position and has max^m acceleration while at extreme position.

(d) Time Period :- (T)

It is the time taken by the oscillating particle to complete one ~~vibration~~ oscillation.

It is given by, $T = \frac{2\pi}{\omega}$

where, $\omega \rightarrow$ Angular velocity.

But we know, $accl^n = -\omega^2 y$

$$\Rightarrow -\frac{accl^n}{y} = \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{accl^n}{disp}}$$

Thus, $T = \frac{2\pi}{\sqrt{\frac{accl^n}{disp}}} = 2\pi\sqrt{\frac{disp}{accl^n}}$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{disp}{accl^n}}}$$