LECTURE NOTES ON

ELECTRICAL MEASUREMENTS & INSTRUMENTATION

FOR

ELECTRICAL ENGINEERING (4TH SEMESTER)

DEPARTMENT OF ELECTRICAL ENGINEERING

GOVERNMENT POLYTECHNIC

BHUBANESWAR, ODISHA,INDIA

SYLLABUS

1. MEASURING INSTRUMENTS

1.1 Define Accuracy, precision, Errors, Resolutions Sensitivity and tolerance.
1.2 Classification of measuring instruments.
1.3 Explain Deflecting, controlling and damping arrangements in indicating type of instruments.
1.4 Calibration of instruments.

2. ANALOG AMMETERS AND VOLTMETERS

2.1. Describe Construction, principle of operation, errors, ranges merits and demerits of:
2.1.1 Moving iron type instruments.
2.1.2 Permanent Magnet Moving coil type instruments.
2.1.3 Dynamometer type instruments
2.1.4 Rectifier type instruments
2.1.5 Induction type instruments
2.2 Extend the range of instruments by use of shunts and Multipliers.
2.3 Solve Numerical

3. WATTMETERS AND MEASUREMENT OF POWER

3.1 Describe Construction, principle of working of Dynamometer type wattmeter. (LPF and UPF type)
3.2 The Errors in Dynamometer type wattmeter and methods of their correction.
3.3 Discuss Induction type watt meters.

4. ENERGYMETERS AND MEASUREMENT OF ENERGY

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6. MEASUREMENT OF RESISTANCE, INDUCTANCE& CAPACITANCE

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6.1..3. Measurement of high resistance by loss of charge method.
6.2 Construction, principle of operations of Megger & Earth tester for insulation resistance and earth resistance measurement respectively.
6.3 Construction and principles of Multimeter. (Analog and Digital)

6.4 Measurement of inductance by Maxewell’s Bridge method.

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7. SENSORS AND TRANSDUCER

7.1. Define Transducer, sensing element or detector element and transduction elements.

7.2. Classify transducer. Give examples of various class of transducer.

7.3. Resistive transducer

7.3.1 Linear and angular motion potentiometer.

7.3.2 Thermistor and Resistance thermometers.

7.3.3 Wire Resistance Strain Gauges

7.4. Inductive Transducer

7.4.1 Principle of linear variable differential Transformer (LVDT)

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7.5. Capacitive Transducer.

7.5.1 General principle of capacitive transducer.

7.5.2 Variable area capacitive transducer.

7.5.3 Change in distance between plate capacitive transducer.

7.6. Piezo electric Transducer and Hall Effect Transducer with their applications.

8. OSCILLOSCOPE

8.1. Principle of operation of Cathode Ray Tube.

8.2. Principle of operation of Oscilloscope (with help of block diagram).

8.3. Measurement of DC Voltage & current.

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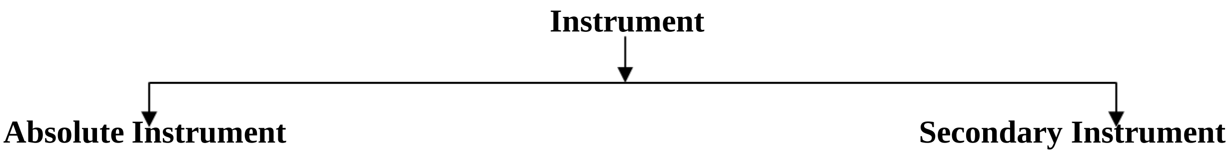
MEASURING INSTRUMENTS

Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.



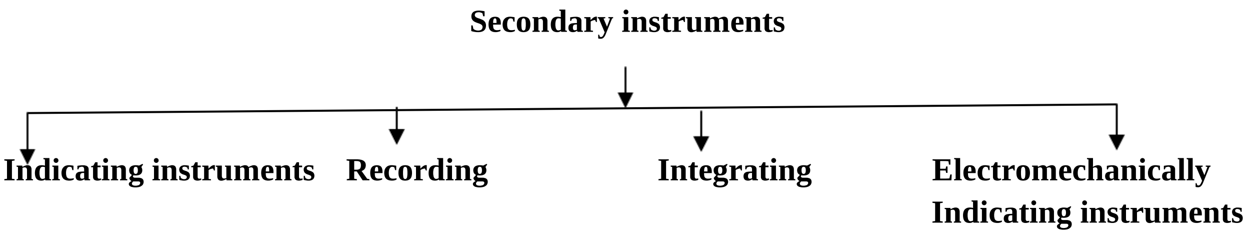
Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary.
They are

(a) Deflecting force

(b) Controlling force

(c)Damping force

Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

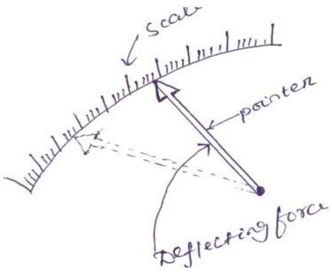


Fig. 1.1 Pointer scale

Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

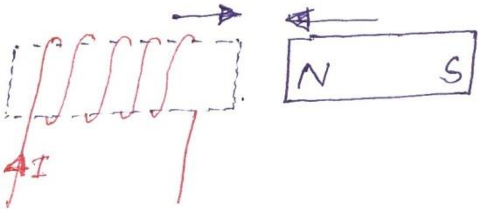


Fig. 1.2

If two soft iron pieces are place near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

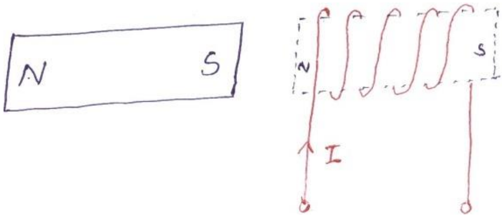


Fig. 1.3

Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electrodynamometer type instrument.

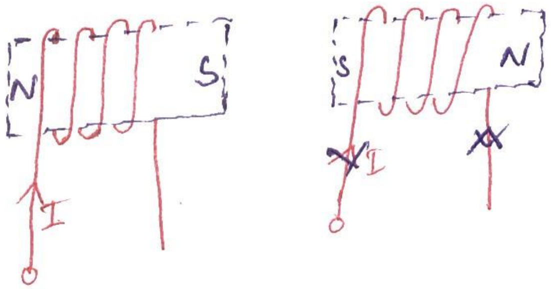


Fig. 1.4

Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \tag{1.1}$$

Spring control

Two springs are attached on either end of spindle (Fig. 1.5).The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection θ.

$$T_C \propto \theta \tag{1.2}$$

The deflecting torque produced T_d proportional to 'I'. When $T_C = T_d$, the pointer will come to a steady position. Therefore

$$\theta \propto I \tag{1.3}$$

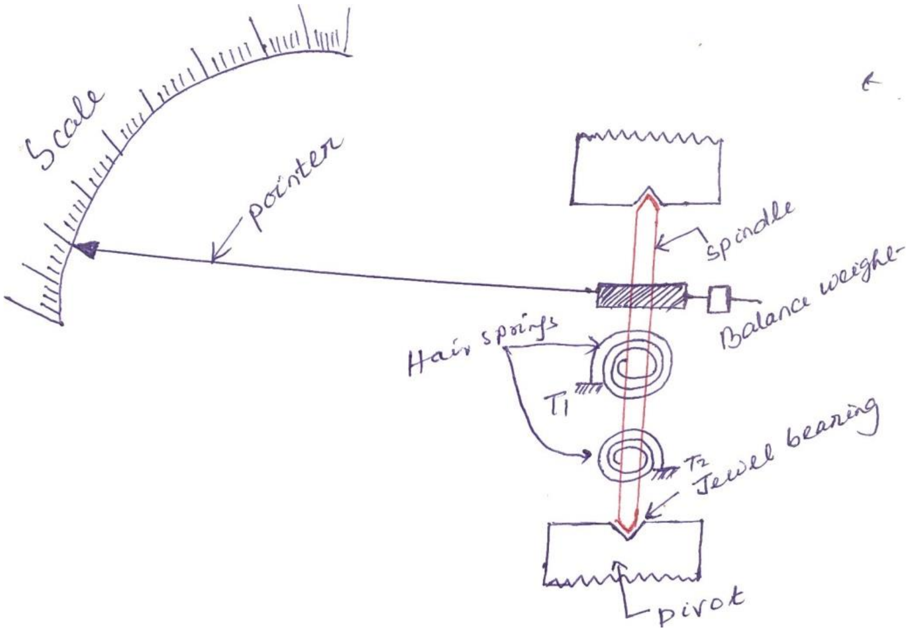


Fig. 1.5

Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform.

Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation is quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

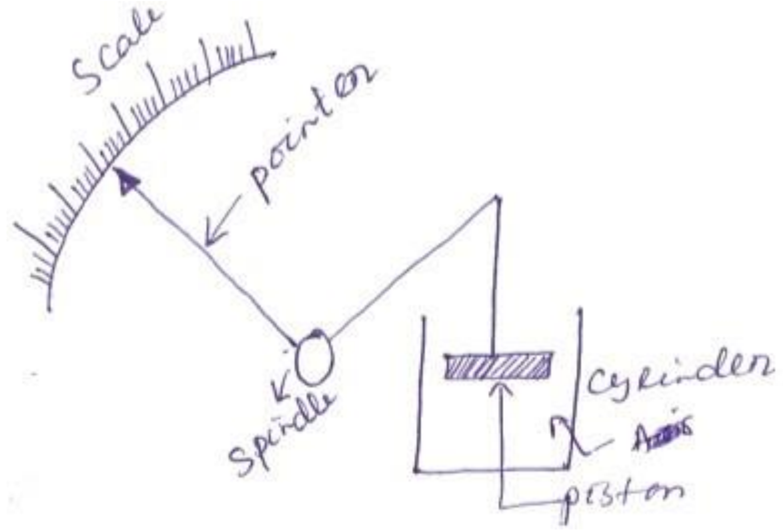


Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

Eddy current damping

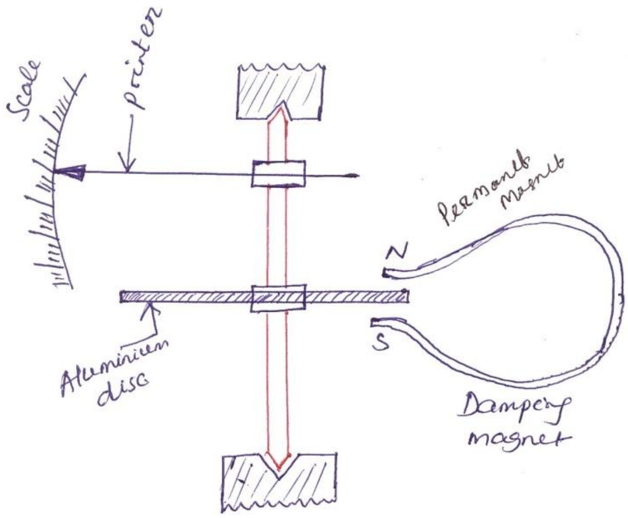


Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

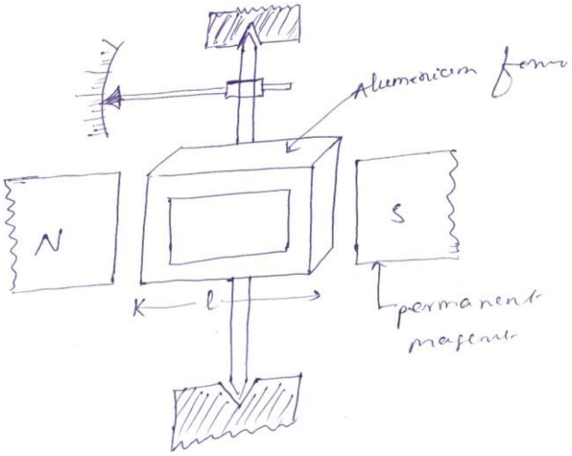


Fig. 1.6 Rectangular type

ANALOG AMMETER AND VOLTMETER

Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

Construction: A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminum former.
Control: Spring control is used.

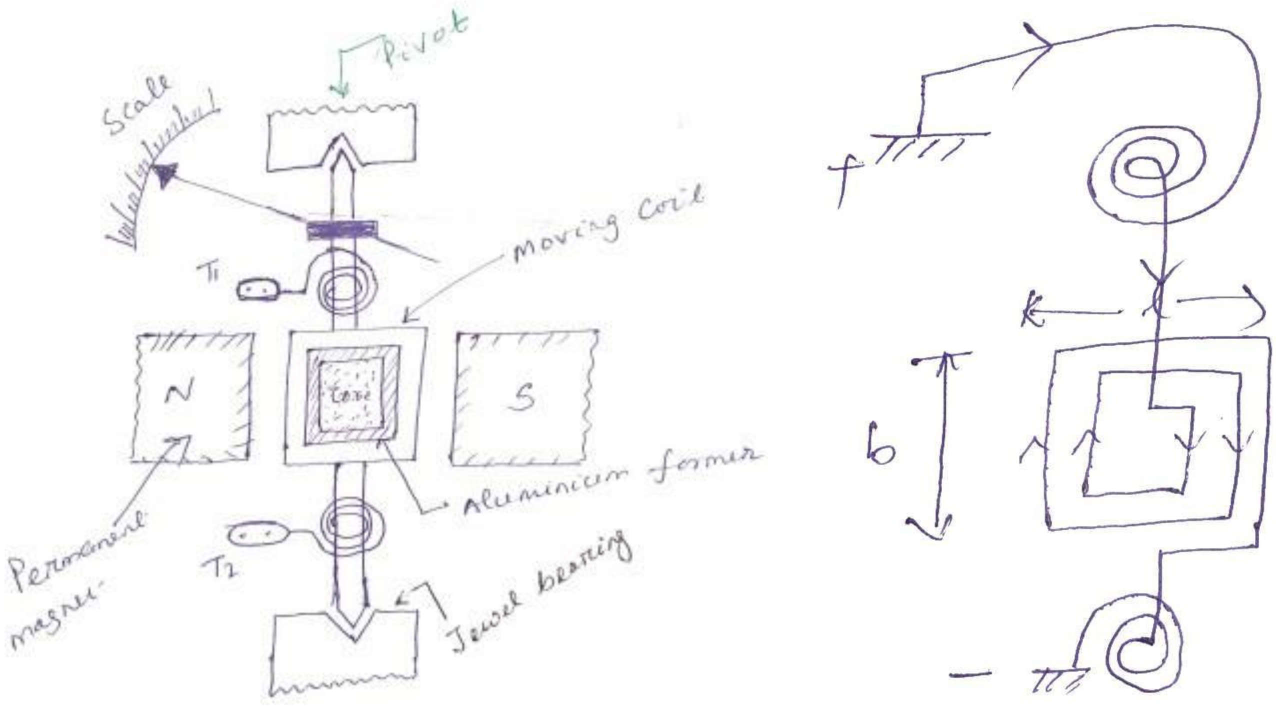


Fig. 1.7

Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.

If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC

Let T_d =deflecting torque

T_C = controlling torque

θ = angle of deflection

K=spring constant

b=width of the coil

l=height of the coil or length of coil

N=No. of turns

I=current

B=Flux density

A=area of the coil

The force produced in the coil is given by

$F = BIL \sin\theta$ (1.4)

When $\theta = 90^\circ$

For N turns, $F = NBIL$ (1.5)

Torque produced $T_d = F \times \perp_r$ distance (1.6)

$T_d = NBIL \times b = BINA$ (1.7)

$T_d = BANI$ (1.8)

$T_d \propto I$ (1.9)

Advantages

- ✓ Torque/weight is high
- ✓ Power consumption is less
- ✓ Scale is uniform
- ✓ Damping is very effective
- ✓ Since operating field is very strong, the effect of stray field is negligible
- ✓ Range of instrument can be extended

Disadvantages

- ✓ Use only for D.C.
- ✓ Cost is high
- ✓ Error is produced due to ageing effect of PMMC
- ✓ Friction and temperature error are present

Extension of range of PMMC instrument Case-

I: Shunt

A low shunt resistance connected in parrel with the ammeter to extent the range of current. Large current can be measured using low current rated ammeter by using a shunt.

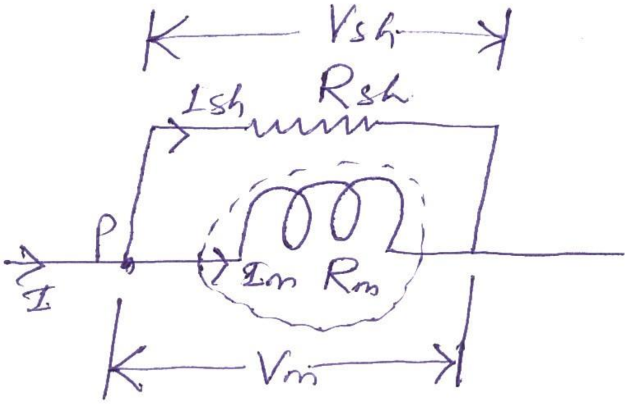


Fig. 1.8

Let R_m =Resistance of meter

R_{sh} =Resistance of shunt

I_m = Current through meter

I_{sh} =current through shunt

I= current to be measure

$$\therefore V_m = V_{sh} \tag{1.10}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m} \tag{1.11}$$

Apply KCL at 'P' $I = I_m + I_{sh}$ (1.12)

Eq^n (1.12) ÷ by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m} \tag{1.13}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \tag{1.14}$$

$$\therefore\ I = I_m\left(1 + \frac{R_m}{R_{sh}}\right) \tag{1.15}$$

$\left(1 + \frac{R_m}{R_{sh}}\right)$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermoelectric emf. The change is resistance, due to change in temperature is negligible.

Case (II): Multiplier

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter of small rating with a multiplier.

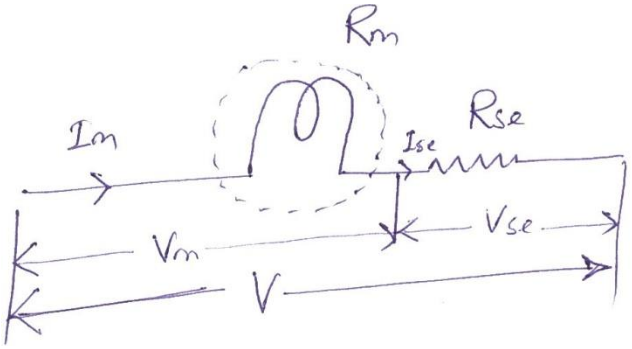


Fig. 1.9

Let R_m =resistance of meter

R_{se} =resistance of multiplier

V_m =Voltage across meter

V_{se} = Voltage across series resistance

V= voltage to be measured

$$I_m = I_{se} \tag{1.16}$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}} \tag{1.17}$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m} \tag{1.18}$$

Apply KVL, $V = V_m + V_{se}$ (1.19)

Eq^n (1.19) $\div V_m$

$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = \left(1 + \frac{R_{se}}{R_m}\right) \tag{1.20}$$

$$\therefore V = V_m\left(1 + \frac{R_{se}}{R_m}\right) \tag{1.21}$$

$$\left(1 + \frac{R_{se}}{R_m}\right) \rightarrow \text{Multiplication factor}$$

Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let 'θ' be the deflection corresponding to a current of 'i' amp

Let the current increases by di, the corresponding deflection is '$\theta + d\theta$'

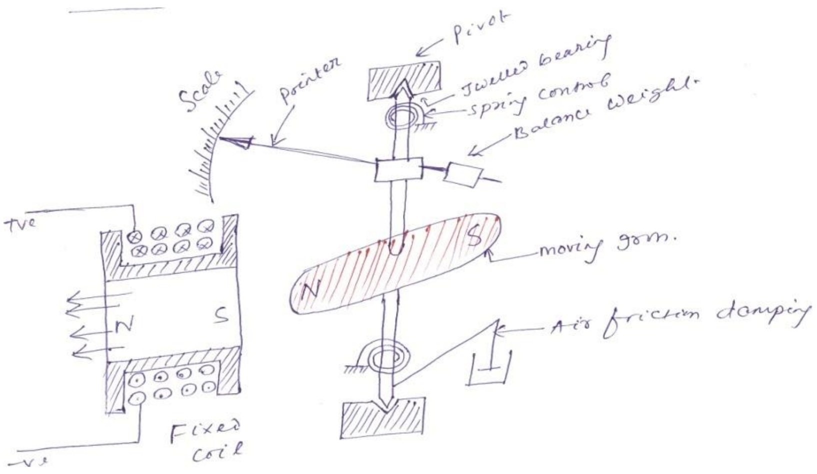


Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ‘L+dL’. The current change by ‘di’ is dt seconds.

Let the emf induced in the coil be ‘e’ volt.

$$e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt} \tag{1.22}$$

Multiplying by ‘idt’ in equation (1.22)

$$e \times idt = L\frac{di}{dt} \times idt + i\frac{dL}{dt} \times idt \tag{1.23}$$

$$e \times idt = Lidi + i^2 dL \tag{1.24}$$

Eq^n (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.

Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2}Li^2$$

$$= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{Li^2 + 2Lidi + i^2dL + 2ididL - Li^2\}$$

$$= \frac{1}{2}\{2Lidi + i^2dL\}$$

$$= Lidi + \frac{1}{2}i^2dL \tag{1.25}$$

Mechanical work to move the pointer by $d\theta$

$$= T_d\, d\theta \tag{1.26}$$

By law of conservation of energy,
Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi + i^2dL = Lidi + \frac{1}{2}i^2dL + T_d\, d\theta \tag{1.27}$$

$$\frac{1}{2}i^2dL = T_d\, d\theta \tag{1.28}$$

$$T_d = \frac{1}{2}i^2\frac{dL}{d\theta} \tag{1.29}$$

At steady state condition $T_d = T_C$

$$\frac{1}{2}i^2\frac{dL}{d\theta} = K\theta \tag{1.30}$$

$$\theta = \frac{1}{2K}i^2\frac{dL}{d\theta} \tag{1.31}$$

$$\theta \propto i^2 \tag{1.32}$$

When the instruments measure AC, $\theta \propto i^2{}_{rms}$

Scale of the instrument is non uniform.

Advantages

- ✓ MI can be used in AC and DC
- ✓ It is cheap
- ✓ Supply is given to a fixed coil, not in moving coil.
- ✓ Simple construction
- ✓ Less friction error.

Disadvantages

- ✓ It suffers from eddy current and hysteresis error
- ✓ Scale is not uniform
- ✓ It consumed more power
- ✓ Calibration is different for AC and DC operation

Repulsion type moving iron instrument

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

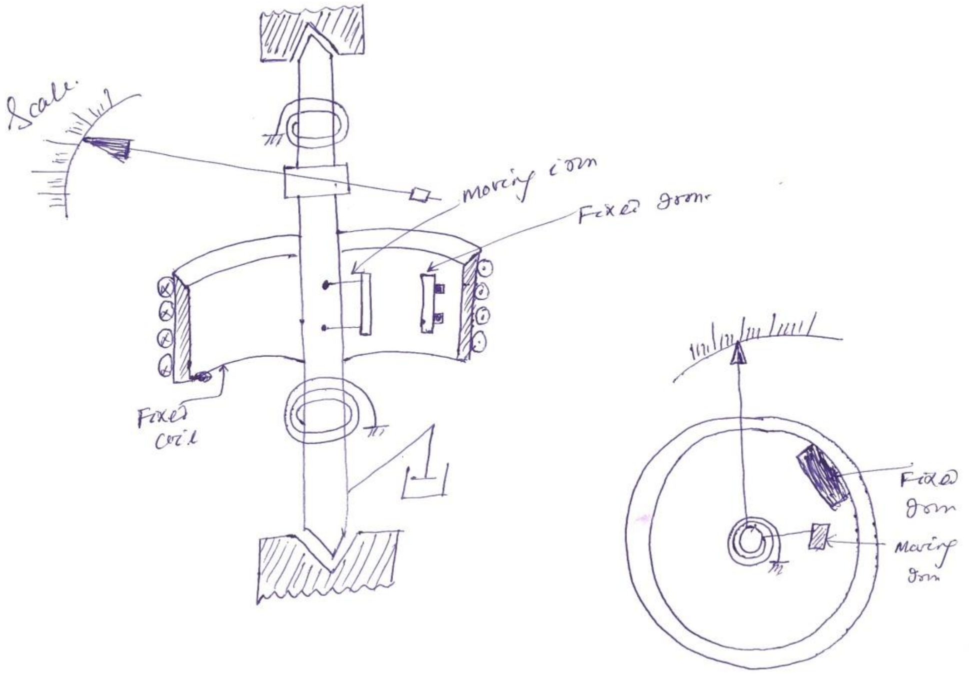


Fig. 1.12

Dynamomeeter (or) Electromagnetic moving coil instrument (EMMC)

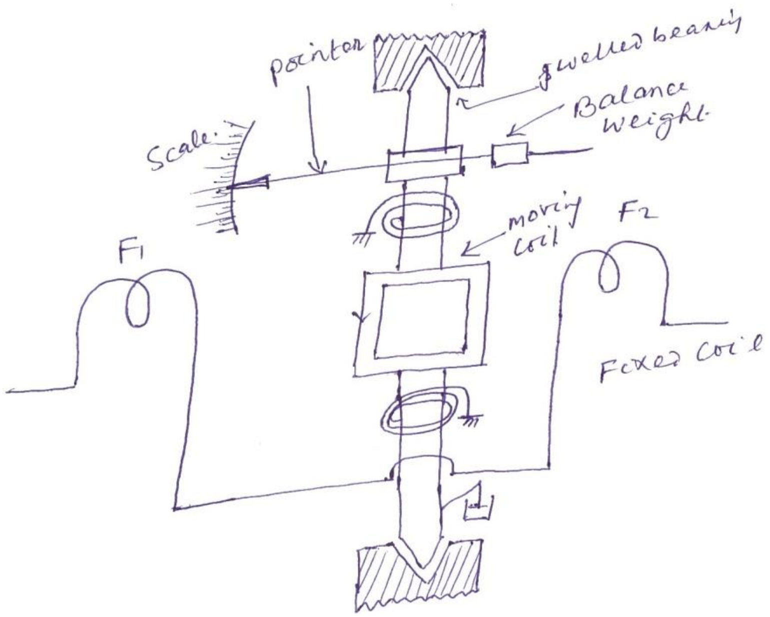


Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction:A fixed coil is divided in to two equal half. The moving coil is placed between the two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving coil is wounded.

Control: Spring control is used.

Damping: Air friction damping is used.

Principle of operation:

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i_1 and i_2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC

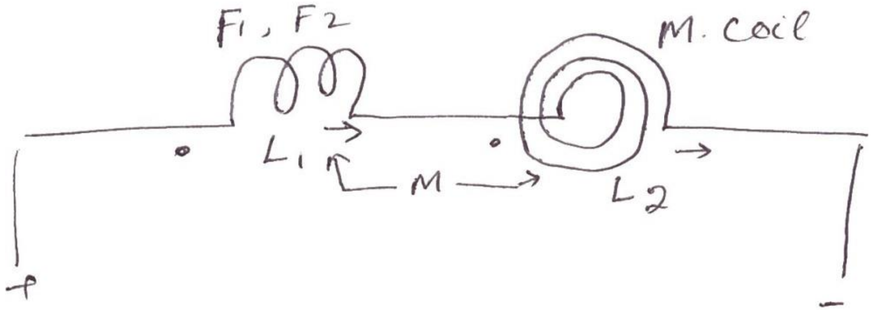


Fig. 1.14

Let

L_1=Self inductance of fixed coil

L_2= Self inductance of moving coil

M=mutual inductance between fixed coil and moving coil

i_1=current through fixed coil

i_2=current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \tag{1.33}$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \tag{1.34}$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta}(L_1 + L_2 + 2M) \tag{1.35}$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \tag{1.36}$$

$$T_d = i^2 \frac{dM}{d\theta} \tag{1.37}$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \tag{1.38}$$

$$T_C = T_d \tag{1.39}$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} \tag{1.40}$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

Extension of EMMC instrument

Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same.

Therefore

$I_1 = I_2 = I$

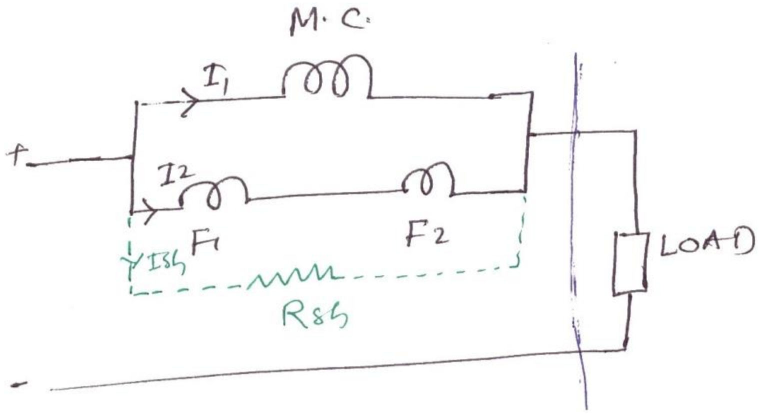


Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value R_{sh} is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \quad (1.41)$$

Or $\therefore T_d = I^2 \frac{dM}{d\theta}$ (1.42)

$$T_C = K\theta \quad (1.43)$$

$$\theta = \frac{I^2}{K}\frac{dM}{d\theta} \quad (1.44)$$

$\therefore \theta \propto I^2$ (Scale is not uniform) (1.45)

Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extent the range of voltmeter.

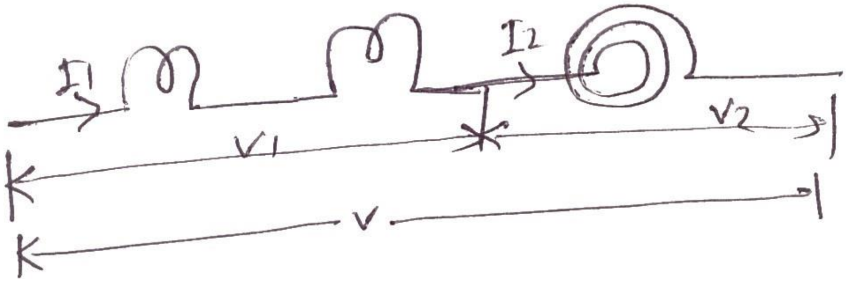


Fig. 1.16

$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \tag{1.46}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \tag{1.47}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \tag{1.48}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \tag{1.49}$$

$$T_d \propto V^2 \tag{1.50}$$

$$\therefore \theta \propto V^2 \quad \text{(Scale in not uniform)} \tag{1.51}$$

Case-III As w**attmeter**

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

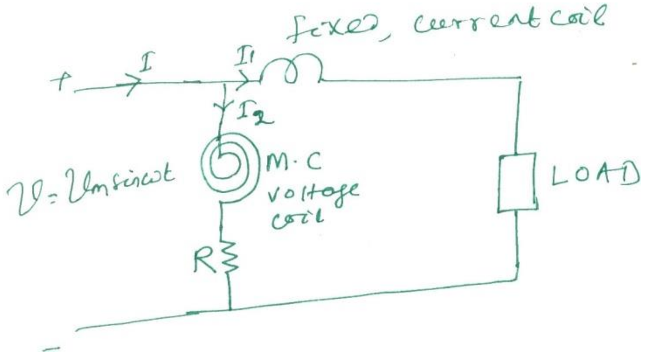


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance ‘R’. The current,

$$I_2 = \frac{v_m \sin wt}{R} \tag{1.52}$$

Let the phase difference between the currents I$_1$ and I$_2$ is ϕ

$$I_1 = I_m \sin(wt - \phi) \tag{1.53}$$

$$T_d = I_1 I_2 \frac{dM}{d\theta} \tag{1.54}$$

$$T_d = I_m \sin(wt - \phi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta} \tag{1.55}$$

$$T_d = \frac{1}{R}(I_m V_m \sin wt \sin(wt - \phi)) \frac{dM}{d\theta} \tag{1.56}$$

$$T_d = \frac{1}{R} I_m V_m \sin wt.\sin(wt - \phi) \frac{dM}{d\theta} \tag{1.57}$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} T_d \times d(wt) \tag{1.58}$$

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} \frac{1}{R} \times I_m V_m \sin wt.\sin(wt - \phi) \frac{dM}{d\theta} \times d(wt) \tag{1.59}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\Pi} \times \frac{1}{R} \times \frac{dM}{d\theta}\left[\int \{\cos\phi - \cos(2wt - \phi)\} dwt\right] \tag{1.60}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\int_0^{2\Pi} \cos\phi.dwt - \int_0^{2\Pi} \cos(2wt - \phi).dwt\right] \tag{1.61}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\cos\phi [wt]_0^{2\Pi}\right] \tag{1.62}$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\cos\phi (2\Pi - 0)\right] \tag{1.63}$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos\phi \tag{1.64}$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos\phi \times \frac{1}{R} \times \frac{dM}{d\theta} \tag{1.65}$$

$$(T_d)_{avg} \propto KVI\cos\phi \tag{1.66}$$

$$T_C \propto \theta \tag{1.67}$$

$$\theta \propto KVI\cos\phi \tag{1.68}$$

$$\theta \propto VI\cos\phi \tag{1.69}$$

Advantages

- ✓ It can be used for voltmeter, ammeter and wattmeter
- ✓ Hysteresis error is nill
- ✓ Eddy current error is nill
- ✓ Damping is effective
- ✓ It can be measure correctively and accurately the rms value of the voltage

Disadvantages

- ✓ Scale is not uniform
- ✓ Power consumption is high(because of high resistance)
- ✓ Cost is more
- ✓ Error is produced due to frequency, temperature and stray field.
- ✓ Torque/weight is low.(Because field strength is very low)

Errors in PMMC

- ✓ The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- ✓ The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- ✓ When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

Difference between attraction and repulsion type instrument

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

Characteristics of meter

Full scale deflection current(I_{FSD})

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between 2 μA to 30mA.

Resistance of the coil(R_m)

This is ohmic resistance of the moving coil. It is due to ρ, L and A. For an ammeter this should be as small as possible.

Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}}(\Omega / volt), \uparrow S = \frac{Z\uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in Ω/volt. When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

Error in M.I instrument

Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_S)^2 + X_L^2} \tag{1.70}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_S)^2 + X_L^2}} \tag{1.71}$$

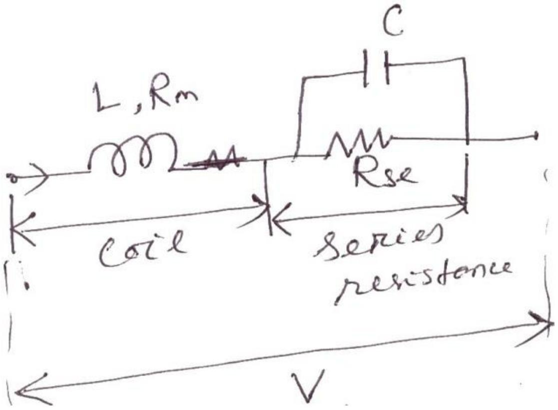


Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance, ($X_L - X_C$) is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_S)^2} \tag{1.72}$$

Electrostatic instrument

In multi cellular construction several vans and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

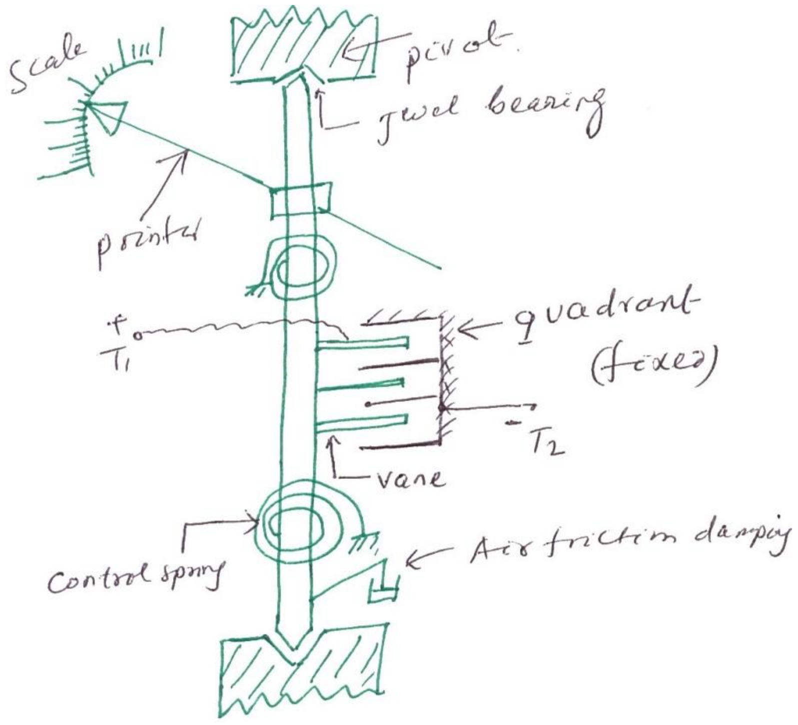


Fig. 1.19

Torque develop by electrostatic instrument

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant

Energy stored= $\frac{1}{2}CV^2$ (1.73)

Let ' θ' be the deflection corresponding to a voltage V.

Let the voltage increases by dv, the corresponding deflection is' $\theta + d\theta$ '

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d\,(CV\,)}{dt} = \frac{dC}{dt}V + C\,\frac{dV}{dt} \qquad (1.74)$$

$V \times dt$ multiply on both side of equation (1.74)

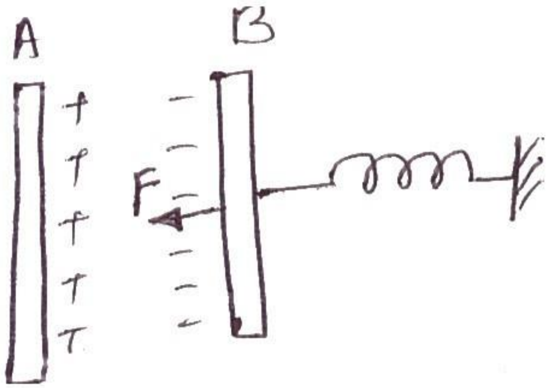


Fig. 1.20

$$Vidt = \frac{dC}{dt}V^2dt + CV\frac{dV}{dt}dt \tag{1.75}$$

$$Vidt = V^2dC + CVdV \tag{1.76}$$

Change in stored energy= $\frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$ (1.77)

$$= \frac{1}{2}\left[(C + dC)V^2 + dV^2 + 2VdV\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left[CV^2 + CdV^2 + 2CVdV + V^2dC + dCdV^2 + 2VdVdC\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2dC + CVdV$$

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + F \times rd\theta \tag{1.78}$$

$$T_d \times d\theta = \frac{1}{2}V^2dC \tag{1.79}$$

$$T_d = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right) \tag{1.80}$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right) \tag{1.81}$$

$$\theta = \frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right) \tag{1.82}$$

Advantages

- It is used in both AC and DC.
- There is no frequency error.
- There is no hysteresis error.
- There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- It is used for high voltage
- Power consumption is negligible.

Disadvantages

- Scale is not uniform
- Large in size
- Cost is more

Multi range Ammeter

When the switch is connected to position (1), the supplied current I_1

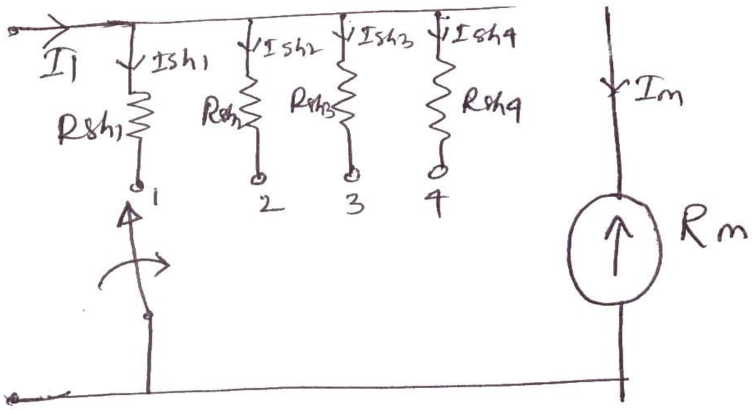


Fig. 1.21

$$I_{sh1} R_{sh1} = I_m R_m \tag{1.83}$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \tag{1.84}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, R_{sh1} = \frac{R_m}{m_1 - 1}, m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m} \tag{1.85}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m} \tag{1.86}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m} \tag{1.87}$$

Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \tag{1.88}$$

$$R_2 = R_{sh2} - R_{sh3} \tag{1.89}$$

$$R_3 = R_{sh3} - R_{sh4} \tag{1.90}$$

$$R_4 = R_{sh4} \tag{1.91}$$

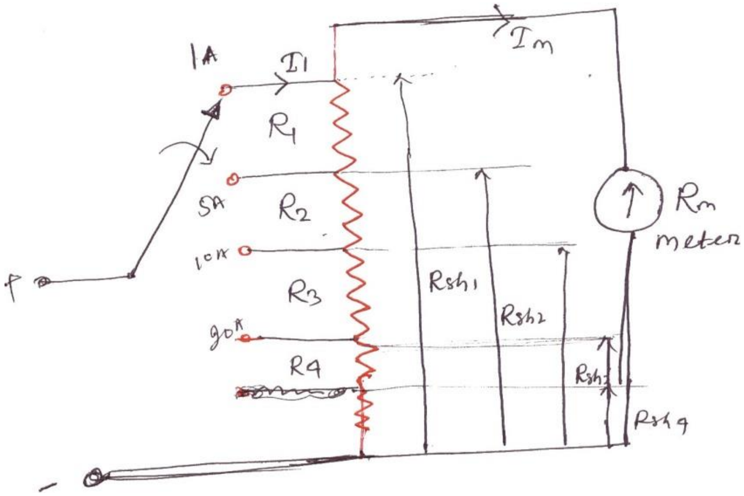


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Aryton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

Multi range D.C. voltmeter

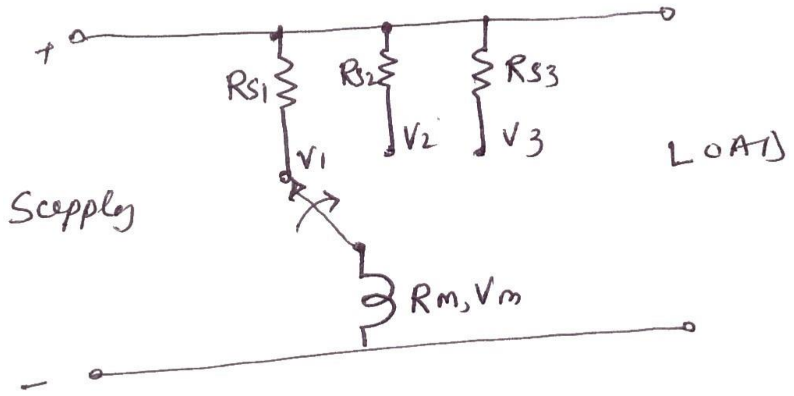


Fig. 1.23

$$R_{s1} = R_m\,(m_1 - 1)$$
$$R_{s2} = R_m\,(m_2 - 1) \tag{1.92}$$
$$R_{s3} = R_m\,(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m},\; m_2 = \frac{V_2}{V_m},\; m_3 = \frac{V_3}{V_m} \tag{1.93}$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required**.**

Potential divider arrangement

The resistance R_1, R_2, R_3 and R_4 is connected in series to obtained the ranges V_1, V_2, V_3 and V_4

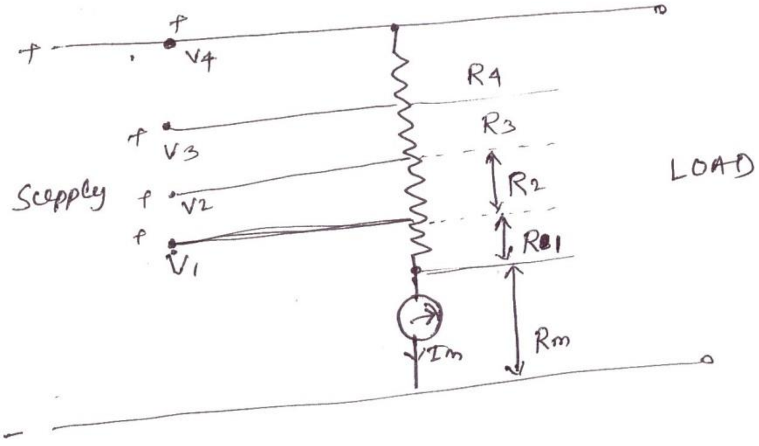


Fig. 1.24

Consider for voltage V1, $(R_1 + R_m)I_m = V_1$

$$\therefore\ R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_m}{R_m}\right)} - R_m = \left(\frac{V_1}{V_m}\right)R_m - R_m \tag{1.94}$$

$$R_1 = (m_1 - 1)R_m \tag{1.95}$$

For V2, $(R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m$ (1.96)

$$R_2 = \frac{V_2}{\left(\frac{V_m}{R_m}\right)} - (m_1 - 1)R_m - R_m \tag{1.97}$$

$$R_2 = m_2 R_m - R_m - (m_1 - 1)R_m$$

$$= R_m (m_2 - 1 - m_1 + 1) \tag{1.98}$$

$$R_2 = (m_2 - m_1)R_m \tag{1.99}$$

For V3 $\left(R_3 + R_2 + R_1 + R_m\right)I_m = V_3$

$$R_3 = \frac{V_3}{I_m} - R_2 - R_1 - R_m$$

$$= \frac{V_3}{V_m}R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_3 = (m_3 - m_2)R_m$$

For V4 $(R_4 + R_3 + R_2 + R_1 + R_m)I_m = V_4$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$= \left(\frac{V_4}{V_m}\right)R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_4 = R_m\left[m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1\right]$$

$$R_4 = (m_4 - m_3)R_m$$

Example: 1.1

A PMMC ammeter has the following specification

Coil dimension are 1cm× 1cm. Spring constant is $0.15 \times 10^{-6}\ N-m/rad$, Flux density is $1.5 \times 10^{-3}\ wb/m^2$ .Determine the no. of turns required to produce a deflection of 90^0 when a current 2mA flows through the coil.

Solution:

At steady state **condition** $T_d = T_C$

$BANI = K\theta$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

A=$1\times10^{-4}\ m^2$

K= $0.15 \times 10^{-6} \frac{N-m}{rad}$

B=$1.5\times10^{-3}\ wb/m^2$

I= $2 \times 10^{-3}\ A$

$\theta = 90^\circ = \frac{\Pi}{2} rad$

N=785 ans.

Example: 1.2

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV.The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

Solution:

Case-1

V_m =400mV

$I_m = 20mA$

I=200A

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20\Omega$$

$$I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$$200 = 20 \times 10^{-3}\left[1 + \frac{20}{R_{sh}}\right]$$

$R_{sh} = 2 \times 10^{-3}\,\Omega$

Case-II

V=1000V

$$V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$4000 = 400 \times 10^{-3}\left(1 + \frac{R_{se}}{20}\right)$$

$R_{se} = 49.98k\Omega$

Example: 1.3

A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

Solution:

$R_m = 3k\Omega$,$V_m = 150V$,$V = 300V$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$300 = 150\left(1+\frac{R_{se}}{3}\right) \Rightarrow R_{se} = 3k\Omega$$

Case-II $V = V_m\left(1+\frac{R_{se}}{R_m}\right)$

$$200 = V_m\left(1+\frac{3}{3}\right)$$

$\therefore V_m = 100V$ Ans

Example: 1.4

What is the value of series resistance to be used to extent ‘0’to 200V range of 20,000**Ω**/volt voltmeter to 0 to 2000 volt?

Solution:

$$V_{se} = V - V = 1800$$

$$I_{FSD} = \frac{1}{20000} = \frac{1}{Sensitivity}$$

$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega$ ans.

Example: 1.5

A moving coil instrument whose resistance is 25**Ω** gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA. Calculate the error caused by a 10^0C rise in temperature when:

(a) Copper moving coil is connected directly across the manganin shunt.

(b) A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is $0.004/^0$C and that of manganin is 0.00015^0/C.

Solution:

Case-1

$$I_m = 1mA$$

$$R_m = 25\Omega$$

I=100mA

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{25}{R_{sh}}\right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for 10^0C rise in temperature, $R_{mt} = 25(1 + 0.004\times 10)$

$$R_t = R_o\,(1+\rho_t \times t)$$

$$R_{m/t=10^\circ} = 26\Omega$$

Shunt resistance for 10^0C, rise in temperature

$$R_{sh/t=10^\circ} = 0.2525(1 + 0.00015\times 10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for 10^0C rise in temperature

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)\Bigg|_{t=10^\circ C}$$

$$100 = I_{mt}\left(1+\frac{26}{0.2529}\right)$$

$$I_m\big|_{t=10} = 0.963mA$$

But normal meter current=1mA

Error due to rise in temperature=(0.963-1)*100=-3.7%

Case-b As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25 + 75 = 100\Omega$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 1\left(1+\frac{100}{R_{sh}}\right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for 10^0C rise in temperature

$$R_m|_{t=10} = 25(1 + 0.004 \times 10) + 75(1 + 0.00015 \times 10) = 101.11\Omega$$

Shunt resistance for 10^0C rise in temperature

$$R_{sh}|_{t=10} = 1.01(1 + 0.00015 \times 10) = 1.0115\Omega$$

$$I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$$100 = I_m\left(1 + \frac{101.11}{1.0115}\right)$$

$$I_m\Big|_{t = 10^\circ} = 0.9905mA$$

Error =(0.9905-1)*100=-0.95%

Example: 1.6

The coil of a 600V M.I meter has an inductance of 1 henery. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

Solution:

$V_m = 600V$, $I_m = 100mA$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\Pi fL = 314\Omega$$

$$R_m = \sqrt{Z_m^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33mA$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39mA$$

Error= $\frac{I_{DC} - I_{AC}}{I_{AC}} \times 100 = \frac{33.39 - 33.33}{33.33} \times 100 = 0.18\%$

Example: 1.7

A 250V M.I. voltmeter has coil resistance of 500**Ω**, coil inductance 0f 1.04 H and series resistance of 2k**Ω**. The meter reads correctively at 250V D.C. What will be the value of capacitance to be used for shunting the series resistance to make the meter read correctly at 50HZ? What is the reading of voltmeter on A.C. without capacitance?

Solution: $C = 0.41 \frac{L}{(R_S)^2}$

$$= 0.41 \times \frac{\square\square 1.04}{(2 \times 10^3)^2} = 0.1 \mu F$$

For A.C $Z = \sqrt{(R_m + R_{Se})^2 + X_L^2}$

$$Z = \sqrt{(500 + 2000)^2 + (314)^2} = 2520\Omega$$

With D.C

$R_{total} = 2500\Omega$

For 2500Ω → 250V

$$1\Omega \rightarrow \frac{250}{2500}$$

$$2520\Omega \rightarrow \frac{250}{2500} \times 2520 = 248V$$

Example: 1.8

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance (μH)	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is 1.5A?

Solution:

For current I=1.5A, θ=55.5 degree=0.96865 rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11\mu H / \deg ree = 6.3\mu H / rad$$

Deflecting torque , $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2} (1.5)^2 \times 6.3 \times 10^{-6} = 7.09 \times 10^{-6}\, N - m$

Spring constant, $K = \frac{T_d}{\theta} = \frac{7.09 \times 10^{-6}}{0.968} = 7.319 \times 10^{-6} \frac{N - m}{rad}$

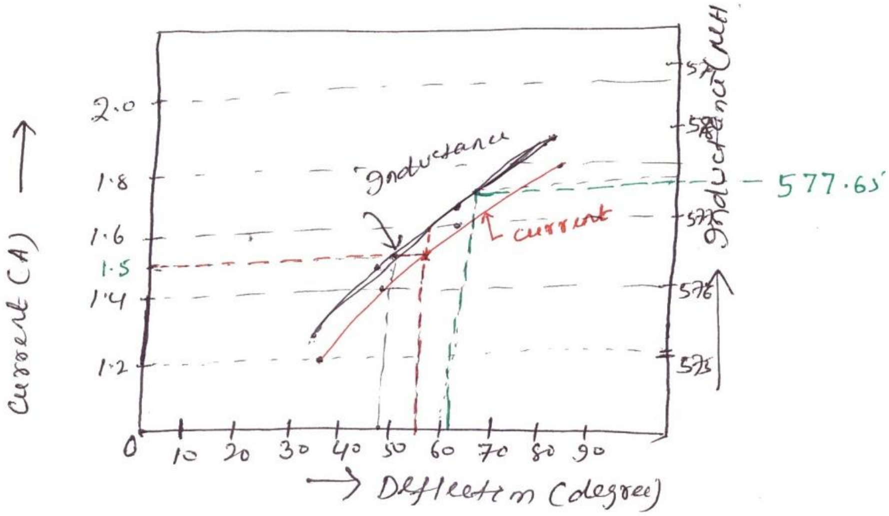


Fig. 1.25

Example: 1.9

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection θ as $M = -6\cos(\theta + 30^\circ) mH$ .Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of 60^0.

Solution:

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6\cos(\theta + 30^\circ)$$

$$\frac{dM}{d\theta} = 6\sin(\theta + 30) mH$$

$$\frac{dM}{d\theta}\Big|_{\theta=60} = 6\sin 90 = 6 mH / \deg$$

$$T_d = I^2 \frac{dM}{d\theta} = (50 \times 10^{-3})^2 \times 6 \times 10^{-3} = 15 \times 10^{-6}\, N - m$$

Example: 1.10

The inductance of a moving iron ammeter with a full scale deflection of 90^0 at 1.5A, is given by the expression $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$, where θ is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

Solution:

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\left.\frac{dL}{d\theta}\right|_{\theta=90^\circ} = 40 - 8 \times \frac{\Pi}{2} - 3\left(\frac{\Pi}{2}\right)^2 \mu H / rad = 20 \mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\frac{\Pi}{2} = \frac{1}{2} \frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

K=Spring constant=$14.32 \times 10^{-6}\ N - m / rad$

For I=1A, $\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}} \left(40 - 8\theta - 3\theta^2\right)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

Example: 1.11

The inductance of a moving iron instrument is given by $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$, where θ is the deflection in radian from zero position. The spring constant is $12 \times 10^{-6}\ N - m / rad$. Estimate the deflection for a current of 5A.

Solution:

$$\frac{dL}{d\theta} = (5-2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5-2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^\circ$$

Example: 1.12

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance (μH)	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a) 30^0 (b) 80^0 . Given that control spring constant is $0.4 \times 10^{-6} N - m / \deg ree$

Solution:

$$\theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

(a) For $\theta = 30^\circ$

The curve is linear

$$\therefore \left(\frac{dL}{d\theta}\right)_{\theta=30} = \frac{355.5-335}{40-20} = 1.075 \mu H / \deg ree = 58.7 \mu H / rad$$

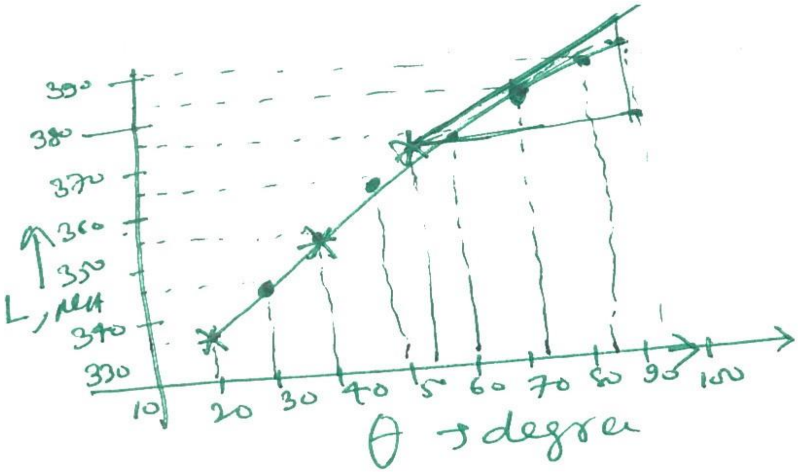


Fig. 1.26

Example: 1.13

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through 90^0. The torsional constant is $10 \times 10^{-6}\ N-m/rad$. The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is 90^0?

Solution:

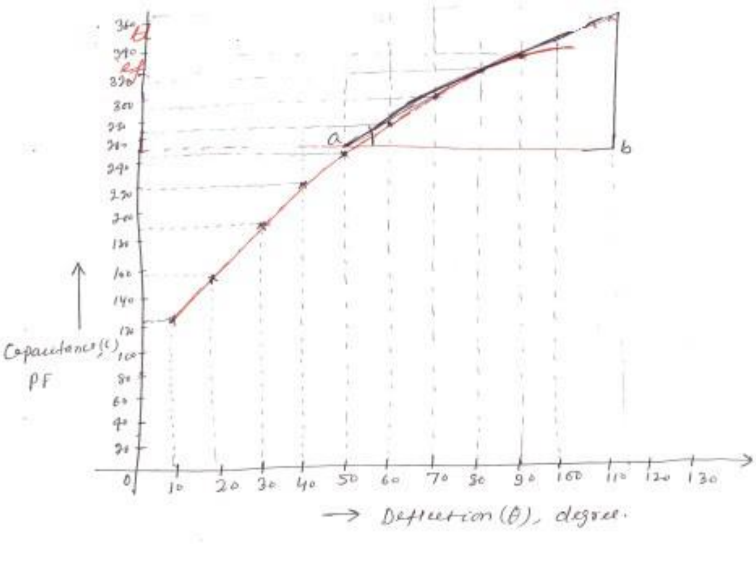


Fig. 1.27

$$\frac{dC}{d\theta} = \tan\theta = \frac{bc}{ab} = \frac{370-250}{110-44} = 1.82PF/\deg ree = 104.2PF/rad$$

Spring constant $K = 10\times10^{-6}\frac{N-m}{rad} = 0.1745\times10^{-6}N-m/\deg ree$

$$\theta = \frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right) \Rightarrow V = \sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V = \sqrt{\frac{2\times0.1745\times10^{-6}\times90}{104.2\times10^{-12}}} = 549volt$$

Example: 1.14

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance $R_m = 50\Omega$ and a full scale deflection current $I_m = 1mA$. The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

Solution:

Case-I 0-10mA

Multiplying power $m = \frac{I}{I_m} = \frac{10}{1} = 10$

$\therefore$ Shunt resistance $R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$

Case-II 0-50mA

$$m = \frac{50}{1} = 50$$

$$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$$

Case-III 0-100mA, $m = \frac{100}{1} = 100\Omega$

$$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$$

Case-IV 0-500mA**,** $m = \frac{500}{1} = 500\Omega$

$$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$$

Example: 1.15

A moving coil voltmeter with a resistance of 20Ω gives a full scale deflection of 120^0, when a potential difference of 100mV is applied across it. The moving coil has dimension of 30mm*25mm and is wounded with 100 turns. The control spring constant is $0.375\times10^{-6} N-m/\deg ree.$ Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper=$1.7\times10^{-8}\Omega m$.

Solution:

Data given

$V_m = 100mV$

$R_m = 20\Omega$

$\theta = 120^\circ$

N=100

$K = 0.375\times10^{-6} N-m/\deg ree$

$R_C = 30\% of R_m$

$\rho = 1.7\times10^{-8}\,\Omega m$

$$I_m = \frac{V_m}{R_m} = 5\times10^{-3} A$$

$T_d = BANI, T_C = K\theta = 0.375\times10^{-6}\times120 = 45\times10^{-6} N-m$

$$B = \frac{T_d}{ANI} = \frac{45\times10^{-6}}{30\times25\times10^{-6}\times100\times5\times10^{-3}} = 0.12 wb/m^2$$

$R_C = 0.3\times20 = 6\Omega$

Length of mean turn path =2(a+b) =2(55)=110mm

$$R_C = N\left(\frac{\rho l}{A}\right)$$

$$A = \frac{N\times\rho\times(l_t)}{R_C} = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6}$$

$= 3.116 \times 10^{-8}\, m^2$

$= 31.16 \times 10^{-3}\, mm^2$

$A = \frac{\Pi}{4} d^2 \Rightarrow d = 0.2mm$

Example: 1.16

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(1) The shunt resistance for a full scale deflection corresponding to 100A

(2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

Solution:

Data given

$I_m = 10mA$

$V_m = 100mV$

$I = 100\,A$

$$I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$$100 = 10 \times 10^{-3}\left(1 + \frac{10}{R_{sh}}\right)$$

$R_{sh} = 1.001 \times 10^{-3}\,\Omega$

$R_{se} = ??, V = 1000V$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$1000 = 100 \times 10^{-3}\left(1 + \frac{R_{se}}{10}\right)$$

$\therefore\ R_{se} = 99.99K\Omega$

Example: 1.17

Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of 50w and a full scale deflection current of 1mA is to be used.

Solution: Data given

$$
\begin{array}{l|l|l}
 & & m_1 = \dfrac{I_1}{I_m} = 1000\,A \\
I_m = 1\times 10^{-3}\,A & I_1 = 1A & m_2 = \dfrac{I_2}{I_m} = 5000\,A \\
 & I_2 = 5A & \\
R_m = 50\Omega & I_3 = 10\,A & m_3 = \dfrac{I_3}{I_m} = 10000\,A \\
 & I_4 = 20\,A & \\
 & & m_4 = \dfrac{I_4}{I_m} = 20000\,A
\end{array}
$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{\square\square 50}{1000 - 1} = 0.05\Omega$$

$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{\square 50}{5000 - 1} = 0.01\Omega$$

$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000 - 1} = 0.005\Omega$$

$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{\square\square 50}{20000 - 1} = 0.0025\Omega$$

$\therefore$ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04\Omega$$

$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005\Omega$$

$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.025 = 0.0025\Omega$$

$$R_4 = R_{sh4} = 0.0025\Omega$$

3.2 Electrodynamometer Type Instruments

The electrodynamometer type instrument is a **transfer instrument**. A transfer instrument is one which is calibrated with a d.c. source and used without any modifications for a.c. measurements. Such a transfer instrument has same accuracy for a.c. and d.c. measurements. The electrodynamometer type instruments are often used in accurate a.c. voltmeters and ammeters, not only at the powerline frequency but also in the lower audiofrequency range. With some little modifications, it can be used as a wattmeter for the power measurements.

Why PMMC instruments can not be used for a.c. measurements ?

The PMMC instrument cannot be used on a.c. currents or voltages. If a.c. supply is given to these instruments, an alternating torque will be developed. Due to moment of inertia of the moving system, the pointer will not follow the rapidly changing alternating torque and will fail to show any reading. In order that the instrument should be able to read a.c. quantities, the magnetic field in the air gap must change along with the change in current. This principle is used in the electrodynamometer type instrument. Instead of a permanent magnet, the electrodynamometer type instrument uses the current under measurement to produce the necessary field flux.

3.2.1 Construction

The Fig. 3.3 shows the construction of the electrodynamometer type instrument.

The various parts of the electrodynamometer type instrument are :

Fixed Coils : The necessary field required for the operation of the instrument is produced by the fixed coils. A uniform field is obtained near the center of coil due to division of coil in two sections. These coils are air cored. Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire. The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

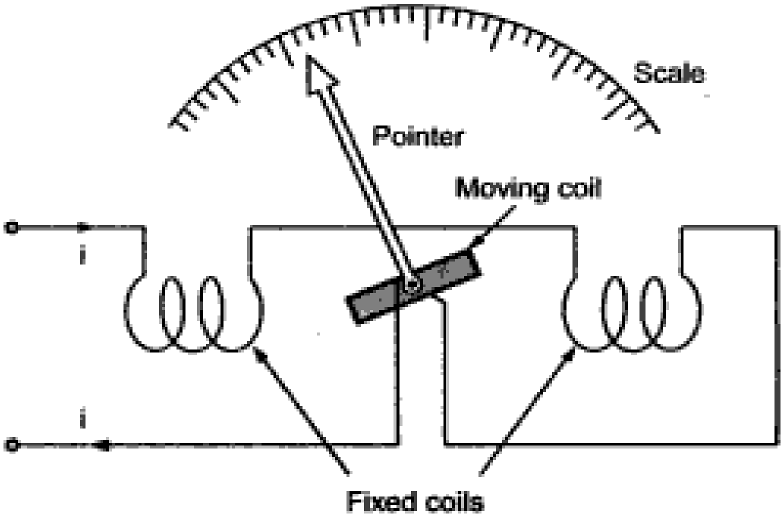


Fig. 3.3 Electrodynamometer type instrument

Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

Moving coil : The moving coil is wound either as a self-sustaining coil or else on a non-metallic former. If metallic former is used, then it would induce eddy currents in it. The construction of moving coil is made light as well as rigid. It is air cored.

Controlling : The controlling torque is provided by springs. These springs act as leads to the moving coil.

Moving system : The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case a high accuracy is desired.

Damping : The damping torque is provided by air friction, by a pair of aluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would be distorted by eddy current damping, it is not employed.

Shielding : The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So shielding is done to protect it from stray magnetic fields. It is done by enclosing in a casing of high permeability alloy.

Cases and Scales : Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid. The case is supported by adjustable levelling screws.

A spirit level may be provided to ensure proper levelling.

For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit.

The electrodynamometer instruments can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

For using electrodynamometer instrument as a wattmeter to measure the power, the fixed coils acts as a current coil and must be connected in series with the load. The moving coil acts as a voltage coil or pressure coil and must be connected across the supply terminals. The wattmeter indicates the supply power. When current passes through the fixed and moving coils, both coils produce the magnetic fields. The field produced by fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

3.2.2 Torque Equation

Let i_1 = Instantaneous value of current in fixed coil

i_2 = Instantaneous value of current in moving coil

L_1 = Self inductance of fixed coils

L_2 = Self inductance of moving coil

M = Mutual inductance between fixed and moving coils

The electrodynamometer instrument can be represented by an equivalent circuit as shown in the Fig. 3.4.

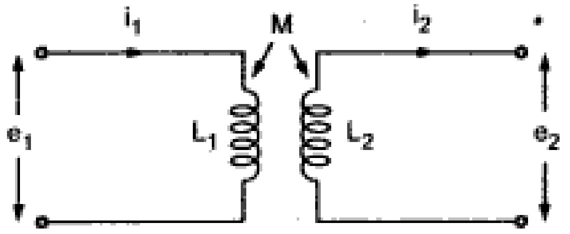


Fig. 3.4

The flux linkages of coil 1 are,

$$\phi_1 = L_1 i_1 + M i_2$$

The flux linkages of coil 2 are,

$$\phi_2 = L_2 i_2 + M i_1$$

Now $e_1 = \dfrac{d\phi_1}{dt}$

and $e_2 = \dfrac{d\phi_2}{dt}$

$$\begin{aligned}\text{Electrical input energy} &= e_1 i_1 dt + e_2 i_2\, dt \\ &= i_1\, d\,\phi_1 + i_2\, d\,\phi_2 \\ &= i_1\, d\,(L_1 i_1 + M i_2) + i_2\, d\,(L_2 i_2 + M i_1) \\ &= i_1 L_1\, d\,i_1 + i_1^2\, d\,L_1 + i_1 i_2\, d\,M + i_1 M\, d\,i_2 + \\ &\quad i_2 L_2 d\,i_2 + i_2^2\, d\,L_2 + i_1 i_2 d\,M + i_2 M\, d\,i_1 \qquad \ldots (1)\end{aligned}$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by,

$$\text{Energy stored} = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$$

$$\begin{aligned}\text{Change in stored energy} &= d\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M\right] \\ &= i_1 L_1 d\,i_1 + \frac{1}{2} i_1^2\, d\,L_1 + i_2 L_2\, d\,i_2 + \frac{1}{2} i_2^2\, d\,L_2 + \\ &\quad i_1 M d\,i_2 + i_2 M d\,i_1 + i_1 i_2\, d\,M \qquad \ldots (2)\end{aligned}$$

From the principle of conservation of energy,

Energy input = Energy stored + Mechanical energy

∴ Mechanical energy = Energy input – Energy stored

Substracting (2) from (1),

$$\text{Mechanical energy} = \frac{1}{2} i_1^2 \, dL_1 + \frac{1}{2} i_2^2 \, dL_2 + i_1 \, i_2 \, dM$$

The self inductances L_1 and L_2 are constants and hence $dL_1$and dL_2 are zero.

$$\therefore \quad \text{Mechanical energy} = i_1 \, i_2 \, dM$$

If T_i is the instantaneous deflecting torque and $d\theta$ is the change in the deflection then

$$\text{Mechanical energy} = \text{Mechanical work done}$$

$$= T_i \, d\theta$$

$$\therefore \quad i_1 \, i_2 \, dM = T_i \, d\theta$$

$$\therefore \quad T_i = i_1 \, i_2 \frac{dM}{d\theta}$$

This is the expression for the instantaneous deflecting torque. Let us see its operation on a.c. and d.c.

D.C. operation : For d.c. currents of I_1 and I_2,

$$T_d = I_1 \, I_2 \frac{dM}{d\theta}$$

The controlling torque is provided by springs hence

$$T_c = K\,\theta$$

In steady state, $T_d = T_c$

$$\therefore \quad I_1 \, I_2 \frac{dM}{d\theta} = K\,\theta$$

$$\therefore \quad \theta = \frac{I_1 \, I_2}{K} \cdot \frac{dM}{d\theta}$$

Thus the deflection is proportional to the product of the two currents and the rate of change of mutual inductance.

A.C. operation : In a.c. operation, the total deflecting torque over a cycle must be obtained by integrating T_i over one period.

Average deflecting torque over one cycle is,

$$T_d = \frac{1}{T} \int_0^T T_i \, dt$$

$$T = \text{Time period of one cycle}$$

$$\therefore \quad T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T i_1 \, i_2 \, dt$$

Now if the two currents are sinusoidal and displaced by a phase angle ϕ then

$$i_1 = I_{m1} \sin \omega t$$

and $$i_2 = I_{m2} \sin (\omega t - \phi)$$

$$\therefore \quad T_d = \frac{dM}{d\theta} \cdot \frac{1}{T} \int_0^T I_{m1} \sin \omega t \cdot I_{m2} \sin (\omega t - \phi) \, d(\omega t)$$

$$= \left(\frac{I_{m1} \, I_{m2}}{2} \right) \cos\phi \, \frac{dM}{d\theta}$$

$$= I_1 \, I_2 \cos\phi \, \frac{dM}{d\theta}$$

where I_1, I_2 are the r.m.s. values of the two currents as,

$$I_1 = \frac{I_{m1}}{\sqrt{2}} \quad \text{and} \quad I_2 = \frac{I_{m2}}{\sqrt{2}}$$

As $$T_c = K\,\theta$$

Hence in steady state, $T_c = T_d$

$$\therefore \quad I_1 \, I_2 \cos\phi \, \frac{dM}{d\theta} = K\,\theta$$

$$\therefore \quad \boxed{\theta = \frac{I_1 \, I_2}{K} \cos\phi \, \frac{d\,M}{d\,\theta}}$$

Thus the deflection is decided by the product of r.m.s. values of two currents, cosine of the phase angle (power factor) and rate of change of mutual inductance.

For d.c. use, the deflection is proportional to square of current and the scale is non-uniform and crowded at the ends. For a.c. use the instantaneous torque is proportional to the square of the instantaneous current. The i^2 is positive and as current varies, the deflecting torque also varies.

But moving system, due to intertia cannot follow rapid variations and thus finally meter shows the average torque.

Thus the deflection is the function of the mean of the squared current. The scale is thus calibrated in terms of the square root of the average current squared i.e. r.m.s. value of the a.c. quantity to be measured.

If an electrodynamometer instrument is calibrated with a d.c. current of 1A and pointer indicates 1 A d.c. on scale then on a.c., the pointer will deflect upto the same mark but 1A in this case will indicate r.m.s. value.

Thus as it is a transfer instrument, there is direct connection between a.c. and d.c. Hence the instrument is often used as a **calibration instrument.**

The instrument can be used as an ammeter to measure currents upto 20 A while using as a voltmeter it can have low sensitivity of about 10 to 30 Ω/V.

The Fig. 3.5 (a), (b) and (c) shows the connections of the electrodynamometer instrument as ammeter, voltmeter and the wattmeter.

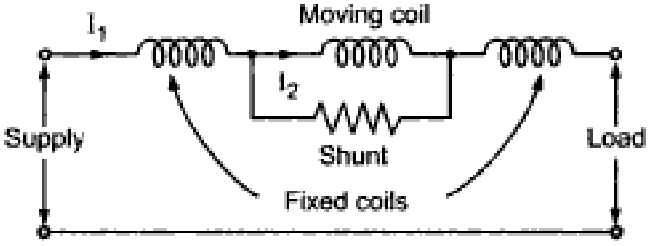


Fig. 3.5 (a) Electrodynamometer ammeter upto 100 mA

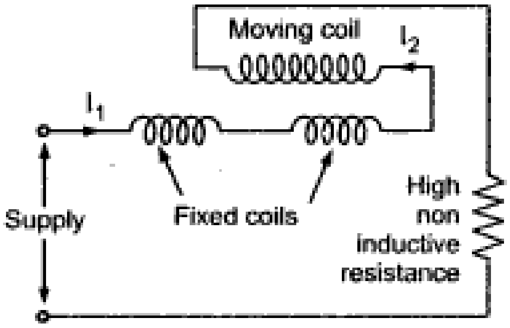


Fig. 3.5 (b) Electrodynamometer voltmeter

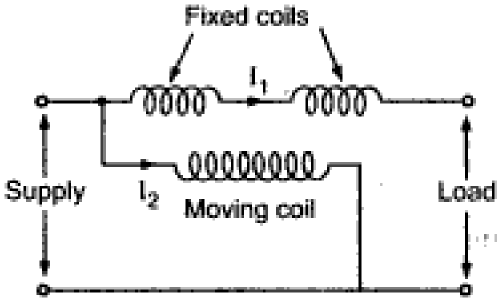


Fig. 3.5 (c) Electrodynamometer wattmeter

3.2.3 Advantages of Electrodynamic Instruments

1) As the coils are air cored, these instruments are free from hysteresis and eddy current losses.
2) They have a precision grade accuracy.
3) These instruments can be used on both a.c. and d.c. They are also used as a transfer instruments.
4) Electrodynamometer voltmeters are very useful where accurate rms values of voltage, irrespective of waveforms, are required.
5) Free from hysteresis errors.
6) Low power consumption.
7) Light in weight.

3.2.4 Disadvantages of Electrodynamic Instruments

1) These instruments have a low sensitivity due to a low torque to weight ratio. Also it introduces increased frictional losses. To get accurate results, these errors must be minimized.
2) They are more expensive than other type of instruments.
3) These instruments are sensitive to overloads and mechanical impacts. Therefore care must be taken while handling them.
4) They have a non-uniform scale.
5) The operating current of these instruments is large due to the fact that they have weak magnetic field.

3.2.5 Errors in Electrodynamometer Instruments

The various errors in electrodynamometer instruments are,

1. Torque to weight ratio : To have reasonable deflecting torque, m.m.f. of the moving coil must be large enough. Thus m.m.f. = NI hence current through moving coil should be high or number of turns should be large. The current can not be made very high because it may cause excessive heating of springs. Large number of turns hence is the only option but it increases weight of the coil. This makes the system heavy reducing torque to weight ratio. This can cause frictional errors in the reading.

2. Frequency errors : The changes in the frequency causes to change self inductances of moving coil and fixed coil. This causes the error in the reading. The frequency error can be reduced by having equal time constants for both fixed and moving coil circuits.

3. Eddy current errors : In metal parts of the instrument the eddy currents get produced. The eddy currents interact with the instrument current, to cause change in the deflecting torque, to cause error. Hence metal parts should be kept as minimum as possible. Also the resistivity of the metal parts used must be high, to reduce the eddy currents.

4. Stray magnetic field error : Similar to moving iron instruments the operating field in electrodynamometer instrument is very weak. Hence external magnetic field can interact with the operating field to cause change in the deflection, causing the error. To reduce the effect of stray magnetic field, the shields must be used for the instruments.

5. Temperature error : The temperature errors are caused due to the self heating of the coil, which causes change in the resistance of the coil. Thus temperature compensating resistors can be used in the precise instrument to eliminate the temperature errors.

3.2.6 Comparison of Various Types of Instruments

The comparison of PMMC, moving iron and electrodynamometer type instruments is summarized in the Table 3.1.

Meter Type	Control	Damping	Suitability	Application
PMMC	Spring	Eddy current	D.C.	Widely used for d.c. current and voltage measurements in low and medium impedance circuits.
Moving Iron	Spring or Gravity	Air friction	D.C. and A.C.	Used for rough indication of currents and voltages. Widely used for the indicator type instruments on panels.
Electrodynamometer	Spring	Air friction	D.C. and A.C.	Used mainly as wattmeter. Also may be used as ammeter or voltmeter. Widely used as a calibration instrument and as a transfer instrument.

Table 3.1

3.3 Single Phase Dynamometer Wattmeter

An electrodynamometer type wattmeter is used to measure power. It has two coils, fixed coil which is **current coil** and moving coil which is **pressure coil** or **voltage coil.** The current coil carries the current of the circuit while pressure coil carries current proportional to the voltage in the circuit. This is achieved by connecting a series resistance in voltage circuit.

The connections of an electrodynamometer wattmeter in the circuit are shown in the Fig. 3.6.

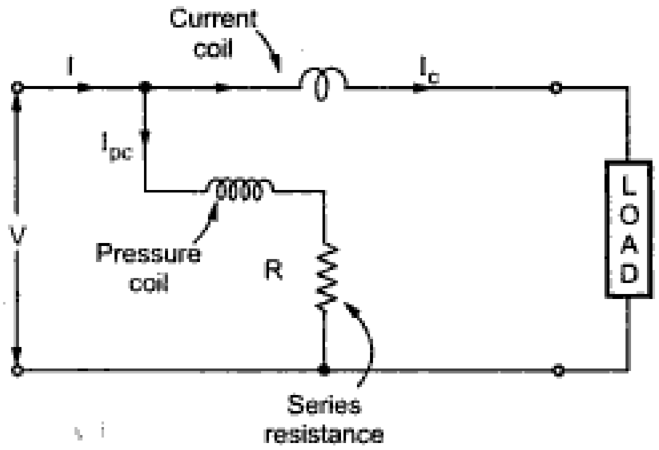


Fig. 3.6 Electrodynamometer wattmeter

I_c = Current through current coil

I_{pc} = Current through pressure coil

R = Series resistance

V = R.M.S. value of supply voltage

I = R.M.S. value of current

3.3.1 Torque Equation

According to theory of electrodynamic instruments,

$$T_i = i_1 \, i_2 \frac{dM}{d\theta} \qquad \dots (1)$$

Let v = Instantaneous voltage = $V_m \sin \omega t = \sqrt{2} \, V \sin \omega t$ ... (2)

Due to high series resistance, pressure coil is treated to be purely resistive.

Key Point: *The current I_{pc} is in phase with V as pressure coil is purely resistive.*

$$i_{pc} = \text{Instantaneous value} = \frac{v}{R_p} \text{ where } R_p = r_{pc} + R$$

$$\therefore \quad i_{pc} = \frac{\sqrt{2}\,V}{R_p} \sin \omega t = \sqrt{2}\, I_{pc} \sin \omega t \qquad \dots (3)$$

If current coil current lags the voltage by angle ϕ then its instantaneous value is,

$$i_c = \sqrt{2} I_c \sin(\omega t - \phi) \qquad \dots (4)$$

Now $i_1 = i_c$ and $i_2 = i_{pc}$ hence,

$$T_i = [\sqrt{2} I_{pc} \sin \omega t][\sqrt{2} I_c \sin(\omega t - \phi)] \frac{dM}{d\theta}$$

$$= 2 I_c I_{pc} \sin(\omega t) \sin(\omega t - \phi) \frac{dM}{d\theta}$$

$$\therefore \quad T_i = I_c I_{pc} [\cos \phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} \quad \ldots (5)$$

Key Point : *Thus instantaneous torque has a component of power which varies as twice the frequency of current and voltage.*

$$\therefore \quad T_d = \text{Average deflecting torque} = \frac{1}{T} \int_0^T T_i \, d(\omega t)$$

$$= \frac{1}{T} \int_0^T I_c I_{pc} [\cos\phi - \cos(2\omega t - \phi)] \frac{dM}{d\theta} d(\omega t)$$

$$\therefore \quad T_d = I_c I_{pc} \cos\phi \frac{dM}{d\theta} \quad \ldots (6)$$

where $I_{pc} = \frac{V}{R_p}$

For a spring controlled wattmeter.

$$T_c = K \theta \quad \ldots (7)$$

But $T_d = T_c$

$$\therefore \quad I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K \theta$$

$$\therefore \quad \boxed{\theta = \frac{1}{K} I_c I_{pc} \cos\phi \frac{dM}{d\theta} = K_1 I_c I_{pc} \cos \phi} \quad \ldots (8)$$

where $K_1 = \frac{1}{K} \frac{dM}{d\theta}$

$$\therefore \quad \boxed{\theta = K_1 I_c \frac{V}{R_p} \cos\phi = K_2 P} \quad \ldots (8)$$

where $K_2 = \frac{K_1}{R_p}$ and $P = V I_c \cos \phi$ = Power

$$\therefore \quad \boxed{\theta \propto P} \quad \ldots (10)$$

Key Point : *Thus the wattmeter deflection when calibrated gives the power consumption of the circuit.*

3.3.2 Reading on Wattmeter

The Fig. 3.7 shows symbolic representation of wattmeter.

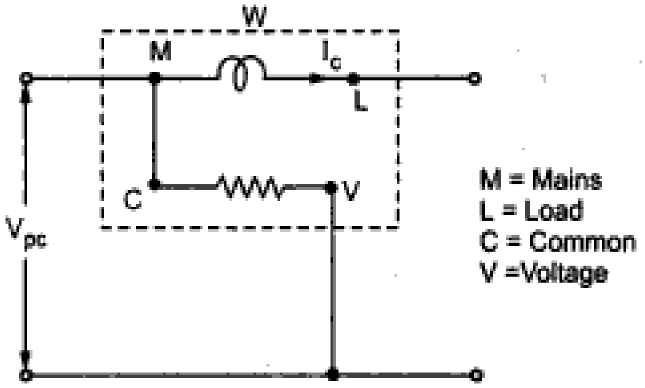


Fig. 3.7 Symbolic representation of wattmeter

Thus if, I_c = Current through current coil

V_{pc} = Voltage across pressure coil

Then wattmeter reading is,

$$\boxed{W = V_{pc} I_c \cos (V_{pc} \wedge I_{pc})}$$

Key Point: *The angle between V_{pc} and I_{pc} may or may not be power factor angle ϕ. It depends on the wattmeter connection in the circuit.*

Thus if wattmeter is connected in a three phase circuit such that $I_c = I_{ph}$ and $V_{pc} = V_{ph}$ then only $V_{pc} \wedge I_{pc} = \phi$.

Key Point : *In a three phase circuit, angle between V_{pc} and I_{pc} is to be obtained from the corresponding phasor diagram.*

3.4.1 Error Due to Pressure Coil Inductance

In case of ideal wattmeter, the current in the pressure coil is in phase with the applied voltage because the pressure coil is assumed to be purely resistive without any reactance. But if it is having inductance the current in the pressure coil lags behind the supply voltage by some angle. Because of this, an error is introduced in the measurement of true power by the wattmeter. Some correction factor must be applied to get exact reading from wattmeter. This can be derived as given below.

Let r_p = Resistance of pressure coil

L = Inductance of pressure coil

R = Resistance in series with pressure coil

R_p = Total resistance of pressure coil circuit = $r_p + R$

V = Voltage supplied to pressure coil

I = Current in current coil circuit

I_p = Current in pressure coil circuit

Z_p = Impedance of pressure coil circuit = $(r_p + R) + j\omega L$

$$= \sqrt{(r_p + R)^2 + (\omega L)^2}$$

Let the current in the pressure coil circuit is lagging behind the supply voltage by an angle β.

$$\therefore \quad \tan\beta = \frac{\omega L}{R_p} = \frac{\omega L}{r_p + R}$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{\omega L}{r_p + R}\right)$$

If we assume that the load power factor to be lagging then the corresponding phasor diagram is as shown in the Fig. 3.9.

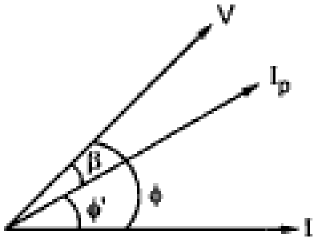


Fig. 3.9

From the Fig. 3.9 we have,

$$\phi = \phi' + \beta$$

$$\therefore \quad \phi' = \phi - \beta$$

Now, Actual wattmeter reading

$$= \frac{I_1 I_2}{K} \cos\phi \, \frac{dM}{d\theta}$$

Here $I_1 = I, \quad I_2 = I_p = \frac{V}{Z_p}, \quad \phi = \phi'$

$$\text{Actual wattmeter reading} = \frac{I_P . I}{K} \cos\phi' \frac{dM}{d\theta}$$

$$= \left(\frac{V}{Z_P K}\right) I \cos(\phi - \beta) \frac{dM}{d\theta}$$

Now, $R_P = Z_P \cos\beta$

$\therefore$ $Z_P = \dfrac{R_P}{\cos\beta}$

$$\text{Actual wattmeter reading} = \frac{V\,I \cos(\phi - \beta)}{\left[\dfrac{R_P}{\cos\beta}\right] \cdot K} \frac{dM}{d\theta} = \frac{V\,I}{KR_P} \cos\beta \cos(\phi - \beta) \frac{dM}{d\theta}$$

In the absence of inductance $Z_P = R_P$ and $\beta = 0$ and the wattmeter gives true power and it reads correctly at all power factors and frequencies.

$$\therefore \quad \boxed{\text{True power} = \frac{I_P \cdot I}{K} \cos\phi \frac{dM}{d\theta} = \frac{V\,I \cos\phi}{K\,R_P} \frac{dM}{d\theta}}$$

If we take ratio of true power and actual wattmeter reading then,

$$\frac{\text{True power}}{\text{Actual wattmeter reading}} = \frac{\left[\dfrac{VI \cos\phi}{K\,R_P}\right] \dfrac{dM}{d\theta}}{\left[\dfrac{VI\cos\beta \cos(\phi - \beta)}{K\,R_P}\right] \dfrac{dM}{d\theta}} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)}$$

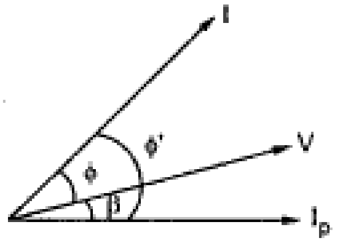
$$\therefore \quad \boxed{\text{True power} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)} \times \text{Actual wattmeter reading}}$$

Thus for lagging loads the correction factor is given by,

$$\boxed{\text{Correction factor} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \beta)}} \quad \textbf{... For lagging p.f.}$$

It can be seen from the phasor diagram that for lagging power factor condition the wattmeter reads high since the inductance of pressure coil will try to bring the current in pressure coil nearly in phase with current in current coil than would be the case if this inductance were zero. Thus error will be involved in wattmeter reading which would be serious at low power factor condition if proper precaution is not taken.

In case of leading power factor condition, the effect of pressure coil inductance is to increase the phase angle between load current and pressure coil current and wattmeter reads low. The corresponding phasor diagram is shown in the Fig. 3.10.



For leading power factor, $\boxed{\text{Correction factor} = \frac{\cos\phi}{\cos\beta\cdot\cos(\phi+\beta)}}$ ... **For leading p.f.**

Now we will find the error which is produced because of inductance of pressure coil at **lagging power factor** condition.

$$\begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix} - \text{True power} = \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix} - \left[\frac{\cos\phi}{\cos\beta\cdot\cos(\phi-\beta)}\right]\begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$= \left[1 - \frac{\cos\phi}{\cos\beta\cdot\cos(\phi-\beta)}\right]\times \text{Acutal wattmeter reading}$$

As β is small, cosβ will be nearly equal to unity.

$$\therefore \qquad \text{Error} = \left[1 - \frac{\cos\phi}{\cos(\phi-\beta)}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$= \left[1 - \frac{\cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$= \left[\frac{\cos\phi\cos\beta + \sin\phi\sin\beta - \cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$= \left[\frac{\cos\phi + \sin\phi\sin\beta - \cos\phi}{\cos\phi\cos\beta + \sin\phi\sin\beta}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$= \left[\frac{\sin\phi\sin\beta}{\cos\phi + \sin\phi\sin\beta}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}$$

$$\therefore \qquad \boxed{\text{Error} = \left[\frac{\sin\beta}{\cot\phi + \sin\beta}\right]\times \begin{matrix}\text{Actual wattmeter}\\ \text{reading}\end{matrix}}$$

Now let us find the percentage error.

Consider the relation $\frac{\text{True power}}{\text{Actual wattmeter reading}} = \frac{\cos\phi}{\cos\beta\cdot\cos(\phi-\beta)}$

$$= \frac{\cos\phi}{\cos\beta\left[\cos\phi\cos\beta + \sin\phi\sin\beta\right]} = \frac{\cos\phi}{\cos\beta\cos\phi\cos\beta\left[1 + \tan\phi\tan\beta\right]}$$

$$= \frac{(1/\cos^2\beta)}{1 + \tan\phi\tan\beta} = \frac{\sec^2\beta}{1 + \tan\phi\cdot\tan\beta} = \frac{1 + \tan^2\beta}{1 + \tan\phi\tan\beta}$$

Now as β is small therefore $\tan^2\beta << 1$.

$$\frac{\text{True power}}{\text{Actual wattmeter reading}} = \frac{1}{1+\tan\phi\cdot\tan\beta}$$

$$\therefore \quad \boxed{\text{Actual wattmeter reading} = [1+\tan\phi\cdot\tan\beta]\times\text{True power}}$$

$$\begin{aligned}\therefore \quad \text{Error} &= \text{Actual wattmeter reading} - \text{True power}\\ &= [1+\tan\phi\cdot\tan\beta]\times\text{True power} - \text{True power}\\ &= [\tan\phi\cdot\tan\beta]\times\text{True power}\end{aligned}$$

$$\%\ \text{Error} = \frac{\text{Actual wattmeter reading} - \text{True power}}{\text{True power}}\times 100$$

$$\boxed{\%\ \text{Error} = [\tan\phi\cdot\tan\beta]\times 100}$$

But true power = VI cos ϕ. Hence the error is given as,

$$\boxed{\text{Error} = [\tan\phi\cdot\tan\beta]\times VI\cos\phi = VI\sin\phi\cdot\tan\beta}$$

From the above equation it is clear that the error is considerable at low power factors.

Compensation of error :

The error due to pressure coil inductance can be compensated as shown in the Fig. 3.11.

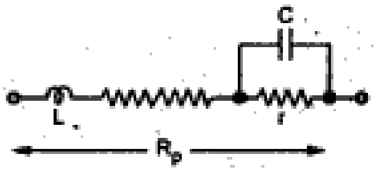


Fig. 3.11

A capacitor C is connected in parallel with a portion of series resistance (multiplier esistance) as shown in the Fig. 3.11.

The total impedance of the circuit shown in above figure is given by,

$$\begin{aligned} Z_p &= (R_p - r) + j\omega L + \frac{(r)(-j/\omega C)}{r - \dfrac{j}{\omega C}}\\ &= (R_p - r) + j\omega L + \frac{(r)(-j)(\omega Cr + j)}{(\omega Cr - j)(\omega Cr + j)}\end{aligned}$$

$$\therefore \quad Z_p = (R_p - r) + j\omega L + \frac{r - j\omega Cr^2}{1+\omega^2C^2r^2}$$

With proper selection of circuit constants at power frequencies $\omega^2C^2r^2 << 1$

$$\begin{aligned}\therefore \quad Z_p &\approx (R_p - r) + j\omega L + r - j\omega Cr^2 \approx R_p + j(\omega L - \omega Cr^2)\\ &\approx R_p + j\omega(L - Cr^2)\end{aligned}$$

If $L = Cr^2$ then $Z_p \approx R_p$ and $\beta = 0°$ which is desired.

Thus the error caused by pressure coil inductance is almost completely eliminated. This type of compensation is slightly affected by change in frequency and can be used for frequencies where $\omega^2C^2r^2 << 1$. It can be applied to frequencies upto 10 kHz.

3.4.2 Error Due to Pressure Coil Capacitance

In addition to inductance of pressure coil there may be capacitance due to interturn capacitance of the series resistance. The effect of capacitance is exactly opposite to that of inductance. Thus the wattmeter reads low on lagging power factors.

The phase angle between pressure coil current and applied voltage depends upon the reactance of the pressure coil circuit. The inductive reactance is normally greater than capacitive reactance, thus the phase angle increases with increase in frequency.

If the capacitive reactance of the pressure coil circuit is equal to its inductive reactance, there will be no error due to these effects since the two errors will neutralize each other.

3.4.3 Error Due to Method of Connection

There are two ways of connecting wattmeter in a given circuit. These are respectively shown in the Fig. 3.12 (a) and (b).

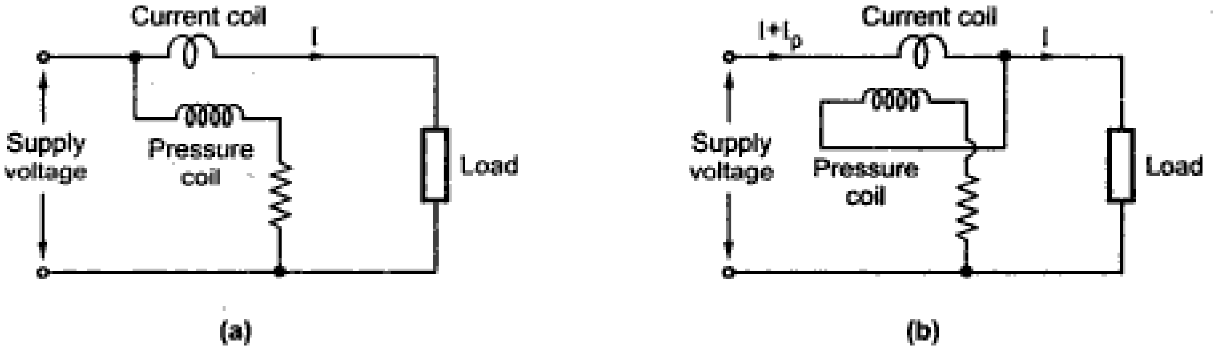


Fig. 3.12

Because of the power loss in the current and pressure coils, error is introduced in the measurement of power.

In connection shown in the Fig. 3.12 (a), pressure coil is connected on the supply side and therefore the voltage applied to the pressure coil is the voltage across the load plus the voltage drop across current coil. Thus wattmeter measures power loss in its current coil in addition to power consumed by load.

Power indicated by wattmeter = Power consumed by load + Power loss in current coil

$\therefore$ Power indicated by wattmeter = Power consumed by load + $I^2 R_c$

If wattmeter connections are as shown in the Fig. 3.12 (b) the current coil is on supply side and hence it carries pressure coil current plus the load current. Thus wattmeter reads in addition to power consumed in load, the power loss in pressure coil.

$$\text{Power indicated by wattmeter} = \underset{\text{by load}}{\text{Power consumed}} + \underset{\text{pressure coil circuit}}{\text{Power loss in}}$$

$$= \underset{\text{by load}}{\text{Power consumed}} + V^2 / R_p$$

With small load current, the voltage drop in current coil is small so connections in Fig. 3.12 (a) introduces small error. Alternatively if load current is large, the pressure coil current is very small as compared with load current. Hence power loss in pressure coil circuit is small as compared with power consumed by load. Thus connection shown in Fig. 3.12 (a) is preferable for small currents while for large currents the connections shown in Fig. 3.12 (b) are preferable.

But if load current is high and the power factor is small, connection shown in Fig. 3.12 (b) results in large error as the total power measured is small. In this case a compensating coil may be used for compensation of error which is explained further in low power factor wattmeters.

3.4.4 Eddy Current Errors

The current coil produces an alternating magnetic field because of which eddy currents are induced in the solid metal parts and within the thickness of the conductors. These currents produce field and tries to reduce flux produced by current coil. The phase angle between fluxes in current coil and potential coil is increased which decreases the deflecting torque produced by the instrument for lagging power factors.

The wattmeter reads low for lagging power factors and reads high for leading power factors. To minimize this error, solid metal parts are avoided as far as possible. Standard conductors are used for the current coil if it carries large current. Thus in practice a special type of wattmeter called low power factor wattmeter (LPF) is used.

➠ **Example 3.1 :** *A wattmeter has a current coil of 0.03 Ω resistance and a pressure coil of 6000 Ω resistance. Calculate the percentage error if the wattmeter is so connected that*

i) The current coil is on the load side ii) The pressure coil is on the load side.

a) If the load takes 20 A at a voltage of 220 V and 0.6 power factor in each case.

b) What load current would give equal errors with the two connections ?

Solution : Power consumed by load,

$$P_T = VI\cos\phi$$

$$= 220 \times 20 \times 0.6$$

$$\therefore \quad P_T = 2640 \text{ watt}$$

i) Consider that current coil is on the load side

Power indicated by wattmeter,

$$P_W = \text{Power consumed by load} + \text{Power loss in current coil}$$

$$= P_T + I^2 R_c$$

$$= 2640 + (20)^2 (0.03)$$

$$= 2652 \text{ watt}$$

$$\% \text{ Error} = \frac{P_W - P_T}{P_T} \times 100 = \frac{2652 - 2640}{2650} \times 100$$

$$\% \text{ Error} = \mathbf{0.45\ \%}$$

ii) Consider the pressure coil is on the load side

Power indicated by wattmeter,

$$P_W = \text{Power consumed by load} + \text{Power loss in pressure coil}$$

$$= P_T + V^2 / R_p$$

$$= 2640 + \frac{(220)^2}{6000}$$

$$= 2648.06 \text{ watt}$$

$$\% \text{ Error} = \frac{P_W - P_T}{P_T} \times 100 = \frac{2648.06 - 2640}{2640} \times 100$$

$$\therefore \quad \% \text{ Error} = \mathbf{0.3055\ \%}$$

iii) Power consumed by load remains same. Thus to get equal error with two connections power indicated in both cases by wattmeter must be same.

$$\therefore \quad \begin{array}{l}\text{Power indicated by wattmeter}\\ \text{with current coil on load side}\end{array} = \begin{array}{l}\text{Power indicated by wattmeter}\\ \text{with pressure coil on load side}\end{array}$$

$$P_T + I^2 R_c = P_T + V^2 / R_p$$

$$I^2 R_c = \frac{V^2}{R_p}$$

$$\therefore \quad I = \frac{1}{\sqrt{R_p \cdot R_c}} \cdot V = \frac{1}{\sqrt{(6000)(0.03)}} (220)$$

$$\therefore \quad I = \mathbf{16.39\ A}$$

At this current equal errors are obtained.

4

Measurement of Energy

4.1 Introduction

The energy is defined as the power delivered over a time interval.

$$\text{energy} = \text{power} \times \text{time}$$

The electrical energy is defined as the work done over a time interval t and mathematically expressed as,

$$\boxed{E = \int_0^t \text{power}(dt) = \int_0^t vi\, dt} \qquad \text{... (1)}$$

where v = voltage in volts and i = current in amperes

The energy is measured in joules (J) or watt-sec (W-s). Thus energy of one joule means the power of 1 watt over a time interval of 1 second.

An electrical energy can also be expressed in the unit watt-hour (Wh) or kilowatt-hour (kWh). Thus one kilowatt-hour energy means the expenditure of 1 kW power over a time interval of 1 hour. The domestic electric energy expenditure is measured in kWh and 1 kWh is called 1 unit of energy.

4.2 Single Phase Energymeter

Induction type instruments are most commonly used as energy meters. Energy meter is an integrating instrument which measures quantity of electricity. Induction type of energy meters are universally used for domestic and industrial applications. These meters record the energy in kilowatt-hours (kWh).

The Fig. 4.1 (See Fig. 4.1 on next page) shows the induction type single phase energymeter.

It works on the principle of induction i.e. on the production of eddy currents in the moving system by the alternating fluxes. These eddy currents induced in the moving system interact with each other to produce a driving torque due to which disc rotates to record the energy.

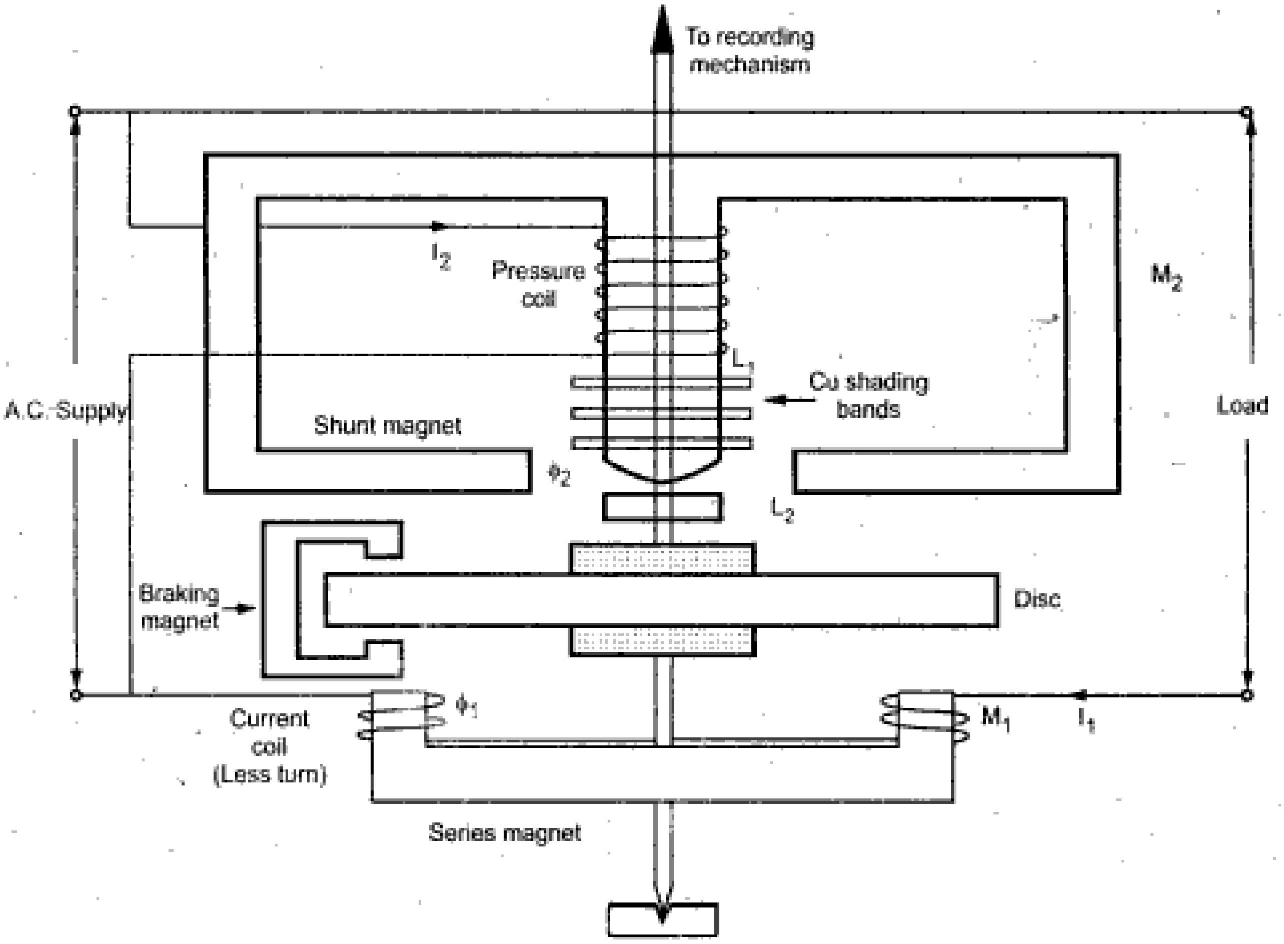


Fig. 4.1 Induction type single phase energymeter

In the energy meter there is no controlling torque and thus due to driving torque only, a continuous rotation of the disc is produced. To have constant speed of rotation braking magnet is provided.

4.2.1 Construction

There are four main parts of operating mechanism,

1) Driving system 2) Moving system 3) Braking system 4) Registering system.

1) Driving system : It consists of two electromagnets whose core is made up of silicon steel laminations. The coil of one of the electromagnets, called **current coil, is** excited by load current which produces flux further. The coil of another electromagnet is connected across the supply and it carries current proportional to supply voltage. This coil is called **pressure coil.** These two electromagnets are called **series** and **shunt** magnets respectively.

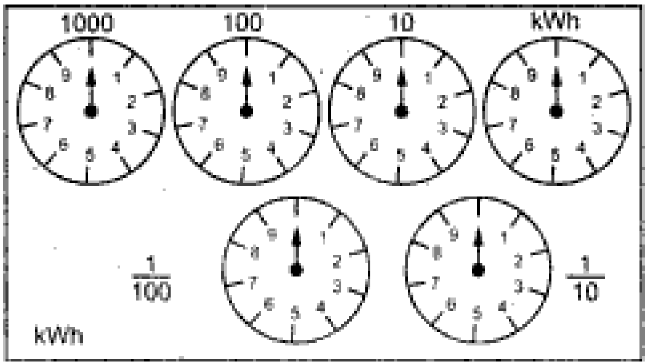
The flux produced by shunt magnet is brought in exact quadrature with supply voltage with the help of copper shading bands whose position is adjustable.

2) Moving system : Light aluminium disc mounted in a light alloy shaft is the main part of moving system. This disc is positioned in between series and shunt magnets. It is supported between jewel bearings. The moving system runs on hardened steel pivot. A pinion engages the shaft with the counting mechanism. There are no springs and no controlling torque.

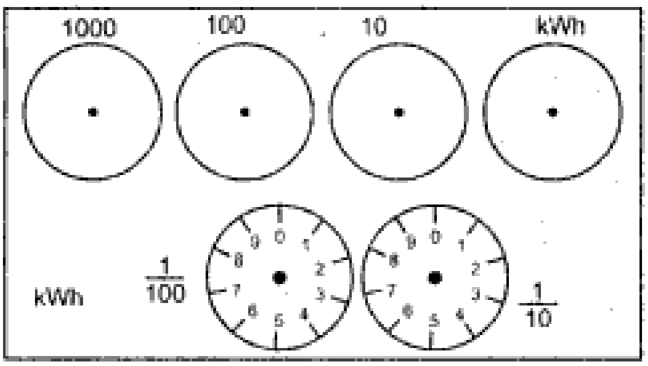
3) Braking system : A permanent magnet is placed near the aluminium disc for braking mechanism. This magnet reproduced its own field. The disc moves in the field of this magnet and a braking torque is obtained. The position of this magnet is adjustable and hence braking torque is adjusted by shifting this magnet to different radial positions. This magnet is called **Braking magnet.**

4) Registering mechanism : It records continuously a number which is proportional to the revolutions made by the aluminium disc. By a suitable system, a train of reduction gears, the pinion on the shaft drives a series of pointers. These pointers rotate on round dials which are equally marked with equal divisions.

Practically the pointer type registering mechanism is used. The pointer indicates one kWh when the disc completes certain number of revolutions. The second dial represents 10 kWh, third 100 kWh while on the other sides, dials measuring 1/100 and 1/10 kWh are also provided. The Fig. 4.2 (a) shows the pointer type register while the Fig. 4.2 (b) shows the cyclometer type register. In some meters the cyclometer type registering mechanism is used.



(a) Pointer register



(b) Cyclometer register

4.3 Theory of Single Phase Induction Type Energymeter

Since the pressure coil is carried by shunt magnet M_2 which is connected across the supply , it carries current proportional to the voltage. Series magnet M_1 carries current coil which carries the load current. Both these coils produce alternating fluxes ϕ_{sh} and ϕ_{se} respectively. These fluxes are proportional to currents in their coils. Parts of each of these fluxes link with the disc and induces e.m.f. in it. Due to these e.m.f.s eddy currents are induceed in the disc. The eddy current induced by the electromagnet M_2 react with magnetic field produced by M_1. Also eddy currents induced by electromagnet M_1 react with magnetic field produced by M_2. Thus each portion of the disc experiences a mechanical force and due to motor action, disc rotates. The speed of disc is controlled by the C shaped magnet called braking magnet. When disc rotates in the air gap, eddy currents are induced in disc which oppose the cause producing them i.e. relative motion of disc with respect to magnet. Hence braking torque T_b is generated. This is proportional to speed N of disc. By adjusting position of this magnet, desired speed of disc is obtained. Spindle is connected to recording mechanism through gears which record the energy supplied.

A simple functional diagram of driving mechanism is shown in the Fig. 4.3.

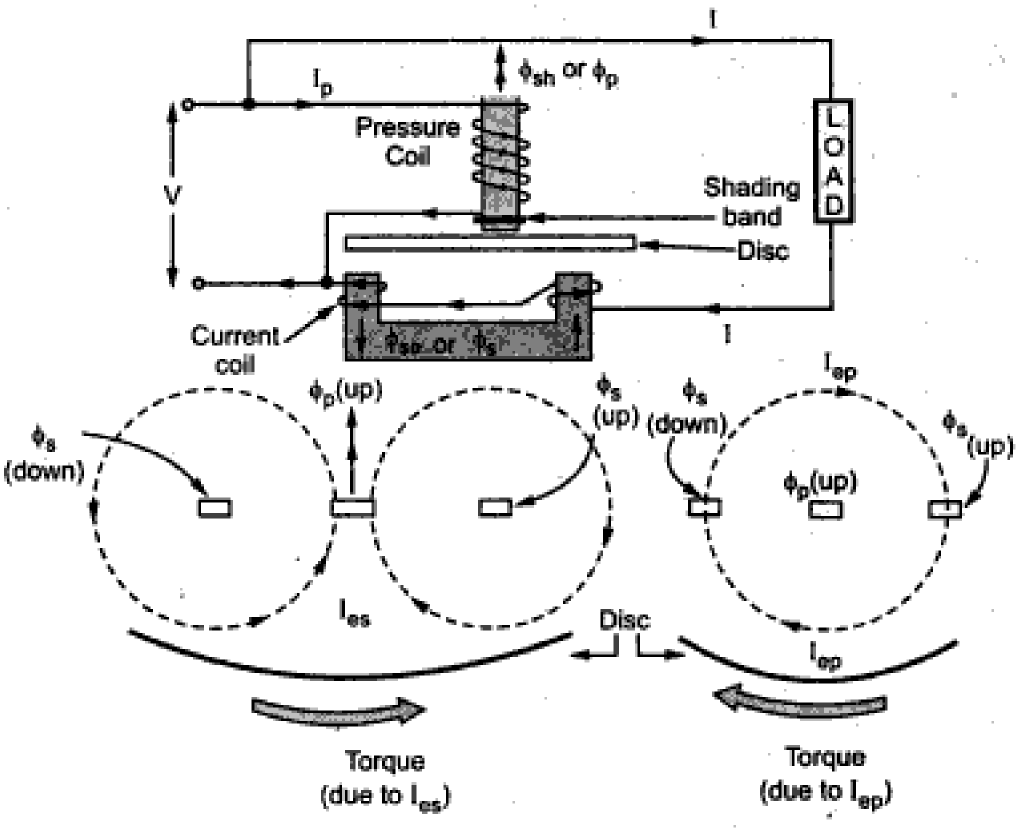


Fig. 4.3 Functional diagram of induction type energymeter

The current I_p produces the total flux ϕ_{pt} which has two components ϕ_g and ϕ_p. The major portion is ϕ_g which flows through the side gaps as the reluctance of this part is very small. While ϕ_p flows across the air gap and across the disc and is responsible to produce the eddy e.m.f. E_{ep} in the disc which produces the eddy current I_{ep} in the disc. The ϕ_p is small and is in phase with I_p. It is proportional to I_p and hence to supply voltage V as it produces I_p through pressure coil. ϕ_p lags the supply voltage V by an angle slightly less than 90°.

The current coil carries the load current I and produces the flux ϕ_s. This is proportional to I and in phase with it. This flux is responsible to induce eddy e.m.f. E_{es} in the disc which produces the eddy current I_{es} in the disc. This interacts with the flux ϕ_p to produce the torque while the eddy current I_{ep} interacts with ϕ_s to produce the torque. These two torques are opposite in direction and the net torque produced is the difference between these two torques.

4.3.1 Torque Equation

The phasor diagram is shown in the Fig. 4.4.

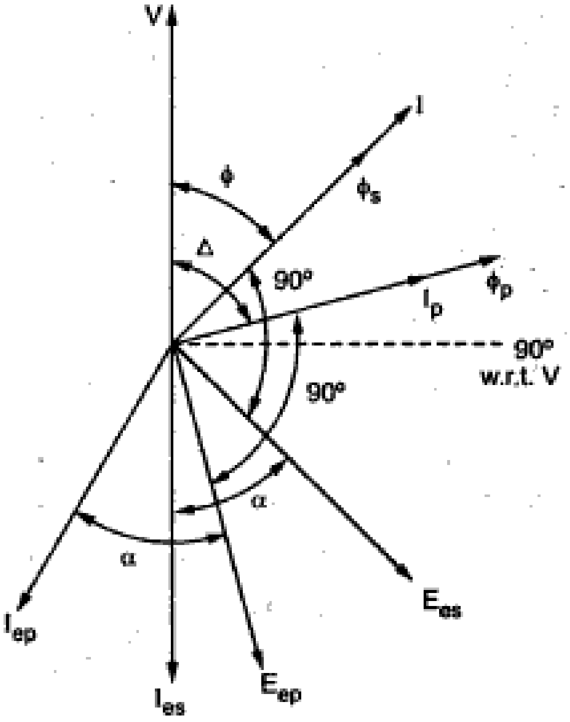


Fig. 4.4 Phasor diagram of single phase induction type energymeter

Let,

V = supply voltage

I = load current

= current coil current

I_p = pressure coil current

Δ = phase angle between V and I_p

$\Delta \approx 90°$

E_{ep} = eddy e.m.f. induced due to ϕ_p

E_{es} = eddy e.m.f. induced due to ϕ_s

α = phase angle of eddy currents

I_{ep} = eddy current due to E_{ep}

I_{es} = eddy current due to E_{es}

The current I_p lags V by Δ and Δ is made 90° using copper shading bands. The current I lags V by ϕ which depends on the load. The flux ϕ_s and I are in phase. The E_{ep} lags ϕ_p by 90° while E_{es} lags ϕ_s by 90°. The eddy currents I_{es} and I_{ep} lags E_{es} and E_{ep} respectively by angle α.

The interaction between ϕ_p and I_{es} produces torque T_1.

While the interaction between ϕ_s and I_{ep} produces torque T_2.

$$\therefore T_1 \propto \phi_p I_{es} \cos(\phi_p \wedge I_{es}) \quad \text{and} \quad T_2 \propto \phi_s I_{ep} \cos(\phi_s \wedge I_{ep})$$

$$\phi_p \wedge I_{es} = \alpha + \phi \quad \text{and} \quad \phi_s \wedge I_{ep} = 180 - \phi + \alpha \qquad \ldots \Delta = 90°$$

$$\therefore T_d \propto T_1 - T_2 \propto \{[\phi_p I_{es} \cos(\alpha + \phi)] - [\phi_s I_{ep} \cos(180 - \phi + \alpha)]\}$$

Now $\phi_p \propto V$, $\phi_s \propto I$, $I_{es} \propto \phi_s \propto I$, $I_{ep} \propto \phi_p \propto V$

$$\therefore T_d \propto V I[\cos(\alpha + \phi) - \cos(180 - \phi + \alpha)]$$

$$\propto VI[(\cos\alpha \cos\phi - \sin\alpha \sin\phi) - (\cos(180 - \phi)\cos\alpha - \sin(180 - \phi)\sin\alpha)]$$

$$\propto VI[\cos\alpha \cos\phi - \sin\alpha \sin\phi - \cos(180 - \phi)\cos\alpha + \sin(180 - \phi)\sin\alpha]$$

$$\propto 2\,VI \cos\alpha \cos\phi \qquad \ldots \cos(180 - \phi) = -\cos\phi,\ \sin(180 - \phi) = \sin\phi$$

$$\therefore \boxed{T_d = K_3\, VI \cos\phi} \qquad (\alpha \text{ is constant}) \qquad \ldots (1)$$

Key Point: *Thus the deflecting torque is proportional to the true power in the circuit.*

If Δ is considered,

$$\boxed{T_d \propto VI \sin(\Delta - \phi)} \qquad \ldots (2)$$

But practically Δ is achieved to be exactly 90° with the help of copper shading bands so that T_d is proportional to power in the circuit.

The braking torque is due to eddy currents induced in the aluminium disc. The magnitude of eddy currents is proportional to the speed N of the disc. Hence the braking torque T_b is also proportional to the speed N.

$$\therefore \quad \boxed{T_b \propto N} \quad \text{i.e.} \quad \boxed{T_b = K_4 N} \qquad \ldots (3)$$

For the steady speed of rotation, $T_d = T_b$.

$$\therefore \quad K_3 \, VI \cos\phi = K_4 N$$

$$\therefore \quad \boxed{N = K \, VI \cos\phi = K[\text{power}]} \qquad \ldots (4)$$

$$\text{Total number of revolutions} = \int_0^t N \, dt = \int_0^t K \, (\text{power}) \, dt$$

$$\therefore \quad \boxed{\text{Total number of revolutions} = K \int_0^t P \, dt = K \times \text{energy}} \qquad \ldots (5)$$

Thus the number of revolutions of the disc in a given time is the energy consumption by the circuit in that time.

$$\boxed{K = \text{Meter constant} = \frac{N}{\text{energy}} = \frac{\text{Number of revolutions}}{\text{kWh}}} \qquad \ldots (6)$$

Thus the number of revolutions of the disc per kWh of energy consumption is called the **meter constant**.

4.4 Errors and Compensations

There are various errors present in the single phase induction type energymeter. The driving system can cause the errors due to inaccurate phase angles, abnormal frequencies, effect of temperature on the resistance and unsymmetrical magnetic circuit. The braking system also can cause error due to change in the strength of the braking magnet, change in resistance of the disc, abnormal friction of moving disc etc. To get accurate reading, these errors are required to be compensated. Hence some adjustments are provided in the energymeter to minimize these errors.

4.4.1 Lag Adjustment or Power Factor Adjustment

It is absolutely necessary that meter should measure correctly for all power factor conditions of the loads. This is possible when the flux produced due to current in the pressure coil lags the applied voltage by 90°. But the iron loss and resistance of winding do not allow the flux to lag by exact 90° with respect to the voltage.

The arrangement used to adjust this angle to be 90° is called lag adjustment. A magnetic shunt circuit is introduced in the device which allows the main portion of the shunt magnet flux to bypass the gap in which the disc rotates. It is possible to produce an m.m.f. in the proper phase relation to the shunt magnet flux to bring ϕ_{sh} i.e. ϕ_p in exact quadrature with the voltage. This is shown in the Fig. 4.5. The lag coil is used in addition to shunt coil. The lag coil is few turns of fairly thick wire placed around the central limb of the shunt magnet. The part of shunt flux i.e. ϕ_p links with the lag coil to induce an e.m.f. in it. This produces the lag coil current I_L. This current produces a m.m.f. AT_L in phase with I_L. Thus now the phase of ϕ_p is decided by the combined m.m.f. of lag coil and shunt coil. This can be adjusted by adjusting the resistance connected across the lag coil. When resistance increases, current and m.m.f. of lag coil decreases which decreases the value of the lag angle of coil hence ϕ_{sh} lags behind the voltage by exactly 90°.

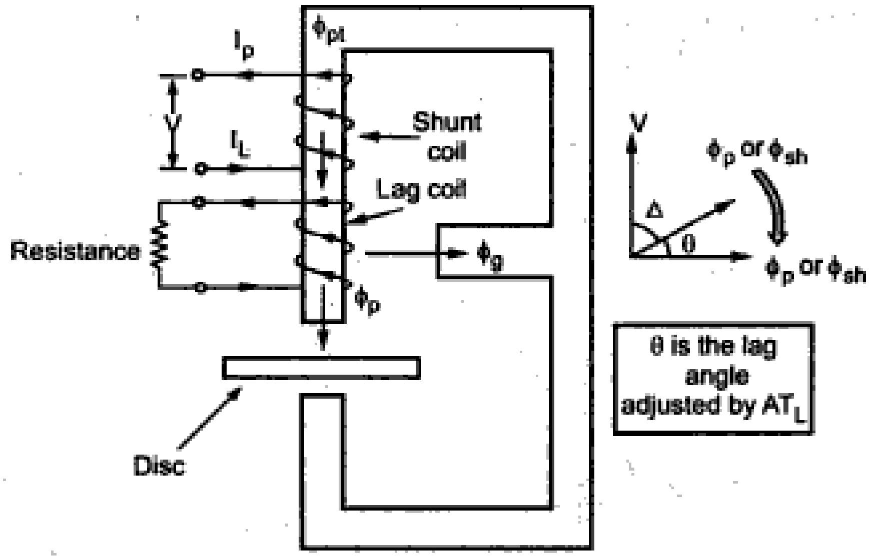


Fig. 4.5 Lag adjustment

Instead of lag coil and resistance, many time copper shading bands are placed on the central limb of the shunt magnet. These bands are adjustable. By moving these bands along the axis of the central limb, the lag adjustment can be achieved. When bands are moved upwards, the e.m.f. induced in them increases increasing the m.m.f. produced, hence lag angle increases. When bands are moved down, the m.m.f. produced by the bands decreases which decreases the lag angle. Thus the ϕ_{sh} can be brought in exact quadrature with the voltage V.

This adjustment is also called **power factor adjustment, quadrature adjustment or inductive load adjustment.**

4.4.2 Light Load Adjustment or Friction Adjustment

Inspite of proper design of the bearings and registering mechanism, there is bound to exist some friction. Due to this, speed of the meter gets affected which cause the error in the measurement of the energy.

To compensate for this, a metallic loop or strip is provided between central limb of shunt magnet and the disc. Due to this strip an additional troque independent of load is produced which acts on the disc in the direction of rotation. This compensates for the friction and meter can be made to read accurately. This is shown as L_2 in the Fig. 4.6.

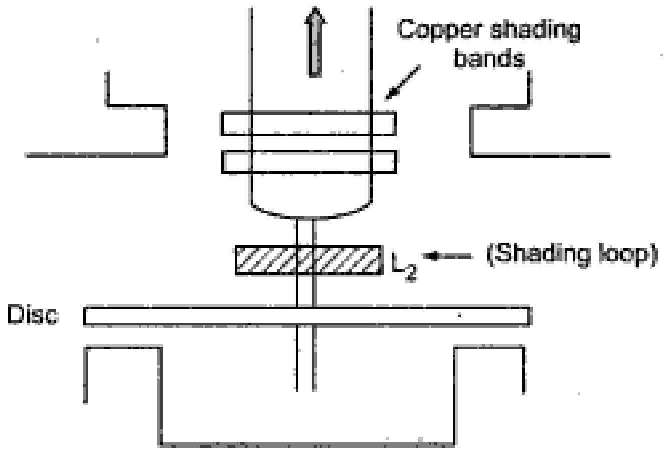


Fig. 4.6 Shading loop for friction adjustment

The shading loop L_2 is also called light load plate.

The interaction between the portions of the flux which are shaded and unshaded by this loop and the currents they induce in the disc generates a small driving torque whose value can be adjusted by lateral movement of the loop L_2. This additional driving torque overcomes the frictional error. This torque is practically independent of the load and depends on line voltage hence remains constant. The friction error is dominant at rated voltage and very low current i.e. at light loads. The shading loop can be moved laterally to adjust the speed to provide necessary compensation.

4.4.3 Creeping Adjustment

In some meters, the disc rotates slowly and continuously when there is no load. The rotation of disc without any current through current coil and only due to excitation of pressure coil is called **creeping.** This is due to friction overcompensation. The torque produced due to light load adjustment may keep disc rotating. To prevent creeping, two holes are drilled in the disc, 180° opposite to each other. When the hole comes under the shunt magnet pole, it gets acted upon by a torque opposite to its rotation. This is shown in the Fig. 4.7.

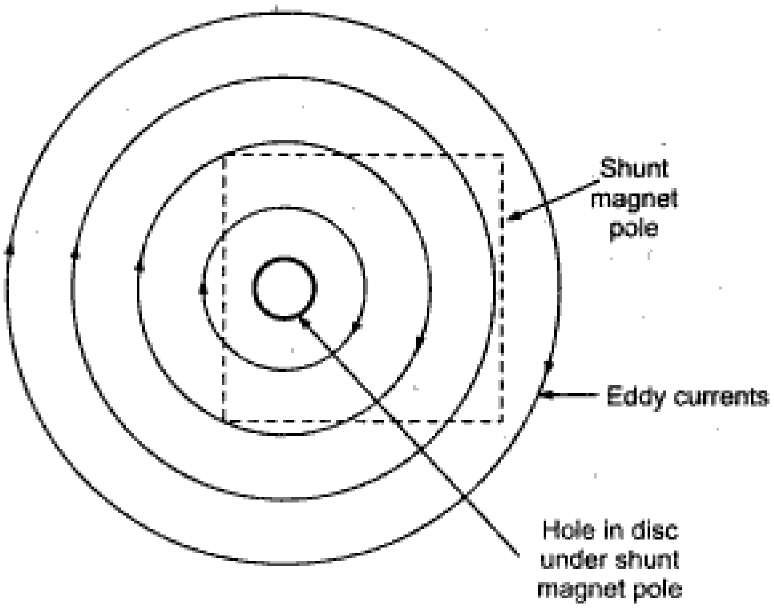


Fig. 4.7 Creeping adjustment

When a hole comes under the shunt magnet, the circular eddy current paths in the disc get distorted. This distortion is responsible to produce torque in opposite direction to the rotation of the disc. This stops creeping. The torque is not very large so as to cause errors under normal operating conditions.

In some cases, a small piece of iron is attached to the edge of the disc. The force of attraction of the braking magnet on the iron piece is responsible to prevent rotation of disc on no loads.

4.4.4 Overload Compensation

When the disc rotates in the field of series magnetic field under load conditions, it cuts the series flux and dynamically, e.m.f. is induced in the disc. This produces eddy currents in the disc which interacts with series magnet flux to produce braking torque. This is proportional to square of the current. This is called self braking torque and at large loads its value is very high, causing serious errors in the measurement. To minimize this braking torque, the full load speed of the disc is kept very low about 40 r.p.m. The current coil series flux is kept minimum compared to shunt magnet flux.

Practically an overload compensating device in the form of a saturable magnetic shunt for the series magnet core is used. This is shown in the Fig. 4.8.

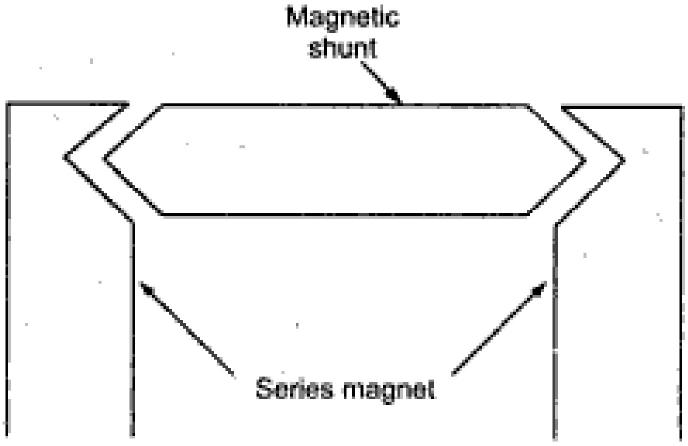


Fig. 4.8 Overload compensation

At high loads, magnetic shunt saturates and diverts some of the series magnetic flux. This compensates for the self braking torque.

4.4.5 Voltage Compensation

When supply voltage varies, the energymeter can cause errors. This is because of two reasons,

i) Nonlinear magnetic characteristics of shunt magnet core.

ii) The braking torque which is proportional to square of the supply voltage.

The voltage compensation is provided by the saturable magnetic shunt which diverts a large proportion of the flux into the active path when the supply voltage increases. The compensation can be provided by increasing the side limb reluctance, by providing holes in the side limbs.

4.4.6 Temperature Compensation

As temperature increases, the resistance of the copper and aluminium parts increases. This has following effects,

i) Small reduction in shunt magnet flux.

ii) Reduction in angle of lag between V and ϕ_p.

iii) Reduction in torque produced by all shading bands.

iv) Increase in eddy current resistance path.

v) Decrease in angle of lag α of the eddy currents.

These various effects neutralize each other and hence errors due to temperature are not serious. But at low lagging power factor loads, such effects may cause serious errors. These effects are compensated by providing a temperature shunt on the brake magnet. Special magnetic materials such as Mutemp is used for the shunt whose permeability decreases considerably as temperature increases. This provides temperature compensation and does not allow the disc to rotate faster as temperature increases.

4.4.7 Main Speed Adjustment

The measurement of energy is dependent on the speed of the rotating disc. For accurate measurement, speed of the disc must be also proportionate. The speed of the meter can be adjusted by means of changing the effective radius of the braking magnet. Moving the braking magnet in the direction of the spindle, decreases the value of the effective radius, decreasing the braking torque. This increases the speed of the meter. While the movement of the braking magnet in the outward direction i.e. away from the centre of the disc, increases the radius, decreasing the speed of the disc. The fine adjustments of the speed can be achieved by providing an additional flux divertor.

4.5 Advantages of Induction Type Energymeter

The various advantages of induction type energymeters are,

1. Its construction is simple and strong.
2. It is cheap in cost.
3. It has high torque to weight ratio, so frictional errors are less and we can get accurate reading.
4. It has more accuracy.
5. It requires less maintenance.
6. Its range can be extended with the help of instrument transformers.

4.6 Disadvantages of Induction Type Energymeter

1. The main disadvantage is that it can be used only for a.c. circuits.
2. The creeping can cause errors.
3. Lack of symmetry in magnetic circuit may cause errors.

4.7 Three Phase Energymeter

In a three phase, four wire system, the measurement of energy is to be carried out by a three phase energy meter. For three phase, three wire system, the energy

Basic construction of power factor meter is similar to a wattmeter. It has two circuits, current circuit and a voltage circuit. The current circuit carries current or fraction of current in the circuit whose power factor is to be measured. The voltage coil is split into two parallel paths, one inductive and one non-inductive. The currents in the two paths are proportional to the voltage of the circuit. Thus the deflection depends upon the phase difference between the main current through current circuit and the currents in the two branches of the voltage circuit i.e. power factor of the circuit.

There are two types of power factor meters,

1) Electrodynamometer type 2) Moving iron type

Let us discuss, these types of power factor meters.

2.15 Single Phase Electrodynamometer Type Power Factor Meter

The construction of electrodynamometer type power factor meter is similar to the construction of electrodynamometer type wattmeter. The basic construction of electrodynamometer type power factor meter is shown in the Fig. 2.14 (a).

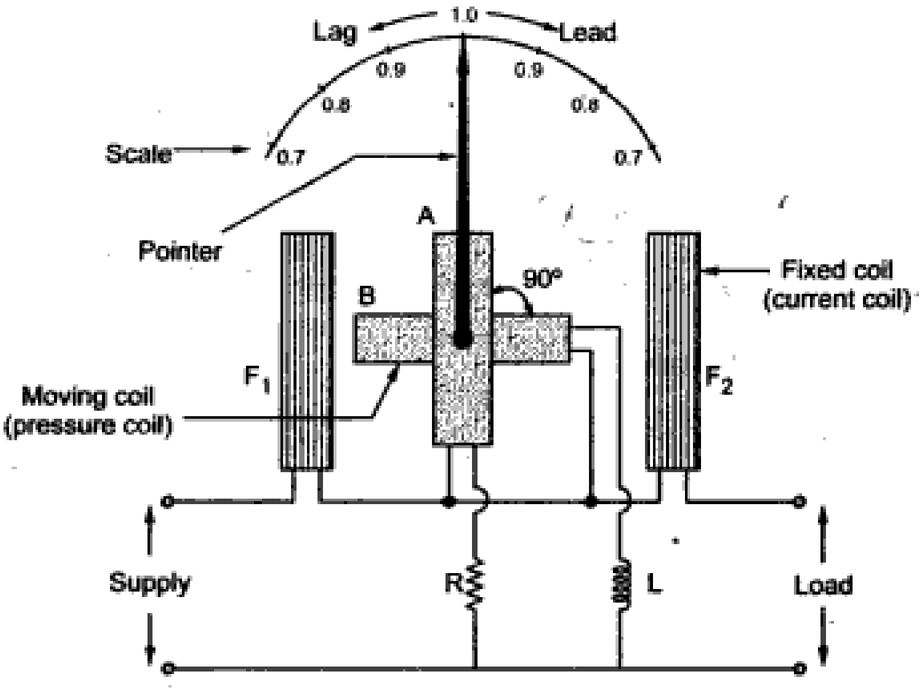


Fig. 2.14 (a) Single phase electrodynamometer type power factor meter

The F_1-F_2 are the two fixed coils which are connected in series. The A-B are the two moving coils which are rigidly connected to each other so that their axes are at 90° to each other. The moving coils A-B move together and carry the pointer which indicates the power factor of the circuit.

The fixed coils F_1-F_2 carry the main current in the circuit. If the current is large, the fraction of the current is passed through the fixed coils. Thus the magnetic field produced by the fixed coils is proportional to the main current.

The moving coils A-B are identical. These are connected in parallel across the supply voltage and hence called pressure coils or voltage coils. The currents through coils A and B are proportional to the supply voltage. The coil A has non-inductive resistance R in series with it while the coil B has an inductance L in series with it. The values of R and L are so adjusted that the coils A and B carry equal currents at normal frequency. So at normal frequency $R = \omega L$. The current through coil A is in phase with the supply voltage while the current through coil B lags the supply voltage by nearly 90° due to highly inductive nature of the circuit. Due to L, current through coil B is frequency dependent while current through coil A is frequency independent.

The currents in the coils A and B are equal and produce the magnetic fields of equal strength, which have phase difference of 90° between them. The coils are also mutually perpendicular to each other.

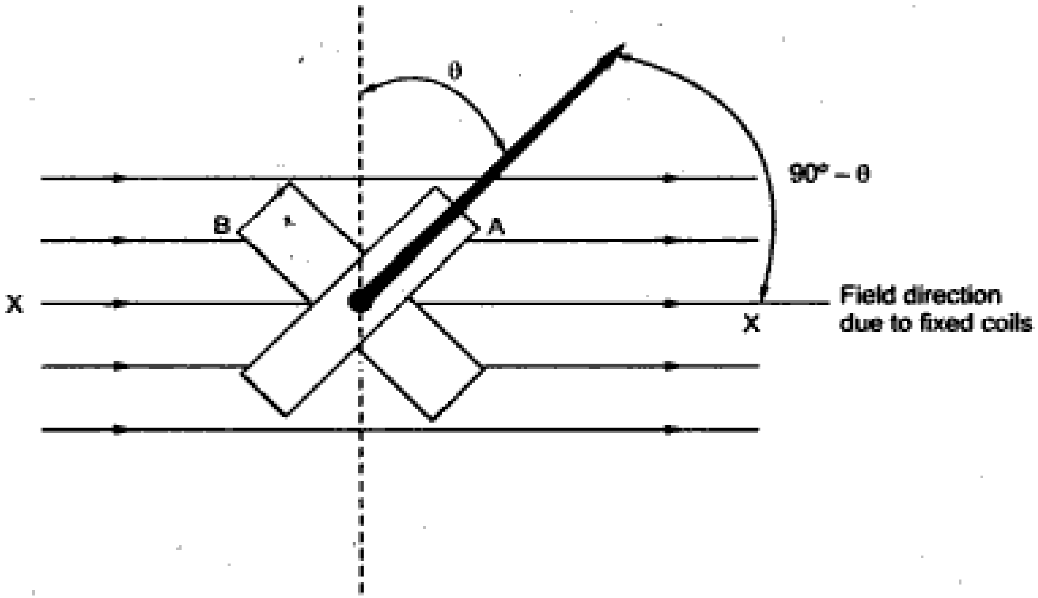
The controlling torque is absent. The contacts to the moving coils are made with the help of extremely fine ligaments which give no controlling effect on the moving system.

2.15.1 Working of Meter

Consider the position of the moving system as shown in the Fig. 2.14 (b).

Assume that the current through coil B lags the voltage exactly by 90°.

Also assume that the field produced by the fixed coils is uniform and in the direction X-X as shown in the Fig. 2.14 (b) .



Due to interaction of the fields produced by the currents through various coils, both the coils A and B experience a torque. The windings are arranged in such a manner that the torques experienced by coil A and B are opposite to each other. Hence the pointer attains an equilibrium position when these two torques are equal.

The torque on each coil, for a given coil current will be maximum when the coil is parallel to the field produced by F_1 - F_2 i.e. direction X-X.

Let ϕ = Power factor angle

θ = Angle of deflection

The θ is measured from the vertical axis, in the equilibrium position.

Similar to a dynamometer type wattmeter, torque on coil A is given by,

$$T_A = K\,VI \cos\phi \cos(90° - \theta) \qquad \text{... (1)}$$

where K = Constant

The equation is similar to the torque equation of a dynamometer type instrument. The current through coil A is in phase with system voltage V and it moves in a magnetic field which is proportional to system current I and dM/dθ which is generally constant for radial field is not constant for parallel field and is proportional to cos (90° – θ).

Similarly current in coil B lags the supply voltage by 90° and it moves in same field. Hence the torque on B is proportional to cos (90° – ϕ) i.e. sin ϕ and cos θ.

$$\therefore \quad T_B = K\,VI \sin\phi \cos\theta \qquad \text{... (2)}$$

In equilibrium position, $T_A = T_B$

$$\therefore \cos\phi \cos(90° - \theta) = \sin\phi \cos\theta$$

$$\therefore \quad \sin\theta = \tan\phi \cos\theta$$

$$\therefore \quad \tan\theta = \tan\phi$$

$$\therefore \quad \boxed{\theta = \phi}$$

Thus the angular position taken up by the moving coils is equal to the system power factor angle. The scale of the instrument can then be calibrated interms of power factor values.

The operation of the instrument is dependent on the specific supply frequency. If the frequency is different or it contains harmonics then inductance of choke coil changes, due to which there will be serious errors in the instruments reading. Thus the operation of the meter is not dependent on the values of current and voltage but dependent on the frequency and the waveform.

2.17 Frequency Meters

The meters which are used in the circuit to indicate the frequency of the supply are called frequency meters.

The frequency meters are classified based on the principle of operation as,

1. Mechanical resonance type frequency meter
2. Electrical resonance type frequency meter
3. Weston type frequency meter

The mechanical resonance type frequency meter is called vibrating reed type frequency meter. The electrical resonance type frequency meter is called ferro-dynamic frequency meter. Let us discuss these frequency meters.

2.17.1 Vibrating Reed Type Frequency Meter

This meter works on the principle of mechanical resonance. The meter consists of number of thin steel strips called reeds. The bottom of the reed is rigidly fixed to an electromagnet. The upper part of the reed is free and bent at right angles. This upper part is called a flag. An electromagnet has a laminated iron core, which carries an excitation coil having large number of turns. This coil is connected across the voltage whose frequency is to be measured. The flags are painted white to have good visibility on the black background. The basic construction of this type of meter and the construction of reed is shown in the Fig. 2.17.

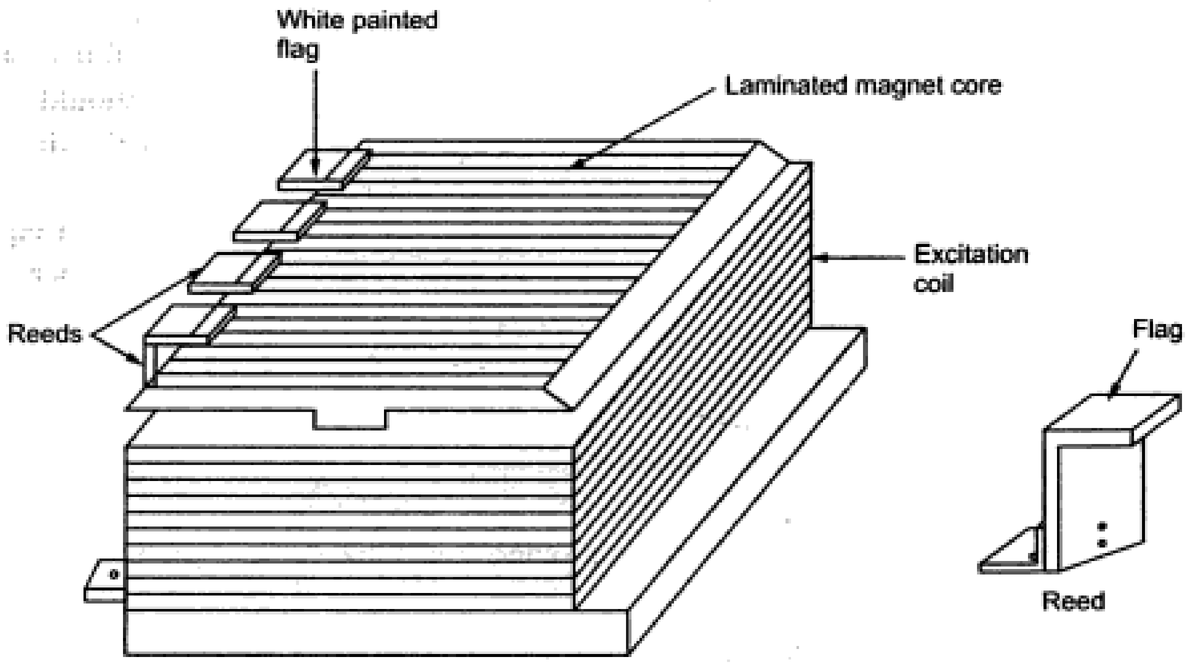


Fig. 2.17 Vibrating reed type frequency meter

The reeds are manufactured such that their weights and dimensions are different. Hence their natural frequencies of vibration are different. The reeds are arranged in the ascending order of their natural frequencies and the natural frequencies are generally differ by half cycle. So natural frequency of first reed may be 48 Hz, next may be 48.5 Hz, next may be 49 Hz and so on.

When meter is connected in the system, the coil carries current i which alternates at the supply frequency. This produces an alternating flux. This produces a force of attraction on the reeds which is proportional to **square of the current** i^2 and hence all the reeds vibrate with a force which varies at twice the supply frequency. But the reed whose natural frequency is twice the frequency of supply voltage will be in resonance and will vibrate most. The tuning in such meters is so precise that for a 1 to 2% change in the frequency away from resonating frequency, the amplitude of vibration decreases drastically and becomes negligible. Thus when a reed corresponding to 50 Hz is vibrating with maximum amplitude, other reeds vibrate but with negligible amplitudes which can not be noticed. This is shown in the Fig. 2.18.

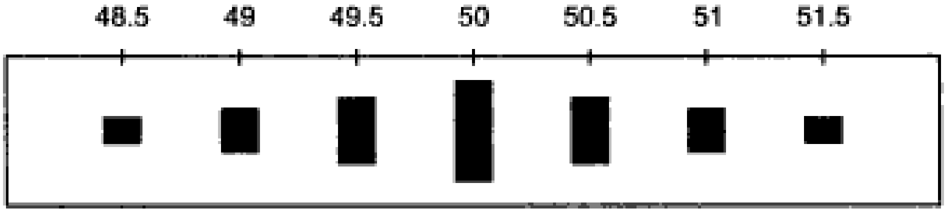


Fig. 2.18 Vibrating reeds

The advantages of this frequency meter are that the reading is not affected by the changes in the waveform of the supply voltage and simple mechanism. But if supply voltage is low, the vibrations may not be noticed. So supply voltage should not be low for the effective operation. One more limitation of the meter is that the difference in the frequencies of the adjacent reeds is 0.5 only. So reading corresponding to less than half the frequency difference can not be obtained. So precise frequency measurement is not possible. The accuracy of the meter depends on the proper tuning of the reeds.

2.17.2 Electrical Resonance Type Frequency Meter

The Fig. 2.19 shows the construction of the electrical resonance type frequency meter.

It consists of a laminated iron core. On one end of the core a fixed coil is wound which is called magnetizing coil. This coil is connected across the supply whose frequency is to be measured. This coil carries current which has same frequency as that of supply. On the same core, a moving coil is pivoted which carries a pointer. A capacitor C is connected across the terminals of the fixed coil.

Let I = Current through magnetizing coil

ϕ = Flux in the iron core

The flux ϕ is assumed to be in phase with the current I.

This flux induces the voltage in the moving coil which always lags flux ϕ by 90°.

Let i = Current through moving coil

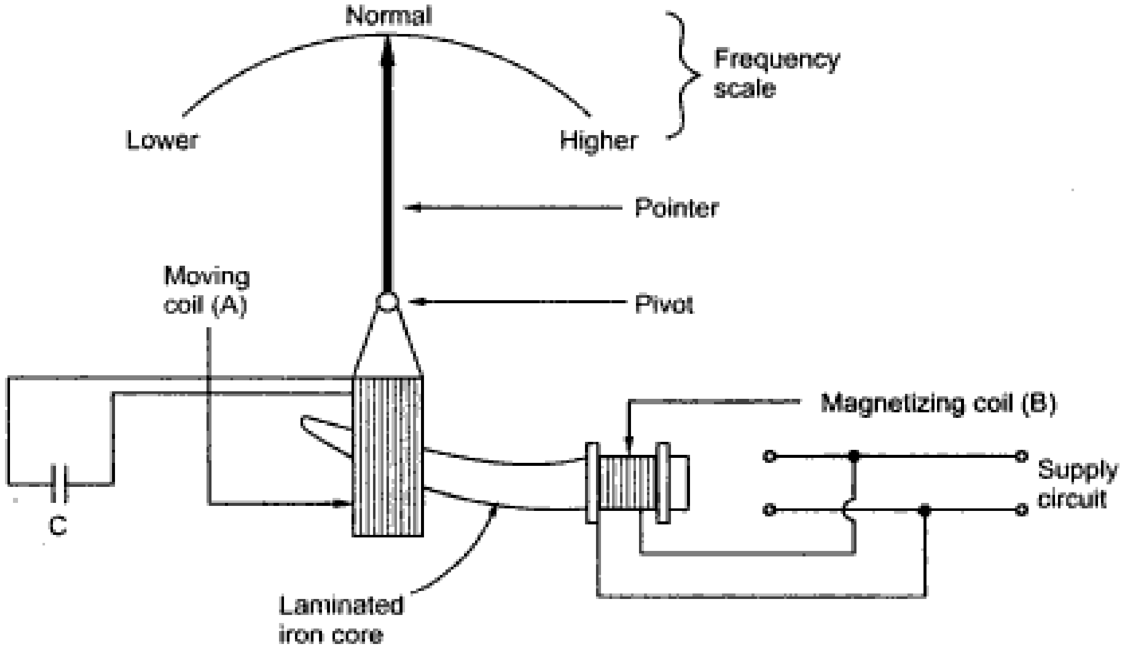
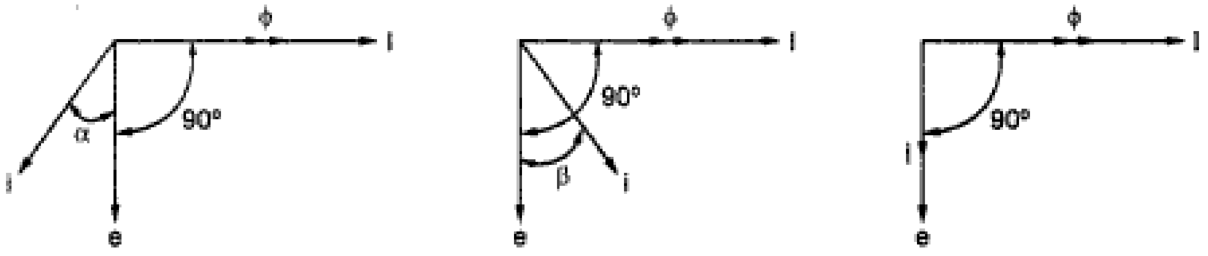


Fig. 2.19 Electrical resonance type frequency meter

The phase of the current i depends on the inductance of the moving coil and the capacitor C.

Consider the different cases and the corresponding phasor diagrams to understand the working of the meter, as shown in the Fig. 2.20 (a), (b) and (c).



(a) (b) (c)

Fig. 2.20

In Fig. 2.20 (a) the circuit of moving coil A is assumed to be inductive, hence current i lags the induced voltage e by angle α. Hence the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90+\alpha) \qquad \text{... (1)}$$

In Fig. 2.20 (b) the circuit of moving coil A is assumed to be largely capacitive, hence current i leads the induced voltage e by angle β. Hence the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90-\beta) \qquad \text{... (2)}$$

This torque is in opposite direction to the torque produced in case of inductive nature of the moving coil circuit.

The Fig. 2.20 (c) shows the resonance condition where the inductive reactance is equal to the capacitive reactance. So current i is in phase with e and the torque acting on the moving coil is given by,

$$T_d \propto I\, i \cos(90^\circ) = 0$$

Hence under resonance condition, torque acting on the moving coil is zero.

Now the capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ is constant for a given frequency. But the inductive reactance $X_L = \omega L$ not only depends on the frequency but also depends on the position of the moving coil on the core. Nearer the moving coil to the magnetizing coil, higher is its inductance. Thus for a given frequency, moving coil moves in such a way to achieve a position where $X_L = X_C$ and electrical resonance is achieved. At this position, torque on the moving coil is zero and the pointer indicates the corresponding frequency. The design of the instrument is such that for a normal frequency, the coil takes a mean position. The capacitor C is chosen such that electrical resonance takes place at this mean position and pointer indicates the normal frequency.

If frequency is higher than the normal value, then $X_C = \frac{1}{2\pi f C}$ decreases. Hence $X_L = 2\pi f L$ must decrease in order to achieve resonance. So moving coil moves away from the magnetizing coil on the core and pointer moves to the right of the mean position, indicating higher frequency.

If frequency is lower than normal value, $X_C = \frac{1}{2\pi f C}$ increases. So to achieve $X_L = X_C$, the moving coil moves towards the magnetizing coil where inductance increases. Thus pointer moves to the left of the mean position, indicating the lower frequency.

An important advantage of the instrument is that the great sensitivity is achieved as the inductance of the moving coil changes slowly with variation of its position on the core. This meter is also called **ferro-dynamic frequency meter.**

5

Potentiometers

5.1 Introduction

A **potentiometer** is an instrument used to measure an unknown e.m.f. which is compared with known e.m.f. Thus it is a device used for measurement of unknown e.m.f. by comparison. The unknown e.m.f. is compared with a known e.m.f. which is obtained from a **standard cell** or any **reference voltage source**. The main advantage of the comparison method of the e.m.f. measurement is that the potentiometer is capable of providing high degree of accuracy as the measurement result is not dependent on the actual deflection of the pointer. Instead of that the accuracy is dependent on the accuracy with which the reference voltage is known.

Basically a potentiometer uses **balance** or **null condition** during the measurement of unknown e.m.f. Actually no current flows in the circuit of the unknown e.m.f. during measurement. Thus no power is consumed in such circuit.

A basic application of the potentiometer is to measure unknown e.m.f. or voltage. It can also be used to determine current. The unknown current can be obtained by using potentiometer by measuring the voltage drop across the standard resistor due to the unknown current.

In the field of the electrical measurements, a potentiometer is most widely used as standard for the calibration of the voltmeters, ammeters and wattmeters. The modern potentiometers are available with high precision and wide range because of the availability of very precise accurate standard e.m.f. in the form of standard cell.

5.2 Principle of Potentiometer

The potentiometer works on the principle of opposing the unknown e.m.f. by a known e.m.f. with the negative terminals of both the e.m.f.s connected together, while the positive terminals connected together through a galvanometer as shown in the Fig. 5.1.

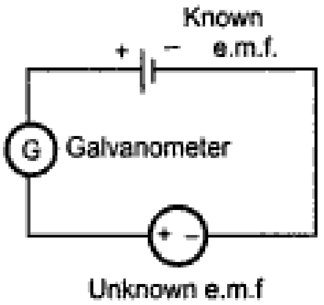


Fig. 5.1 Representation of method of balancing e.m.f.

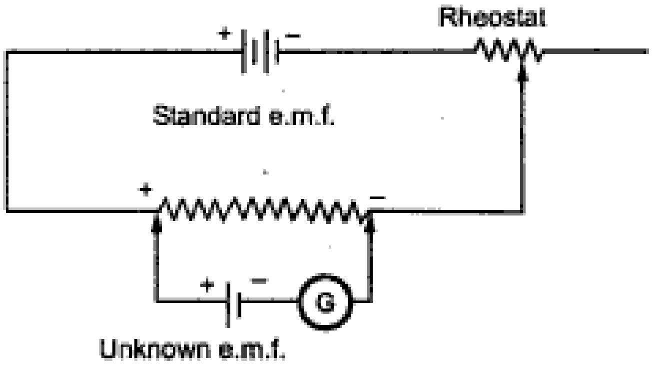


Fig. 5.2 Alternative method of balancing e.m.f.

When the e.m.f.s are of same values, there is no deflection on galvanometer. Thus to measure the unknown e.m.f. by using above method, the known e.m.f. used must be variable. Another important requirement is that known e.m.f. should be varied to give a larger number of known values but it is practically very difficult.

Hence alternatively, the unknown e.m.f. is connected in parallel with and in opposition to a voltage drop measured across the resistor is shown in the Fig. 5.2.

The main advantage of this method is that the current in the resistor can be varied easily to obtain any desired voltage with very fine adjustment. The voltage drop across resistor can be determined by calibrating the resistor with standard cell.

The potentiometers are classified as d.c. potentiometers and a.c. potentiometers. There are various forms of the d.c. potentiometers used widely practically. The basic, simplest type of the d.c. potentiometer is the slide wire potentiometer. Let us study the slide wire potentiometer in detail.

5.3 Slide Wire D.C. Potentiometer

The slide wire d.c. potentiometer is the basic and simplest type of the d.c. potentiometer as shown in the Fig. 5.3.

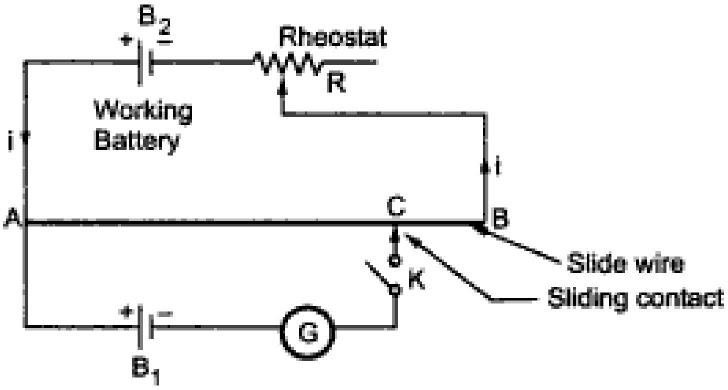


Fig. 5.3 Basic slide wire potentiometer

A basic potentiometer circuit consists of a slide wire AB of uniform cross section and unit length. Generally slide wire is made up of manganin. Let r be the resistance per unit length of slide wire. The battery B_2 supplies a current through the slide wire which is limited with the help of regulating resistance i.e. rheostat. The battery B_1, whose e.m.f. is to be measured is connected in series with a galvanometer G and switch K.

When the switch K is opened, the current through slide wire is i. If the sliding contact is at position C, let the length AC be l units, then the voltage drop across AC is given by i r l.

Consider that switch K is closed which puts the battery B_1 in the circuit. The battery B_1 whose e.m.f. is to be measured is connected such that the voltage drop along the slide wire and e.m.f. of B_1 oppose each other. The deflection in the galvanometer G depends on the magnitudes of voltage drop across the slide wire portion AC and e.m.f. of B_1. If the voltage drop across length l of the slide wire is greater than e.m.f. of battery B_1, then the current will flow in the direction A to C through the galvanometer. Similarly if the e.m.f. of battery B_1 is greater than the voltage drop across the length l of the slide wire, then the current will flow in the direction C to A through the galvanometer. The most important condition exhibiting the basic principle of the potentiometer is that no current flows through the galvanometer when the two e.m.f.s are equal.

Generally a scale is provided along with the slide wire which enables to measure the length of portion AC.

Hence to measure e.m.f. of a battery, first adjust a current through slide wire with switch K open. Then insert battery whose e.m.f. is to be measured. By closing switch K, adjust sliding contact such that the galvanometer shows zero deflection. Measure the length of the portion of the slide wire with the help of scale provided. Then the unknown e.m.f. E of battery is given by,

$$E = i\,(r\,l) \qquad \text{... (5.1)}$$

where r is the resistance per unit length, i is the working current adjusted using rheostat R.

If e.m.f.s of the two batteries B_1 and B_2 are to be compared then insert the first battery B_1 in series with the galvanometer and then adjust the sliding contact such that no current flows through the galvanometer. Measure the length of the slide wire portion say l_1. Repeat the same procedure with battery B_2 in the circuit. Let the length measured be l_2. Let e.m.f. of batteries B_1 and B_2 be E_1 and E_2 respectively, then we can write,

$$E_1 = i\,(r\,l_1) \qquad \text{... (5.2)}$$

$$E_2 = i\,(r\,l_2) \qquad \text{... (5.3)}$$

Thus, we can write,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \qquad \text{... (5.4)}$$

From above equation it is clear that the ratio of two lengths gives the ratio of the two e.m.f.s.

If one of the batteries used is a standard cell, say battery B_2 of known voltage, the e.m.f. of battery B_1 is given by,

$$E_1 = E_2 \left(\frac{l_1}{l_2}\right) \qquad \text{... (5.5)}$$

While using basic slide wire potentiometer following precautions must be taken.

1. The supply battery B_2 should be of high capacity so that a constant current flows through the slide wire throughout the measurement.
2. A small resistance should be used in series with the galvanometer to protect it during the initial adjustments of contact C. It also takes care that no appreciable current is taken from the standard cell.
3. The accuracy of the measurement depends on how accurately ratio (l_1 / l_2) is determined. Hence in such elementary potentiometers, the length of the slide wire used should be large enough so that percentage error in measurement reduces. Now a days in the potentiometers used for precise measurement, by connecting a number of resistance coils in series with short slide wire the effect of larger length can be obtained.

5.3.1 Standardisation of Potentiometer

Standardisation of a potentiometer is a process of adjusting the working current supplied by the supply battery such that the voltage drop across a portion of sliding wire matches with the standard reference source.

The practical set up for standardising a d.c. potentiometer is as shown in the Fig. 5.4.

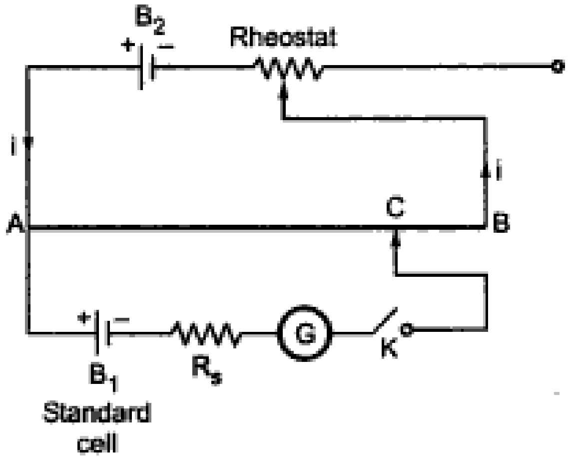


Fig. 5.4 Practical set up for d.c. potentiometer standardisation

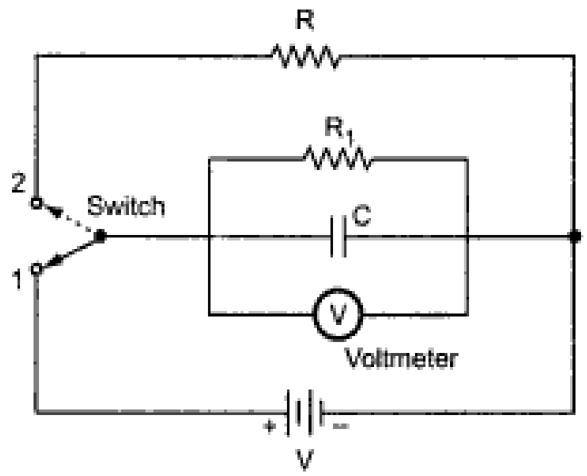


Fig. 6.28 Measurement of high resistance using loss of charge method include leakage resistance of C

The circuit consists high insulation resistance R to be measured alongwith capacitor of known value C shunted with electrostatic voltmeter and leakage resistance R_1.

Initially, capacitor C is charged to suitable voltage say V_1 by moving switch to position 1. Then switch is moved to position 2. The capacitor starts discharging through parallel combination of R and R_1. At certain instant t voltage across capacitor C i.e. V_2 is measured using electrostatic voltmeter. Thus in time t, voltage across capacitor drops down from V_1 to V_2. Let the equivalent resistance through which C discharges be denoted by R′ where $R \| R_2$.

The expression for current any instant t is given by,

$$i = -\frac{dq}{dt} = -C\frac{dV}{dt} \quad \text{... (2)}$$

But

$$i = \frac{\text{Potential drop across } R'}{R'} = \frac{V}{R'} \quad \text{... (3)}$$

Comparing equations (2) and (3), we can write,

$$\frac{V}{R'} = -C\frac{dV}{dt} \quad \text{or} \quad \frac{dV}{V} = -\frac{dt}{RC}$$

Integrating both sides,

$$\left[ln\ V\right]_{V_1}^{V_2} = \left[\frac{-t}{R'C}\right]_0^t$$

i.e.

$$ln\frac{V_2}{V_1} = -\frac{t}{R'C}$$

∴

$$V_2 = V_1\, e^{-\frac{t}{R'C}} \quad \text{... (4)}$$

Thus if time t is known, the resistance R′ can be obtaining by measuring voltages V_1 and V_2. The same test is repeated with unknown resistance R removed. Then C discharges through only R_1. Then expression is given by,

$$\boxed{V_2 = V_1 \, e^{-\frac{t}{R_1 C}}} \quad \text{... (5)}$$

Thus the value of the leakage resistance of the capacitor can also be found out using this method. Note that in this method, leakage resistance of the voltmeter can be neglected if its value of low. If it is high, then it must be considered alongwith R_1.

6.16.3 Megohm Bridge

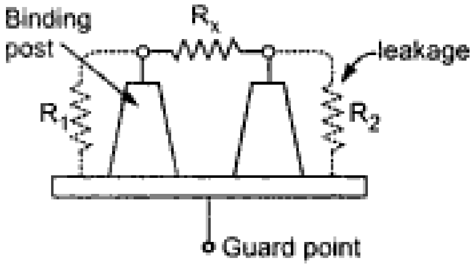


Fig. 6.29 Three terminal resistance

To avoid the leakage current external to the bridge, the junction of ratio arms R_A and R_B is brought out as a separate guard terminal on the front panel of the instrument. This can be used to connect **three terminal resistance**. The three terminal resistance is shown in the Fig. 6.29.

The high resistance is connected between two binding posts which are fixed to metal plate. The two main terminals of the resistor are connected to the R_x terminals in the bridge. The third terminal is the common point of resistances R_1 and R_2, which represents the leakage paths, from the main terminal along the insulating post of the metal plate. The guard is connected to the guard terminal on the front panel of the bridge as shown in the Fig. 6.30.

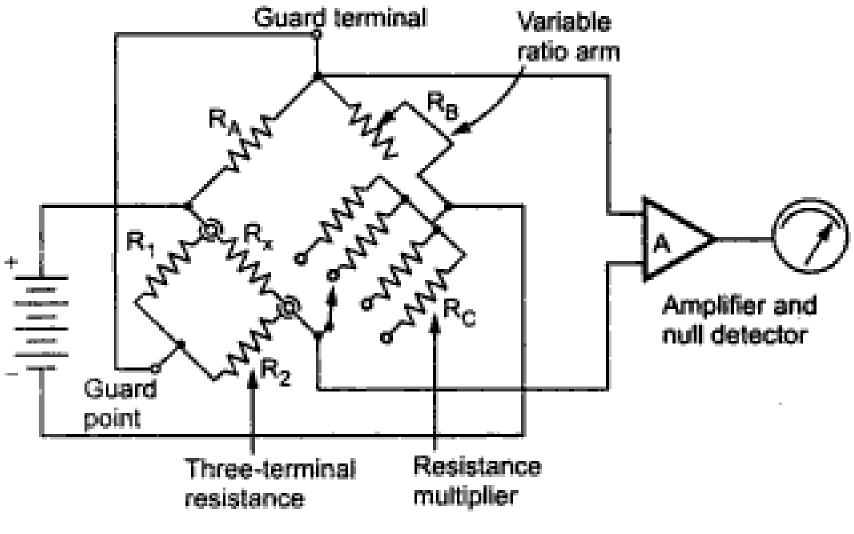


Fig. 6.30 Guarded bridge with three terminal resistor

This connection puts R_1 in parallel with ratio arm resistance R_A but since R_1 is very much larger than R_A, its shunting effect is negligible. Similarly R_2 in parallel with galvanometer has no effect as R_2 is much higher than galvanometer. It slightly reduces the sensitivity of the galvanometer. Thus the effect of external leakage path can be removed by using the guard circuit on the three terminal resistance.

If guard circuit is absent, leakage resistances R_1 and R_2 would be placed directly across R_x and giving large error in the measurement.

6.16.4 Megger

Resistances of the order of 0.1 MΩ and upwards are classified as high resistances.These high resistances are measured by portable, instrument known as **megger**. It is also used for testing the insulation resistance of cables.

6.16.4.1 Principle of Operation

It is based on the principle of electromagnetic induction. The Fig 6.31 shows the construction of megger.

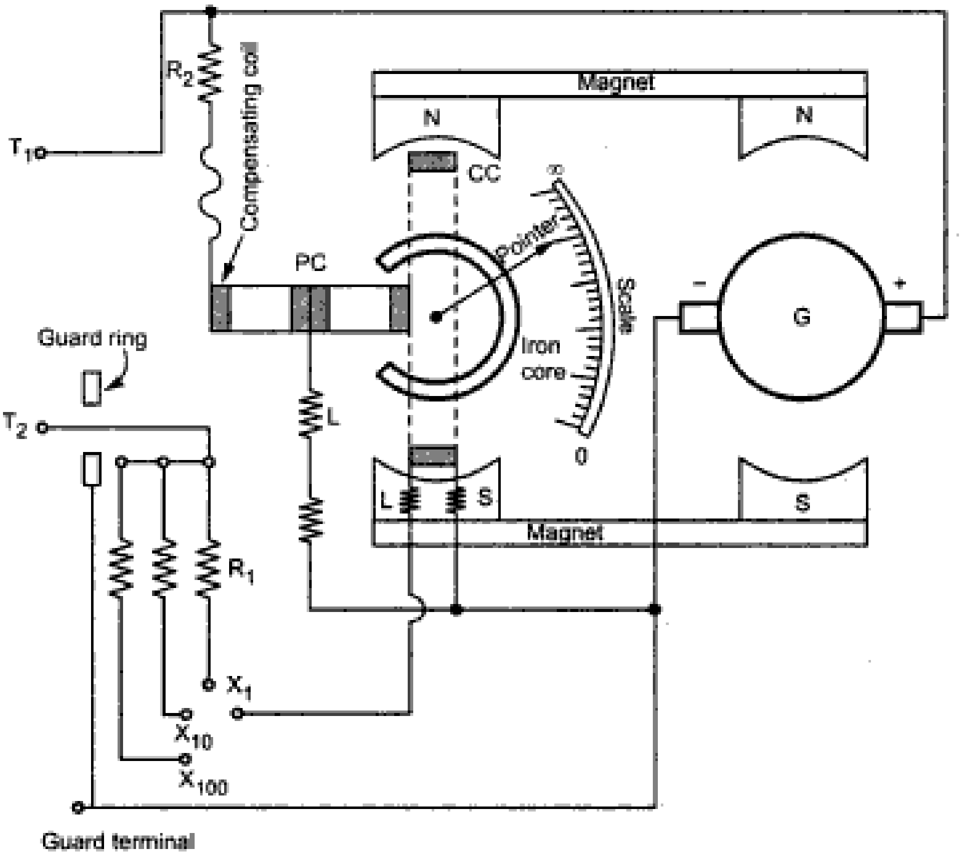


Fig. 6.31 Megger

When a current carrying conductor is placed in a uniform magnetic field it experiences a mechanical force whose magnitude depends upon the strength of current and magnetic field. While its direction depends on the direction of current and magnetic field.

6.16.4.2 Construction

It consists of a permanent magnet which provides the field for both the generator G and ohmmeter. The moving element of the ohmmeter consist of three coil viz. current or deflection coil, pressure or control coil and compensating coil. These coils are mounted on a central shaft which are free to rotate over a stationary C-shaped iron core.

The coils are connected to the circuit through flexible leads called ligaments which do not produce a restoring torque on the moving element, consequently the moving element takes up any position over the scale when the generator handle is stationary.

The current coil is connected in series with resistance R_1 between one generator terminal and the test terminal T_2. The series resistance R_1 protects the current coil in the event of the test terminals getting short circuited and also controls the range of the instrument. The pressure coil, in series with a compensating coil and protection resistance R_2 is connected across the generator terminals. The compensating coil is included in the circuit to ensure better scale proportions. The scale is calibrated reversely means the normal position of pointer indicates infinity while full scale deflection indicates zero resistance.

6.16.4.3 Working

When the current flows from the generator, through the pressure coil, the coil tends to set itself at right angles to the field of the permanent magnet.

When the test terminals are open, corresponding to infinite resistance, no current flows through deflection coil. Thus the pressure coil governs the motion of the moving element making it move to its extreme anticlockwise position. The pointer comes to rest at the infinity end of the scale.

When the test terminals are short circuited i.e. corresponding to zero resistance, the current from the generator flowing through the current coil is large enough to produce sufficient torque to overcome the counter-clockwise torque of the pressure coil. Due to this, pointer moves over a scale showing zero resistance.

When the high resistance to be tested is connected between terminals T_1 and T_2 the opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale. The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.

7

A.C. Bridges

7.1 Introduction

The bridges are used for not only the measurement of resistances but also used for the measurement of various component values like capacitance, inductance etc.

A **bridge circuit** in its simplest form consists of a network of four resistance arms forming a closed circuit. A source of current is applied to two opposite junctions. The current detector is connected to other two junctions.

The bridge circuits use the **comparison measurement** methods and operate on **null-indication principle**. The bridge circuit compares the value of an unknown component with that of an accurately known standard component. Thus the accuracy depends on the bridge components and not on the null indicator. Hence high degree of accuracy can be obtained.

In a bridge circuit, when no current flows through the null detector which is generally galvanometer, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called **balancing condition** or **balancing equation**. This equation gives us the value of the unknown component.

7.1.1 Advantages of Bridge Circuit

The various **advantages** of the bridge circuit are,

1) The balance equation is independent of the magnitude of the input voltage or its source impedance. These quantities do not appear in the balance equation expression.

2) The measurement accuracy is high as the measurement is done by comparing the unknown value with the standard value.

3) The accuracy is independent of the characteristics of a null detector and is dependent on the component values.

4) The balance equation is independent of the sensitivity of the null detector, the impedance of the detector or any impedance shunting the detector.

5) The balance condition remains unchanged if the source and detector are interchanged.

6) The bridge circuit can be used in the control circuits. When used in such control applications, one arm of the bridge contains a resistive element that is sensitive to the physical parameter like pressure, temperature etc. which is to be controlled.

7.2 Types of Bridges

The two types of bridges are,

1) D.C. bridges and 2) A.C. bridges

The **d.c. bridges** are used to measure the **resistances** while the **a.c. bridges** are used to measure the **impedances** consisting capacitances and inductances. The d.c. bridges use the d.c. voltage as the excitation voltage while the a.c. bridges use the alternating voltage as the excitation voltage.

The two types of d.c. bridges are,

1. Wheatstone bridge 2. Kelvin bridge

The various types of a.c. bridges are,

1. Capacitance comparison bridge
2. Inductance comparison bridge
3. Maxwell's bridge
4. Hay's bridge
5. Anderson bridge
6. Schering bridge
7. Wien bridge

Let us now discuss the various types of the bridges in detail.

7.3 A.C. Bridges

An a.c. bridge in its basic form consists of four arms, a source of excitation and a balance detector. Each arm consists of an impedance. The source is an a.c. supply which supplies a.c. voltage at the required frequency. For high frequencies, the electronic oscillators are used as the source. The balance detectors commonly used for a.c. bridges are head phones, tunable amplifier circuits or vibration galvanometers. The headphones are used as detectors at the frequencies of 250 Hz to 3 to 4 kHz. While working with single frequency a tuned detector is the most sensitive detector. The vibration galvanometers are useful for low audio frequency range from 5 Hz to 1000 Hz but are commonly used below 200 Hz. Tunable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.

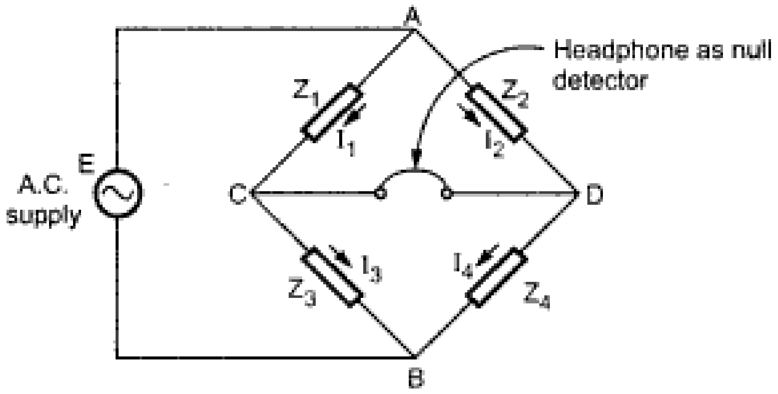


Fig. 7.1 A.C. Wheatstone bridge

The simple a.c. bridge is the outcome of the Wheatstone bridge. The impedances at audio and radio frequency range can be easily determined by such simple a.c. Wheatstone bridge. It is shown in the Fig. 7.1.

This is similar to d.c. Wheatstone bridge. The bridge arms are impedances. The bridge is excited by a.c. supply and pair of headphones is used as a null detector. The null response is obtained by varying one of the bridge arms.

7.3.1 Sources and Detectors

For bridge measurements at very low frequencies, the power line itself may act as a source of supply to the bridge circuit. For bridge measurements at higher frequencies electronic oscillators are used as a source of supply to the bridge circuit. These electronic oscillators are used as a source of supply universely because,

i) The output waveform is very close to sine wave.

ii) The output frequency is very stable.

iii) The output frequency is easily determinable with accuracy and also it is easily adjustable.

iv) The output power is sufficient to drive the bridge circuits.

A typical oscillator has a frequency range of 40 Hz to 125 kHz with power output of 7 W.

For the a.c. bridges commonly used detectors are as follows.

i) **Headphones :** These are the most commonly used as detectors in a.c. bridges. The frequency range over which headphones can be used as detector effectively is 250 Hz upto 3 to 4 kHz.

ii) **Vibration galvanometers :** For low audio frequency ranges and power ranges, these detectors are extremely effective. Even though these type of detectors are manufactured to work at various frequency ranges starting from 5 Hz to 1000 Hz. These detectors can be effectively used below 200 Hz with greater sensitivity than the headphones.

iii) **Tuneable amplifier detectors :** The transistor amplifier can be tuned electrically to any desired frequency and then it can be made to respond to a narrow bandwidth at a bridge frequency. The output of such amplifier is connected to the indicating instruments. The frequency range for these detectors is 10 Hz to 100 kHz.

7.3.2 Bridge Balance Equation

For bridge balance, the potential of point C must be same as the potential of point D. These potentials must be equal interms of amplitude as well as phase.

Thus the drop from A to C must be equal to drop across A to D, in both magnitude and phase for the bridge balance.

$$\therefore \quad \bar{E}_{AC} = \bar{E}_{AD} \quad \dots (1)$$

The vector notation indicates, both amplitude and phase to be considered.

$$\therefore \quad \overline{I_1 Z_1} = \overline{I_2 Z_2} \quad \dots (2)$$

When the bridge is balanced, no current flows through the headphones.

$$\therefore \quad \bar{I}_3 = \bar{I}_1 \quad \text{and} \quad \bar{I}_4 = \bar{I}_2$$

$$\text{Now} \quad \bar{I}_1 = \frac{\bar{E}}{\bar{Z}_1 + \bar{Z}_3} \quad \dots (3)$$

$$\text{and} \quad \bar{I}_2 = \frac{\bar{E}}{\bar{Z}_2 + \bar{Z}_4} \quad \dots (4)$$

Substituting (3) and (4) into (2) we get,

$$\frac{\bar{E} \cdot \bar{Z}_1}{\bar{Z}_1 + \bar{Z}_3} = \frac{\bar{E} \cdot \bar{Z}_2}{\bar{Z}_2 + \bar{Z}_4}$$

$$\therefore \quad \overline{Z_1 Z_2} + \overline{Z_1 Z_4} = \overline{Z_1 Z_2} + \overline{Z_2 Z_3}$$

$$\therefore \quad \overline{Z_1 Z_4} = \overline{Z_2 Z_3} \quad \dots (5)$$

The equation (5) is the **balancing equation** in the impedance form.

In the **admittance** form the condition can be expressed as,

$$\overline{Y_1 Y_4} = \overline{Y_2 Y_3} \quad \dots (6)$$

The admittance is the reciprocal of the impedance.

Now in the polar form the impedances are expressed as,

$$\overline{Z_1} = Z_1 \angle \theta_1$$

$$\overline{Z_2} = Z_2 \angle \theta_2$$

SENSORS AND TRANSDUCERS

Sensor

It is defined as an element which produces signal relating to the quantity being measured [1]. According to the Instrument Society of America, sensor can be defined as "A device which provides a usable output in response to a specified measurand." Here, the output is usually an 'electrical quantity' and measurand is a 'physical quantity, property or condition which is to be measured'. Thus in the case of, say, a variable inductance displacement element, the quantity being measured is displacement and the sensor transforms an input of displacement into a change in inductance.

Transducer

It is defined as an element when subjected to some physical change experiences a related change [1] or an element which converts a specified measurand into a usable output by using a transduction principle.

It can also be defined as a device that converts a signal from one form of energy to another form.

A wire of Constantan alloy (copper-nickel 55-45% alloy) can be called as a sensor because variation in mechanical displacement (tension or compression) can be sensed as change in electric resistance. This wire becomes a transducer with appropriate electrodes and input-output mechanism attached to it. Thus we can say that 'sensors are transducers'.

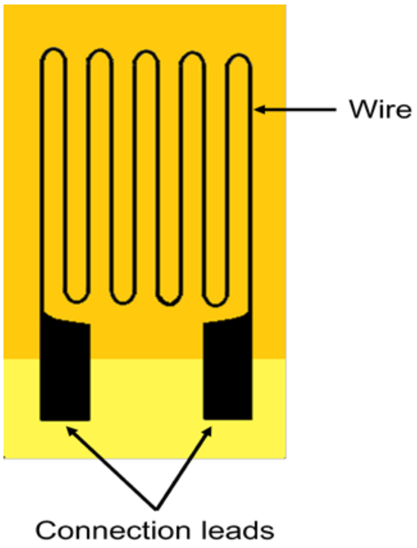
STRAIN GAUGES

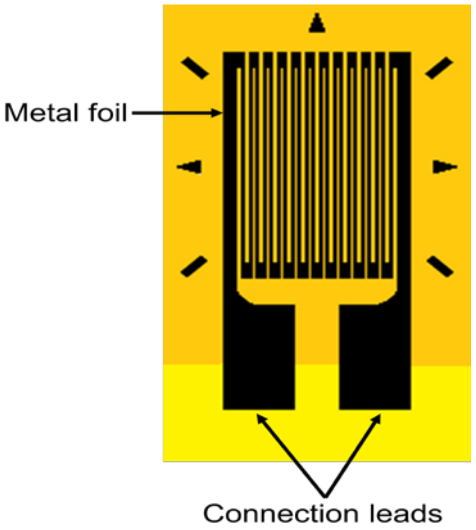
The strain in an element is a ratio of change in length in the direction of applied load to the original length of an element. The strain changes the resistance *R* of the element. Therefore, we can say

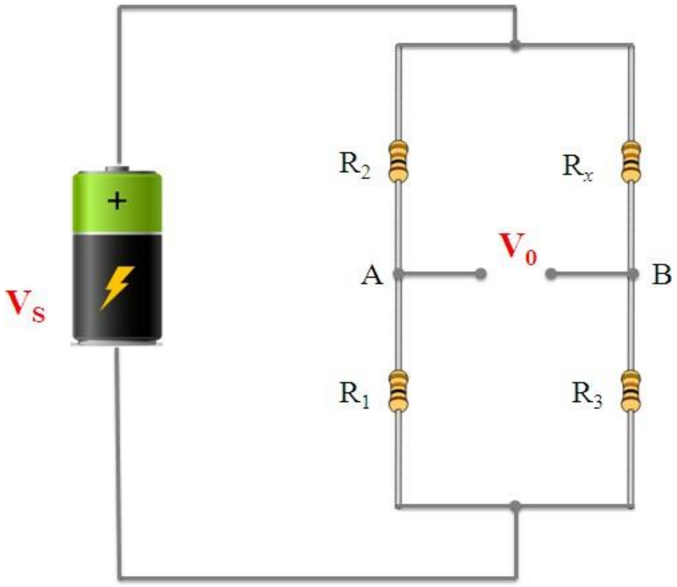
$\Delta R/R \propto \varepsilon$;

$$\Delta R/R = G\,\varepsilon \qquad (2.2.5)$$

where G is the constant of proportionality and is called as gauge factor. In general, the value of G is considered in between 2 to 4 and the resistances are taken of the order of 100 Ω.







Resistance strain gauge follows the principle of change in resistance as per the equation 2.2.5. It comprises of a pattern of resistive foil arranged as shown in Figure 2.2.3. These foils are made of Constantan alloy (copper-nickel 55-45% alloy) and are bonded to a backing material plastic (ployimide), epoxy or glass fiber reinforced epoxy. The strain gauges are secured to the workpiece by using epoxy or Cyanoacrylate cement Eastman 910 SL. As the workpiece undergoes change in its shape due to external loading, the resistance of strain gauge element changes. This change in resistance can be detected by a using a Wheatstone's resistance bridge as shown in Figure 2.2.4. In the balanced bridge we can have a relation,

$$R_2 / R_1 = R_x / R_3 \qquad (2.2.6)$$

where R_x is resistance of strain gauge element, R_2 is balancing/adjustable resistor, R_1 and R_3 are known constant value resistors. The measured deformation or displacement by the stain gauge is calibrated against change in resistance of adjustable resistor R_2 which makes the voltage across nodes A and B equal to zero.

APPLICATIONS OF STRAIN GAUGES

Strain gauges are widely used in experimental stress analysis and diagnosis on machines and failure analysis. They are basically used for multi-axial stress fatigue testing, proof testing, residual stress and vibration measurement, torque measurement, bending and deflection measurement, compression and tension measurement and strain measurement.

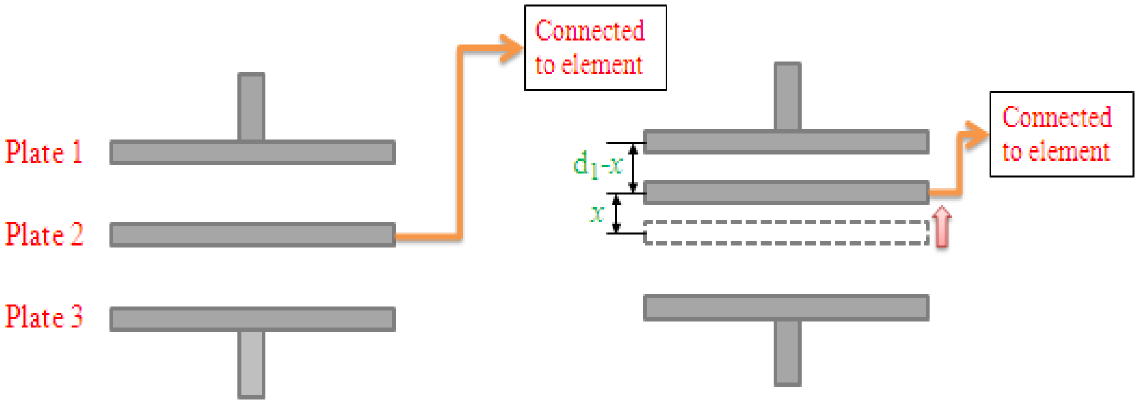
Strain gauges are primarily used as sensors for machine tools and safety in automotives. In particular, they are employed for force measurement in machine tools, hydraulic or pneumatic press and as impact sensors in aerospace vehicles.

CAPACITIVE ELEMENT BASED SENSOR

Capacitive sensor is of non-contact type sensor and is primarily used to measure the linear displacements from few millimeters to hundreds of millimeters. It comprises of three plates, with the upper pair forming one capacitor and the lower pair another. The linear displacement might take in two forms:

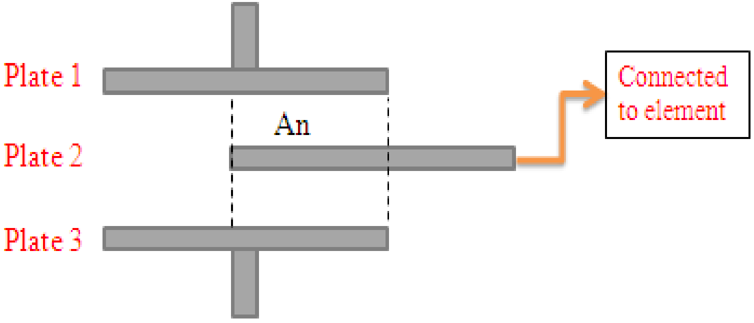
a. one of the plates is moved by the displacement so that the plate separation changes.

b. area of overlap changes due to the displacement.



(a) Three-plates capacitive element sensor

(b) Displacement measurement thru change in plate separation



(c) Displacement measurement thru change in area of overlap

The capacitance C of a parallel plate capacitor is given by,

$$C = \varepsilon_r \varepsilon_o A / d \tag{2.2.7}$$

where ε_r is the relative permittivity of the dielectric between the plates, ε_o permittivity of free space, *A* area of overlap between two plates and *d* the plate separation.

As the central plate moves near to top plate or bottom one due to the movement of the element/workpiece of which displacement is to be measured, separation in between the plate changes. This can be given as,

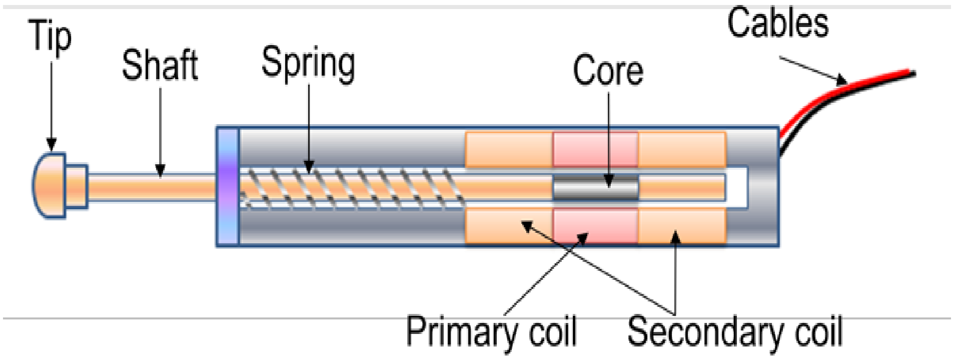
$$C_1 = (\varepsilon_r \varepsilon_o A) / (d + x) \tag{2.2.8}$$

$$C_2 = (\varepsilon_r \varepsilon_o A) / (d - x) \tag{2.2.9}$$

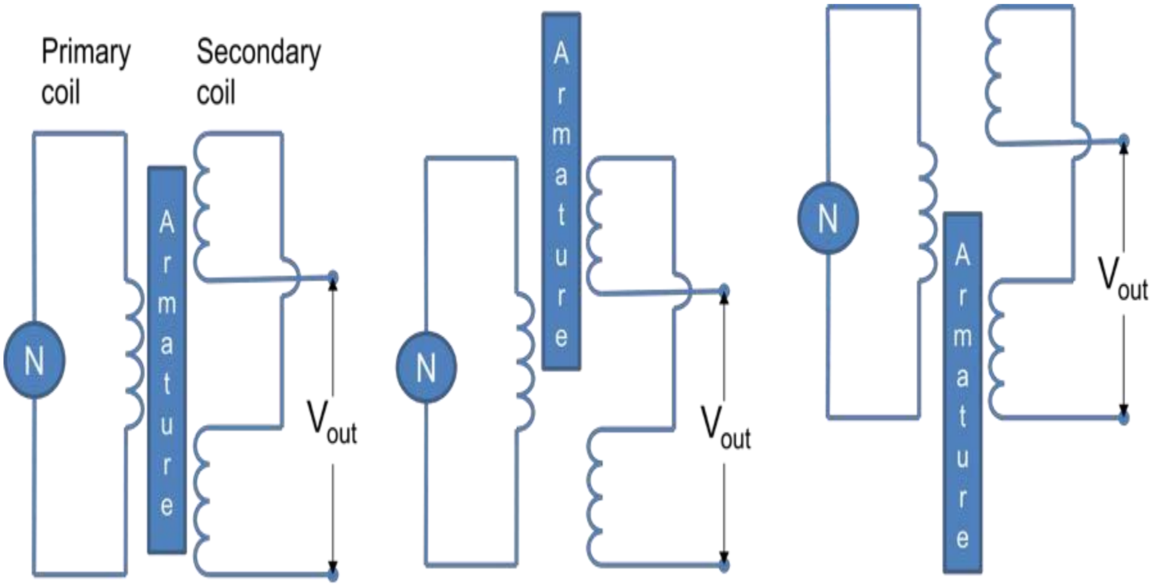
When C1 and C2 are connected to a Wheatsone's bridge, then the resulting out-of-balance voltage would be in proportional to displacement x.

Capacitive elements can also be used as proximity sensor. The approach of the object towards the sensor plate is used for induction of change in plate separation. This changes the capacitance which is used to detect the object.

LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)



Linear variable differential transformer (LVDT) is a primary transducer used for measurement of linear displacement with an input range of about ± 2 to ± 400 mm in general. It has non-linearity error ± 0.25% of full range. Figure 2.2.6 shows the construction of a LVDT sensor. It has three coils symmetrically spaced along an insulated tube. The central coil is primary coil and the other two are secondary coils. Secondary coils are connected in series in such a way that their outputs oppose each other. A magnetic core attached to the element of which displacement is to be monitored is placed inside the insulated tube.



Due to an alternating voltage input to the primary coil, alternating electro- magnetic forces (emfs) are generated in secondary coils. When the magnetic core is centrally placed with its half portion in each of the secondary coil regions then the resultant voltage is zero. If the core is displaced from the central position as shown in Figure 2.2.7, say, more in secondary coil 1 than in coil 2, then more emf is generated in one coil i.e. coil 1 than the other, and there is a resultant voltage from the coils. If the magnetic core is further displaced, then the value of resultant voltage increases in proportion with the displacement. With the help of signal processing devices such as low pass filters and demodulators, precise displacement can be measured by using LVDT sensors.

LVDT exhibits good repeatability and reproducibility. It is generally used as an absolute position sensor. Since there is no contact or sliding between the constituent elements of the sensor, it is highly reliable. These sensors are completely sealed and are widely used in Servomechanisms, automated measurement in machine tools.

HALL EFFECT SENSOR

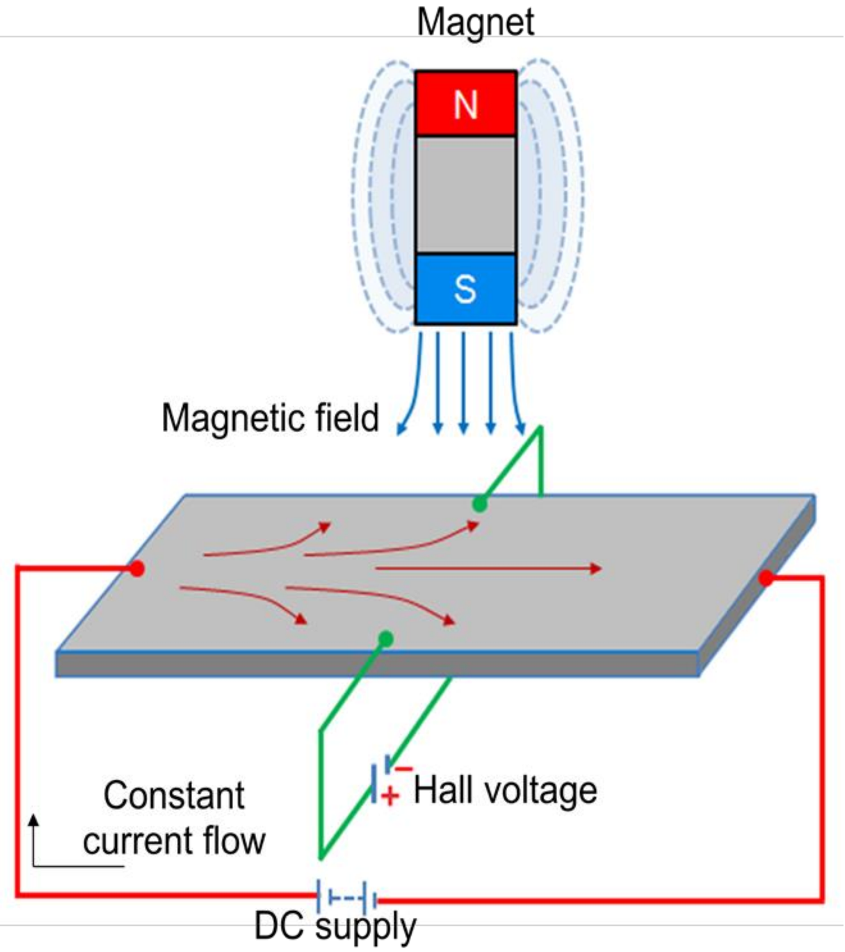


Figure 2.3.8 shows the principle of working of Hall effect sensor. Hall effect sensors work on the principle that when a beam of charge particles passes through a magnetic field, forces act on the particles and the current beam is deflected from its straight line path. Thus one side of the disc will become negatively charged and the other side will be of positive charge. This charge separation generates a potential difference which is the measure of distance of magnetic field from the disc carrying current.

The typical application of Hall effect sensor is the measurement of fluid level in a container. The container comprises of a float with a permanent magnet attached at its top. An electric circuit with a current carrying disc is mounted in the casing. When the fluid level increases, the magnet will come close to the disc and a potential difference generates. This voltage triggers a switch to stop the fluid to come inside the container.

These sensors are used for the measurement of displacement and the detection of position of an object. Hall effect sensors need necessary signal conditioning circuitry. They can be operated at 100 kHz. Their non-contact nature of operation, good immunity to environment contaminants and ability to sustain in severe conditions make them quite popular in industrial automation.

TRANSDUCERS

Transducer is a device which transforms energy from one form to another. It also converts any non-electrical quantity to electrical quantity.

i.e. (1) Mechanical energy to Electrical signal
(2) Heat to Electrical signal.
(3) Liquid level to Electrical Signal.
(4) Flow rate to Electrical Signal.

There are two types of transducers:
(1) Electrical Transducers
(2) Mechanical Transducers

Electrical Transducers

Non-electrical quantities such as temperature, pressure, displacement, speed, fluid flow cannot be measured directly. The electrical Quantities such as voltage, current, resistance, inductance capacitance can be measured directly. For measurement of nonelectrical

quantities these quantities are to be converted into the electrical quantities and then measured. This function of converting non-electrical quantities to electrical quantities is accomplished by a device called as the Electrical Transducers.

Advantages of Electrical Transducers

- The electrical output from the transducer can be amplified into any desired level of voltage and current.
- Very small power is required for the controlling of the electrical and electronic systems.
- Frictional effect is reduced to minimum.
- The output can be measured at a distance from the sensing medium through wired of wireless medium.

Disadvantages of Electrical Transducers

- The electrical transducers are comparatively costly than the other types of transducers.
- They have low reliability in comparison to that of the mechanical transducers.

1.2 CLASSIFICATION OF TRANSDUCERS

Classification of Transducers on the basis of method of application

On the basis of method of application, the transducers can be classified as:
(1) Primary Transducers
(2) Secondary Transducers

Primary Transducers

When the input is sensed by transducer and the physical phenomenon is converted into electrical signal then such transducers is called as primary transducers.

e.g.: Thermistors: It senses the temperature directly and causes change in resistance with the change in temperature.

Secondary Transducers

When the input is first sensed by a sensor, output being of some form other than input signal is given to the transducer for conversion into electrical form. Such transducer is called as secondary transducers. It consists of two parts: sensing element and transduction element.

E.g.: LVDT (Linear Variable Differential Transducers): Displacement is converted to the electrical signal by LVDT.

Sensing element: It corresponds to the change in physical phenomenon.

Transduction element: It changes the output of the sensing element into electrical signal.

Classification of Transducers on the basis of energy conversion:

On the basis of energy conversion, the transducers can be classified as:

(1) Active Transducers

(2) Passive Transducers

Active Transducers

An active transducer does not require any external power source for its operation.

They develop their own voltage or current. An active transducer can generate an electrical signal directly in response to the physical quantity to be measured.

E.g.: Thermocouple, Tacho-generator, Photovoltaic cell.

Passive Transducers

These transducers require an external power source for its conversion. In these tranducers electrical parameters such as resistance, Inductance, Capacitance changes and it causes a change in voltage and current.

E.g.: Resistive transducer, Inductive transducer and capacitive transducer.

Classification of Transducers on the basis of nature of output signal

On the basis of energy conversion, the transducers can be classified as:

(1) Analog Transducers

(2) Digital Transducers

Analog Transducers

It converts input signal to output signal which is continuous function of time.

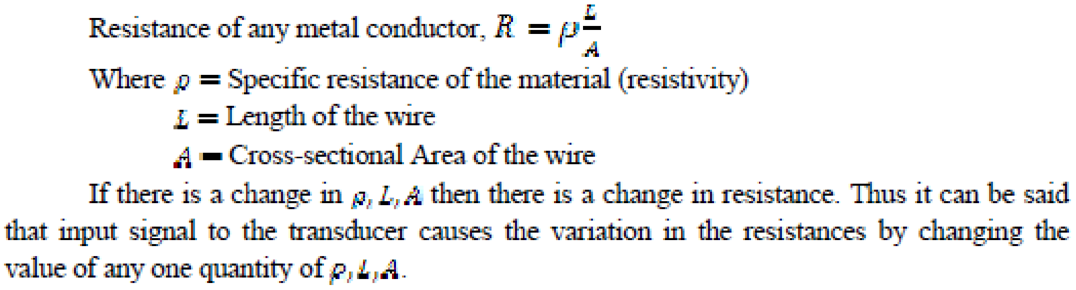
E.g.: Strain gauge, thermocouple, strain gauge.

Digital Transducers

It converts input signal into output signal in the form of pulses which gives discrete output. These are more popular in use in the form of digital measuring instruments.

1.3 RESISTIVE TRANSDUCERS

In resistive transducers, resistance changes due to change in some of the physical phenomenon.



Resistance of any metal conductor, $R = \rho \frac{L}{A}$

Where ρ = Specific resistance of the material (resistivity)

L = Length of the wire

A = Cross-sectional Area of the wire

If there is a change in ρ, L, A then there is a change in resistance. Thus it can be said that input signal to the transducer causes the variation in the resistances by changing the value of any one quantity of ρ, L, A.

E.g.: Potentiometer: Length varies so there is a change in resistance.

Strain Gauges: When the gauge is strained then there is a change in cross sectional area and hence there is a change in resistance.

For temperature measurement there is a change in resistivity ‘ρ’.

1.3.1. POTENTIOMETER

Potentiometer is a very simple device where there si a change in output voltage due to change in displacement. It is a passive transducer which requires an external power source for its operation. It is a very cheap form of transducer and is very widely used.

It converts linear displacement to the voltage by change in resistance and also rotational displacement to voltage by change ion resistance.

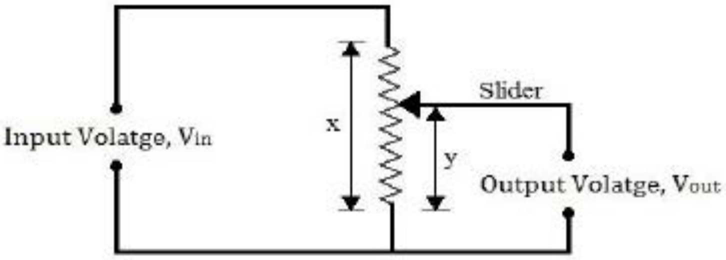
It is of the following types:

- Linear potentiometer.

- Rotational Potentiometer
- Helipot

Linear Potentiometer

Linear potentiometers are used for the measurement of displacement. It is simple and cheap. Its output voltage is a function of linear displacement.



x = Total length of resistive material.
y = Relative position of the slider on the resistive material.

Then,

$$Vout = \frac{y}{x} Vin$$

Fig. 1.1: Potetiometric arrangemet

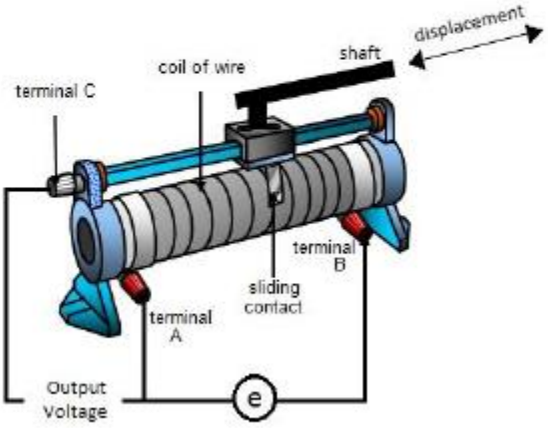


Fig. 1.2: Potentiometer connection using a rheostat

Construction:

It consists of a resistance element provided with a sliding contact called as wiper. The resistance element is a wire made up of platinum or nickel alloy. This resistance element is wound uniformly throughout the length of the sliding contact. Displacement of the slider/wiper determines the change in the voltage drop related to the position of slider. It is mostly used in the laboratory demonstrations and is not suitable for industrial use.

Rotary Potentiometer:

This kind of potentiometer operates in the same way as that of the linear potentiometer. In rotary potentiometer the resistance element is connected a circular manner in case of rotary potentiometer. This kind of potentiometer has a rotary movement and is useful in the measurement of angular displacement. The angular measurement can be measured between 10 and 3570.

When the motion of the wiper is in both translational and rotational way then the potentiometer is called as helipot.

Advantage of potentiometer:

- It can measure large displacement.
- It has high electrical efficiency.
- It is simple in construction.
- It is cheap and easy to operate.

Disadvantages of potentiometer:

- Sliding contact may wear out and get contaminated or generate noise.
- They require large force to move their sliding contact.

1.3.2. STRAIN GAUGES

If a metal conductor is stretched or compressed, its resistances changes due to the change in both length and diameter of the conductor.

The change in the value of the resistance by straining of the gauge may be partially explained. As the gauge is under a positive strain, the length of it increases and its area of cross section decreases. Since the resistance of the conductor is proportional to the length and inversely proportional to its area of cross section, the resistance of the gauge increases with positive strain.

Let us consider a strain gauge made up of circular wire

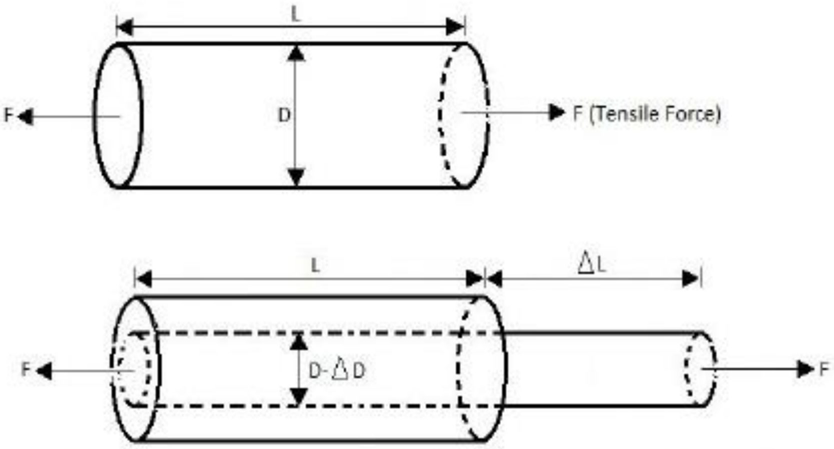


Fig. 1.3: Change in dimensions of a strain gauge due to force 'F'

L = Original length of wire
D = Original Diameter of wire
F = Tensile Force

ΔL = Change in length
ΔD = Change in Diameter
ΔR = Change in Resistance
ΔA = Change in cross sectional area

We know,

$$R = \rho \frac{L}{A}$$

Differentiating with respect to stress s,

$$\frac{dR}{ds} = \frac{\rho}{A}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2}\frac{\partial A}{\partial s} + \frac{L}{A}\frac{\partial \rho}{\partial s}$$

Dividing by $R = \rho \frac{L}{A}$

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$

Per unit change in length = $\frac{\Delta L}{L}$

Per unit change in area = $\frac{\Delta A}{A}$

Per unit change in Resistivity = $\frac{\Delta \rho}{\rho}$

$$A = \frac{\pi}{4} D^2$$

$$\frac{\partial A}{\partial s} = 2 \frac{\pi}{4} D \frac{\partial D}{\partial s}$$

$$\frac{1}{A}\frac{\partial A}{\partial s} = \frac{\left(\frac{2\pi}{4}\right)}{\left(\frac{\pi}{4}\right)} \frac{D}{D^2} \frac{\partial D}{\partial s} = \frac{2}{D}\frac{\partial D}{\partial s}$$

We know,

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$

Poisson's ratio, $\mathcal{V} = \frac{Lateral\ Strain}{Longitudinal\ Strain} = -\frac{\frac{\partial D}{d}}{\frac{\partial L}{L}}$

$$\frac{\partial D}{D} = -\nu \times \frac{\partial L}{L}$$

$$\frac{1}{R}\frac{dR}{ds} = \frac{1}{L}\frac{\partial L}{\partial s} - \nu \frac{2}{L}\frac{\partial L}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s}$$

$$\frac{\Delta R}{\mathrm{R}} = \frac{\Delta L}{\mathrm{L}} + 2\nu \frac{\Delta L}{\mathrm{L}} + \frac{\Delta \rho}{\rho}$$

Gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

$$G_f = \frac{\frac{\Delta R}{\mathrm{R}}}{\frac{\Delta L}{\mathrm{L}}}$$

Therefore, $G_f = 1 + 2\nu + \frac{\Delta\rho/\rho}{\Delta L/L}$

$\frac{\Delta L}{L} = \epsilon =$Strain

Thus $G_f = 1 + 2\nu + \frac{\Delta\rho/\rho}{\epsilon}$

Unbonded Metal Strain Gauges

This gauge consists of a wire stretched between two points in a insulating medium

such as air. The wires may be made up of copper-nickel, chrome-nickel or nickel-iron alloys. The wires are tensioned to avoid buckling when they experience a compressive force.
The unbounded metal wire gauges used almost exclusively in transducer applications, employ preload resistance wires connected in a wheat-stone bridge. At initial period, the strains and resistances of the four arms are normally equal, with this result, output voltage of the bridge, . When displacement occurs, it increases tension in two wires and decreases in the other two, thereby increase the resistance of the two wires which are in tension and decreasing the resistance of the two wires. This causes an unbalance of the bridge producing an output voltage which is proportional to the input displacement and hence to the applied pressure.

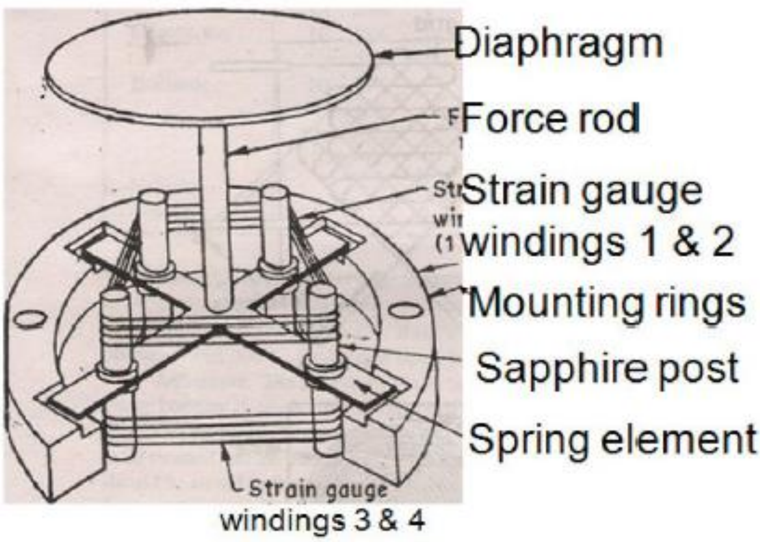


Fig. 1.4: Constructional diagram of Unbonded metal strain gauge

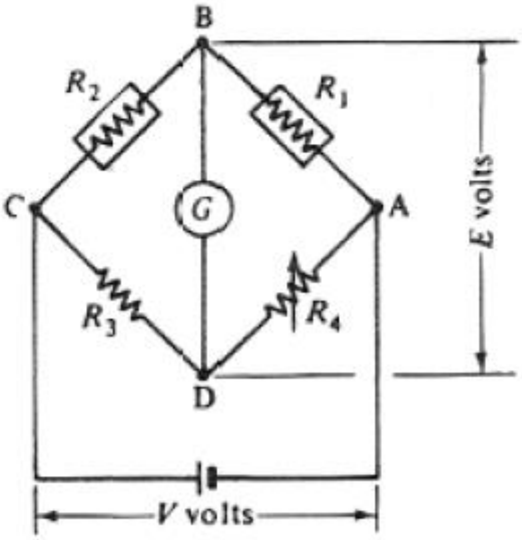


Fig. 1.5: Schematic diagram of unbonded metal strain gauge

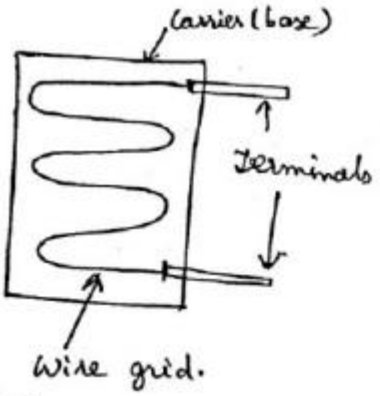
Bonded Wire Strain Gauges
It is used for both stress analysis and for constructing transducers.

A resistance wire strain gauge consists of grid of fine resistances wire of about 0.025mm in diameter or less. The grid is cemented to carrier (base) which may be thin sheet of paper, a thin sheet of Bakelite or sheet of Teflon. The wire is covered on top with a thin sheet of material so as to prevent it from any mechanical damage. The spreading of the wire permits an uniform distribution of stress over the grid. The carrier is bonded with an adhesive material to the specimen under study. The wires cannot buckle as they are embedded in a matrix of cement and hence faithfully follow both the tensile and the compressive strains of the specimen. The materials and the wire sizes used for the banded wire strain gauges are the same as used for the unbonded wire strain gauges.

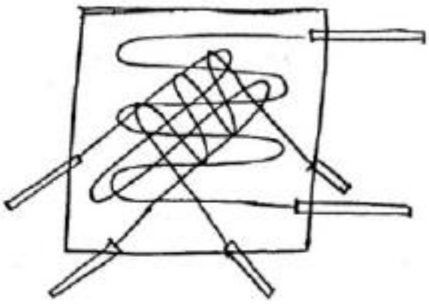
It is desirable that resistance wire strain gauges should have the following characteristics:

(1) Strain gauge should have a high value of gauge factor Gf . A high gauge factor indicates high sensitivity.

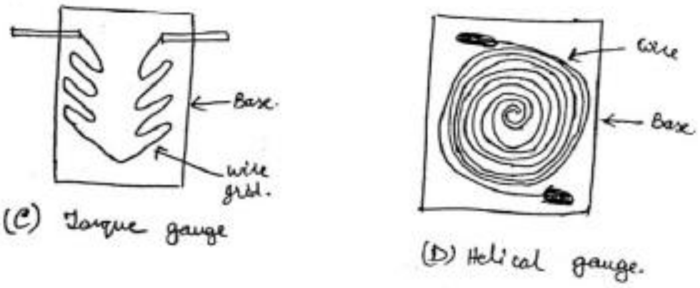
(2) The resistances of the strain gauges should be as high as possible since this minimizes the effects of undesirable variations of resistance in the measurement circuit.



(a) Linear Strain gauge.



(b) Rosette.



(C) Torque gauge

(D) Helical gauge.

Fig. 1.6: Different types of bonded strain gauge

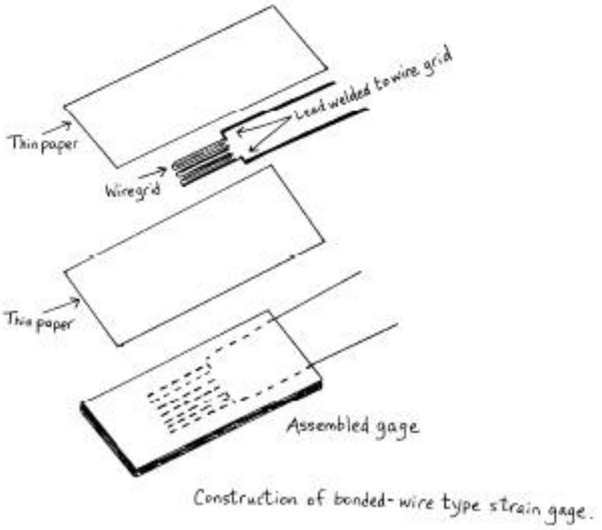


Fig. 1.7: Construction of Bonded strain gauge

PLATINUM RESISTANCE THERMOMETER

The resistance type thermometer bulbs are sensing elements in the form of the wires or foils. The films deposited on insulating surfaces are also used for temperature sensing. In the wire type, the arrangement are commonly a helical coil wound as a double wire to avoid inductive effects. The laboratory type resistance thermometers have the temperature sensing element wound on a cross mica former and enclosed in Pyrex tube shown in fig. 1.

The industrial type of thermometer shown in fig. 2. Here the former is of grooved ceramic and the wire is being protected by a glass coating or by a stainless steel tube. The element is normally sealed in glass when used for temperatures upto 150oC and ceramic for use in temperatures upto 8500C.

Resistive elements are also made up of thin etched grids of metal foils similar in shape of the foil type strain gauges. They are constructed of the platinum and may be bonded to plastic booking for attachment to a surface.

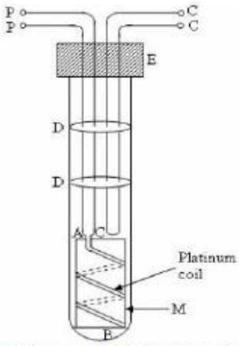


Fig. 1.8: Schematic diagram of Platinum Resistance Thermometer

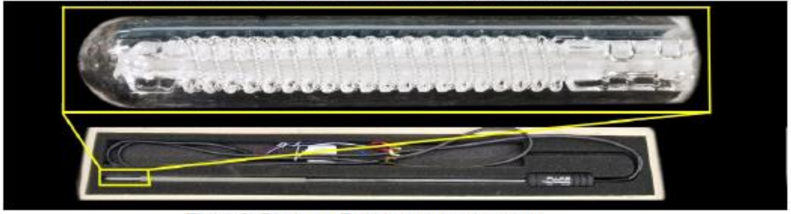


Fig. 1.9: Platinum Resistance thermometer

THERMISTOR

Thermistors are general composed of semi-conductors. Most thermistors have a negative temperature coefficient i.e. their resistance decreases with increase of temperature. This allows thermistors circuits to detect very small changes in temperature which could be observed using a RTD or a thermocouple.

Thermistors are widely used to measure temperature ranging from -600C to 150C.

Resistance of thermistors ranges from 0.5Ω to 0.75MΩ.

Construction of thermistors:

These are composed of sintered mixture of metallic oxides such as manganese, nickel, cobalt, copper, iron and uranium. They are available in variety of shapes and sizes. They may be in the form of beads, rods and disks. Thermistors in the form of beads are the smallest in size.

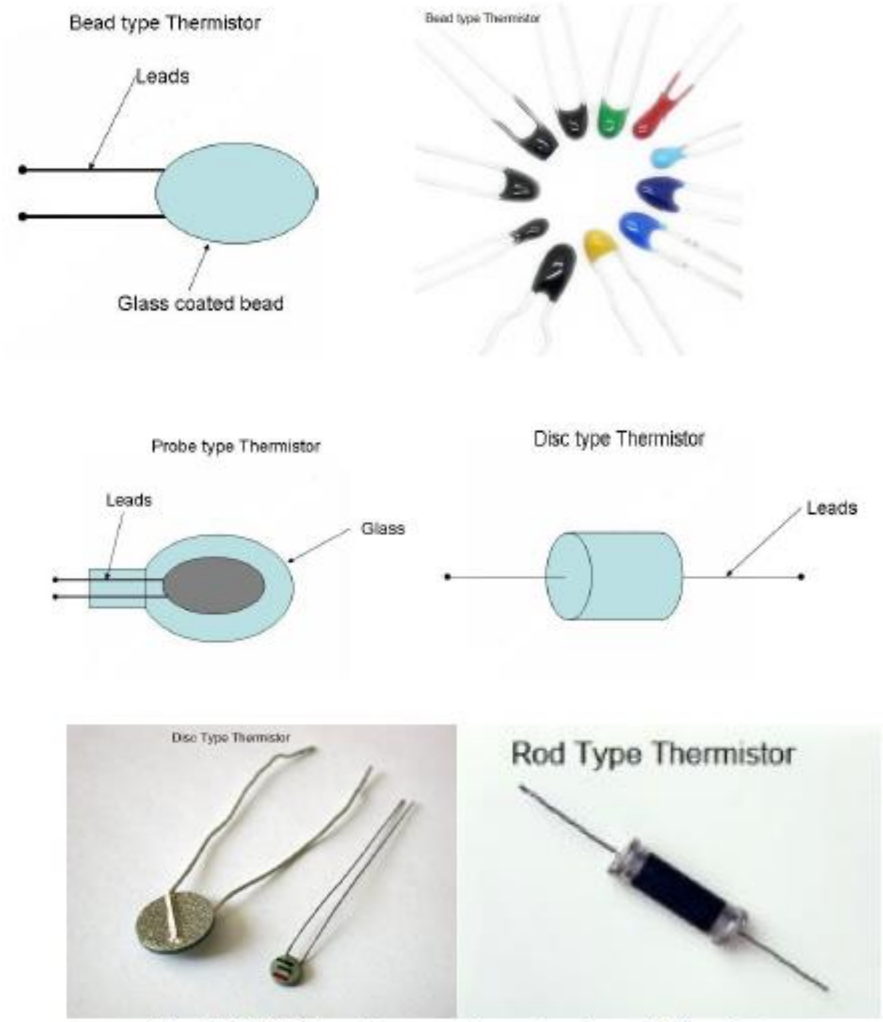


Fig. 1.10: Different types and construction of Thermistors

Resistance temperature characteristics of thermistor

It is given by,

$$R_{T1} = R_{T2} exp[\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)]$$

R_{T1} =Resistance of the thermistor at temperature T_1 (0K)

R_{T1} =Resistance of the thermistor at temperature T_1 (0K)

β = Constant, depending upon the material of thermistor.

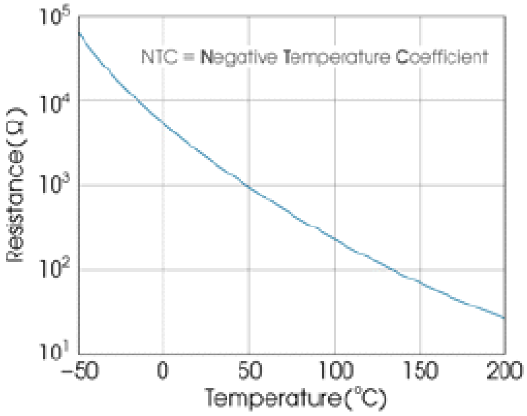


Fig. 1.11: Resistance temperature characteristics of thermistor

Thus a thermistor has a very high negative temperature coefficient of resistance making it an ideal temperature transducer. Between -100^0C to 400^0C, the thermistor changes between its resistivity from 10^4 to 10^{-4}Ωm which explains high sensitivity of thermistors for measurement of temperature.

Inductive Transducer Definition:

Inductive Transducer Definition may be either of the self generating or the passive type. The self generating type utilises the basic electrical generator principle, i.e. a motion between a conductor and magnetic field induces a voltage in the conductor (generator action). This relative motion between the field and the conductor is supplied by changes in the measured.

An inductive electromechanical transducer is a device that converts physical motion (position change) into a change in inductance.

Transducers of the variable inductance type work upon one of the following principles.

1. **Variation of self inductance**
2. **Variation of mutual inductance**

Inductive Transducer Definition are mainly used for the measurement of displacement. The displacement to be measured is arranged to cause variation in any of three variables

1. **Number of turns**
2. **Geometric configuration**
3. **Permeability of the magnetic material or magnetic circuits**

For example, let us consider the case of a general inductive transducer. The Inductive Transducer Definition has N turns and a reluctance R. When a current i is passed through it, the flux is

$$\phi = \frac{Ni}{R}$$

$$\frac{d\phi}{dt} = \frac{N}{2} \times \frac{di}{dt} - \frac{Ni}{R^2} \times \frac{dR}{dt}$$

If the current varies very rapidly,

$$\frac{d\phi}{dt} = \frac{N}{2} \times \frac{di}{dt}$$

But emf induced in the coil is given by

$$e = N \times d\phi/dt$$

$$e = N \times \frac{N}{2} \times \frac{di}{dt} = \frac{N^2}{R} \times \frac{di}{dt}$$

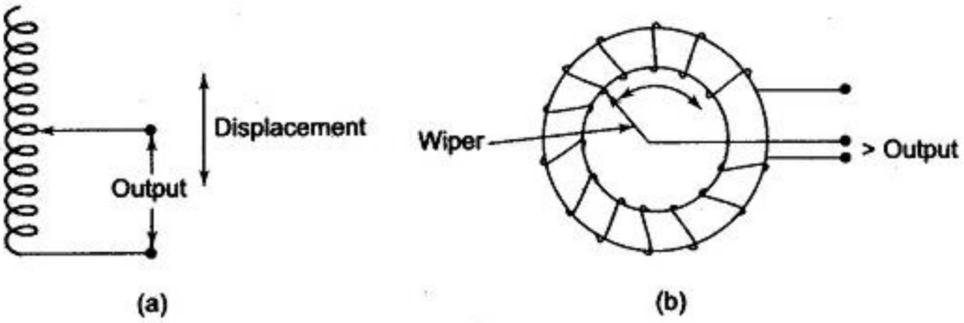
Also the self inductance is given by

$$L = \frac{e}{di/dt} = \frac{N^2}{R}$$

Therefore, the output from an inductive transducer can be in the form of either a change in voltage or a change in inductance.

Change in Self Inductance with Numbers of Turns

The output may be caused by a change in the number of turns. Figures (a) and (b) are transducers used for, the measurement of displacement of linear and angular movement respectively.



(a) (b)

Figure (a) is an air cored transducer for measurement of linear displacement.

Figure(b) is an iron cored coil used for the measurement of angular displacement.

In both cases, as the number of turns are changed, the self inductance and the output also changes.

Transducer Working on the Principle of Change in Self Inductance with Change in Permeability

Figure-1 shows an Inductive Transducer Definition which works on the principle of the variation of permeability causing a change in self inductance. The iron core is surrounded by a winding. If the iron core is inside the winding, its permeability is increased, and so is the inductance. When the iron core is moved out of the winding, the permeability decreases, resulting in a reduction of the self inductance of the coil. This transducer can be used for measuring displacement.

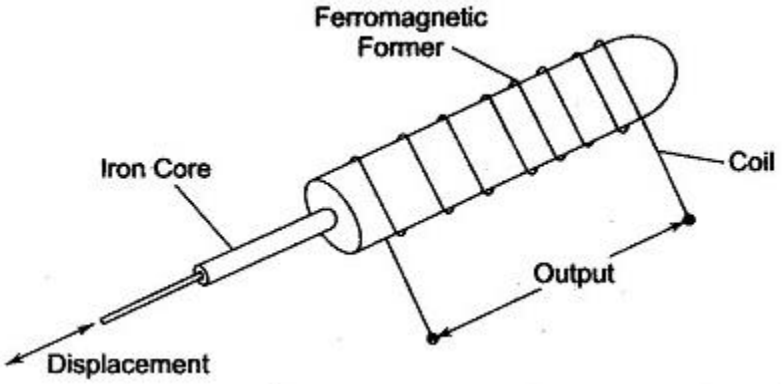


Figure-1

Variable Reluctance Type Transducer

A transducer of the variable type consists of a coil wound on a ferromagnetic core. The displacement which is to be measured is applied to a ferromagnetic target. The target does not have any physical contact with the core on which it is mounted. The core and the target are separated by an air gap, as shown in Fig 2.

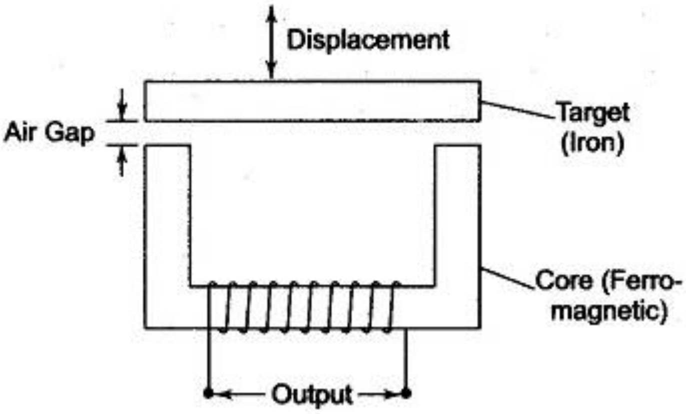


Fig-2

The reluctance of the magnetic path is determined by the size of the air gap. The inductance of the coil depends upon the reluctance of the magnetic circuits. The self inductance of the coil is given by

$$L = \frac{N^2}{R_i + R_g}$$

where N = number of turns

R_i = reluctance of iron parts

R_g = reluctance of air gap

The reluctance of the iron part is negligible compared to that of the air gap.

Therefore

$$L = N^2/R_g$$

But reluctance of the air gap is given by

$$R_g = \frac{l_g}{\mu_o \times A_g}$$

where

l_g = length of the air gap

A_g = area of the flux path through air

μ_o = permeability

R_g is proportional to l_g, as μ_o and A_g are constants.

Hence L is proportional to l/lg, i.e. the self inductance of the coil is inversely proportional to the length of the air gap.

When the target is near the core, the length is small and therefore the self inductance large. But when the target is away from the core the reluctance is large, resulting in a smaller self inductance value. Hence the inductance of the coil is a function of the distance of the target from the core, i.e. the length of the air gap.

Since it is the displacement which changes the length of the air gap, the self inductance is a function of displacement, albeit a non-linear one.

A variable reluctance bridge is shown in Fig-3.

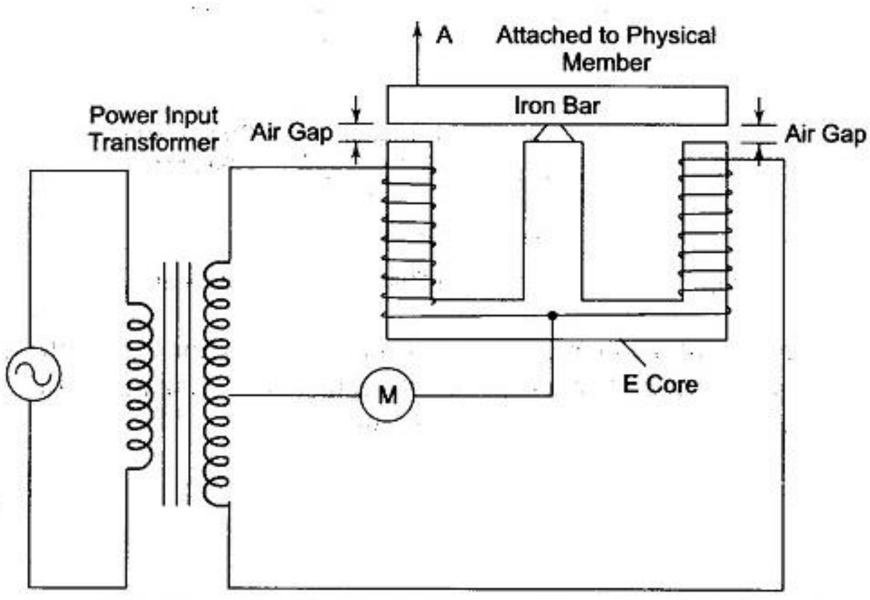


Fig-3

A separate coil is wound on each outside leg of an E core and an iron bar is pivoted on the centre leg. A magnet extends from each outside_leg through an air gap and through the iron bar to the centre leg.

The moving member is attached to one end of the iron bar and causes the bar to wobble back and forth, thereby varying the size of each air gap.

The bridge consists of two transducer coils and a tapped secondary of the input power transformers. It is balanced only when the inductance of the two transducer coils are equal, i.e. when the iron bar is in a nearly exact horizontal position and the air gaps are equal.

Whenever the iron bar at point A moves and alters the air gap, the bridge becomes unbalanced by an amount proportional to the change in inductance, which in turn is proportional to the displacement of the moving member.

The increase and decrease of the inductance with varying air gap sizes is non-linear, and so is the output. Also, the flux density within the air gaps is easily affected by external fields.

Linear Variable Differential Transducer:

The differential transformer is a passive inductive transformer. It is also known as a Linear Variable Differential Transducer (LVDT). The basic construction is as shown in Fig-1.

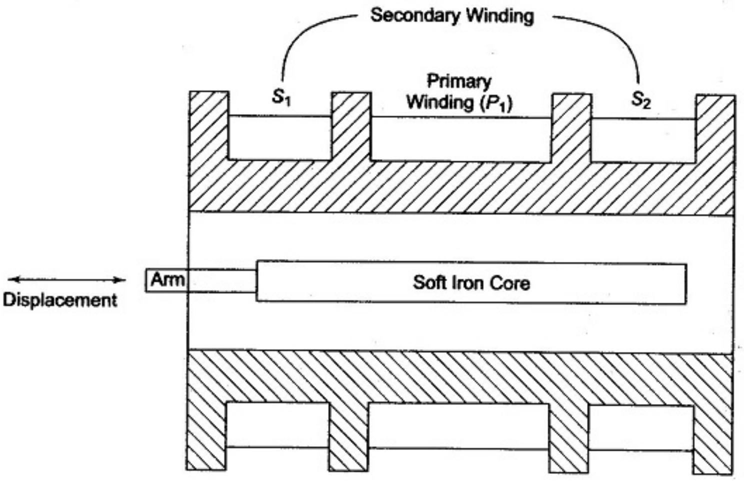


Fig-1

The transformer consists of a single primary winding P_1 and two secondary windings S_1 and S_2 wound on a hollow cylindrical former. The secondary windings have an equal number of turns and are identically placed on either side of the primary windings. The primary winding is connected to an ac source.

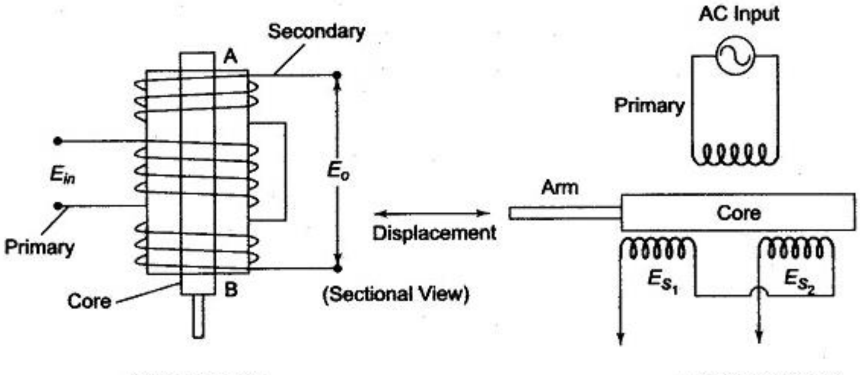
An movable soft iron core slides within the hollow former and therefore affects the magnetic coupling between the primary and the two secondaries.

The displacement to be measured is applied to an arm attached to the soft iron core. (In practice, the core is made up of a nickel-iron alloy which is slotted longitudinally to reduce eddy current losses.)

When the core is in its normal (null) position, equal voltages are induced in the two secondary windings. The frequency of the ac applied to the primary winding ranges from 50 Hz to 20 kHz.

The output voltage of the secondary windings S_1 is E_{s1} and that of secondary winding S_2 is E_{s2}.

In order to convert the output from S_1 to S_2 into a single voltage signal, the two secondaries S_1 and S_2 are connected in series opposition, as shown in Fig.



(a) Construction

(b) Basic Circuit

Hence the output voltage of the transducer is the difference of the two voltages. Therefore the differential output voltage $E_o = E_{s1} \sim E_{s2}$.

When the core is at its normal position, the flux linking with both secondary windings is equal, and hence equal emfs are induced in them. Hence, at null position $E_{s1} = E_{s2}$. Since the output voltage of the transducer is the difference of the two voltages, the output voltage E_o is zero at null position.

Now, if the core is moved to the left of the null position, more flux links with winding S_1 and less with winding S_2. Hence, output voltage E_{s1} of the secondary winding S_1 is greater than E_{s2} . The magnitude of the output voltage of the secondary is then $E_{s1} — E_{s2}$, in phase with E_{s1} (the output voltage of secondary winding S_1).

Similarly, if the core is moved to the right of the null position, the flux linking with winding S_2 becomes greater than that linked with winding S_1. This results in E_{s2} becoming larger than E_{s1}. The output voltage in this case is $E_o = E_{s2} — E_{s1}$ and is in phase with E_{s2}.

The amount of voltage change in either secondary winding is proportional to the amount of movement of the core. Hence, we have an indication of the amount of linear motion. By noting which output is increasing or decreasing, the direction of motion can be determined. The output ac voltage inverts as the core passes the centre position. The farther the core moves from the centre, the greater the difference in value between E_{s1} and E_{s2} and consequently the greater the value of E_o. Hence, the amplitude is function of the distance the core has moved, and the polarity or phase indicates the direction of motion, as shown in Fig.

As the core is moved in one direction from the null position, the difference voltage, i.e. the difference of the two secondary voltages increases, while maintaining an in-phase relation with the voltage from the input source. In the other direction from the null position, the difference voltage increases but is 180° out of phase with the voltage from the source.

By comparing the magnitude and phase of the difference output voltage with that of the source, the amount and direction of the movement of the core and hence of the displacement may be determined.

The amount of output voltage may be measured to determine the displacement. The output signal may also be applied to a recorder or to a controller that can restore the moving system to its normal position.

The output voltage of an Linear Variable Differential Transducer is a linear function of the core displacement within a limited range of motion (say 5 mm from the null position).

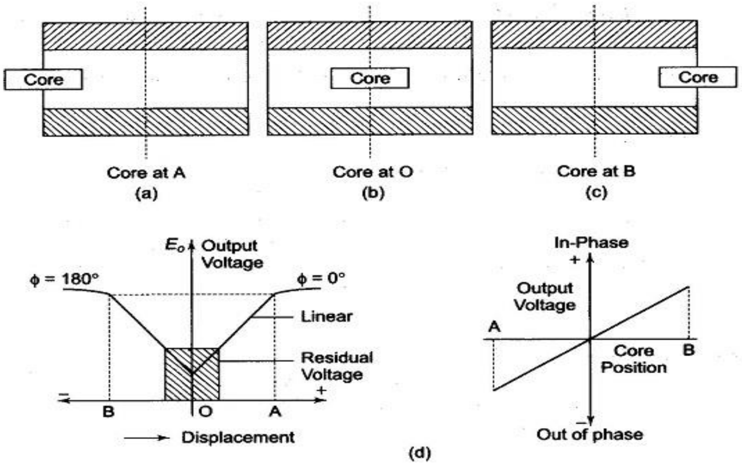
Figure(d) shows the variation of the output voltage against displacement for various position of the core. The curve is practically linear for small displacements (up to 5 mm). Beyond this range, the curve starts to deviate.

The diagram in Figs(a), (b) and (c) shows the core of an Linear Variable Differential Transducer at three different positions.

In Fig. 13.21(b), the core is at 0, which is the central zero or null position. Therefore, $E_{s1} = E_{s2}$, and $E_o = 0$.

When the core is moved to the left, as in Fig. 13.21(a) and is at A, E_{s1} is more than E_{s2} and E_o is positive. This movement represents a positive value and therefore the phase angle, is $\Phi = 0°$.

When the core is moved to the right towards B, E_{s2} is greater than E_{s1} and hence E_o is negative. Therefore, S_2 the output voltage is 180° out of phase with the voltage which is obtained when the core is moved to the left. The characteristics are linear from 0 — A and 0 — B, but after that they become non-linear.One advantage of an Linear Variable Differential Transducer over, the inductive bridge type is that it produces higher output voltage for small changes in core position. Several commercial models that produce 50 mV/mm to 300 mV/mm are available. 300 mV/mm implies that a 1 mm displacement of the core produces a voltage output of 300 mV.



(a), (b), (c) Various Core Position of LVDT
(d) Variation of Output Voltage vs Displacement

Linear Variable Differential Transducer are available with ranges as low as ± 0.05 in. to as high as ± 25 in. and are sensitive enough to be used to measure displacements of well below 0.001 in. They can be obtained for operation at temperatures as low as — 265°C and as high as + 600°C and are also available in radiation resistance designs for nuclear operations.

Advantages of Linear Variable Differential Transducer

1. **Linearity:** The output voltage of this transducer is practically linear for displacements upto 5 mm (a linearity of 0.05% is available in commercial LVDTs).
2. **Infinite resolution:** The change in output voltage is stepless. The effective resolution depends more on the test equipment than on the
3. **High output:** It gives a high output (therefore there is frequently no need for intermediate amplification devices).
4. **High sensitivity:** The transducer possesses a sensitivity as high as 40 V/mm.
5. **Ruggedness:** These transducers can usually tolerate a high degree of vibration and shock.
6. **Less friction:** There are no sliding contacts.
7. **Low hysteresis:** This transducer has a low hysteresis, hence repeatability is excellent under all conditions.
8. **Low power:** consumption Most LVDTs consume less than 1 W of

Disadvantages of Linear Variable Differential Transducer

1. Large displacements are required for appreciable differential output.
2. They are sensitive to stray magnetic fields (but shielding is possible).
3. The receiving instrument must be selected to operate on ac signals, or a demodulator network must be used if a dc output is required.
4. The dynamic response is limited mechanically by the mass of the core and electrically by the applied voltage.

5. Temperature also affects the transducer.

Capacitive Transducer

Definition: The capacitive transducer is used for measuring the displacement, pressure and other physical quantities. It is a passive transducer that means it requires external power for operation. The capacitive transducer works on the principle of variable capacitances. The capacitance of the capacitive transducer changes because of many reasons like overlapping of plates, change in distance between the plates and dielectric constant.

The capacitive transducer contains two parallel metal plates. These plates are separated by the dielectric medium which is either air, material, gas or liquid. In the normal capacitor the distance between the plates are fixed, but in capacitive transducer the distance between them are varied.

The capacitive transducer uses the electrical quantity of capacitance for converting the mechanical movement into an electrical signal. The input quantity causes the change of the capacitance which is directly measured by the capacitive transducer.

The capacitors measure both the static and dynamic changes. The displacement is also measured directly by connecting the measurable devices to the movable plate of the capacitor. It works on with both the contacting and non-contacting modes.

Principle of Operation

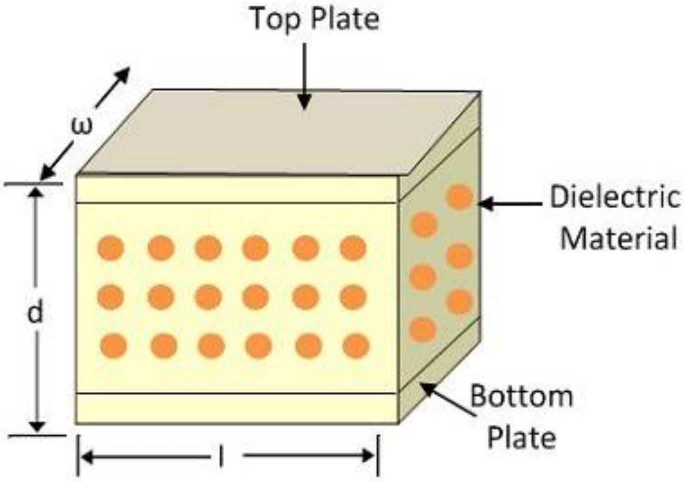
The equations below express the capacitance between the plates of a capacitor

$$C = \varepsilon A/d$$

$$C = \varepsilon_r \varepsilon_0 A/d$$

Where A – overlapping area of plates in m^2
d – the distance between two plates in meter
ε – permittivity of the medium in F/m
ε_r – relative permittivity
ε_0 – the permittivity of free space

The schematic diagram of a parallel plate capacitive transducer is shown in the figure below.



Parallel Plate Capacitive Transducer

The change in capacitance occurs because of the physicals variables like displacement, force, pressure, etc. The capacitance of the transducer also changes by the variation in their dielectric constant which is usually because of the measurement of liquid or gas level.

The capacitance of the transducer is measured with the bridge circuit. The output impedance of transducer is given as $X_c = 1/2\pi f c$

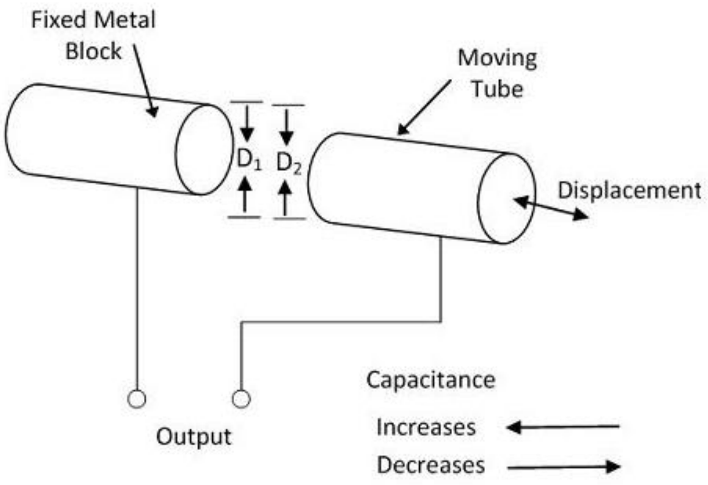
Where, C – capacitance
f – frequency of excitation in Hz.

The capacitive transducer is mainly used for measurement of linear displacement. The capacitive transducer uses the following three effects.

1. Variation in capacitance of transducer is because of the overlapping of capacitor plates.
2. The change in capacitance is because of the change in distances between the plates.
3. The capacitance changes because of dielectric constant.

The following methods are used for the measuring displacement.

1. **A transducer using the change in the Area of Plates** – The equation below shows that the capacitance is directly proportional to the area of the plates. The capacitance changes correspondingly with the change in the position of the plates.



Capacitive transducer

The capacitive transducers are used for measuring the large displacement approximately from 1mm to several cms. The area of the capacitive transducer changes linearly with the capacitance and the displacement. Initially, the nonlinearity occurs in the system because of the edges. Otherwise, it gives the linear response.

The capacitance of the parallel plates is given as

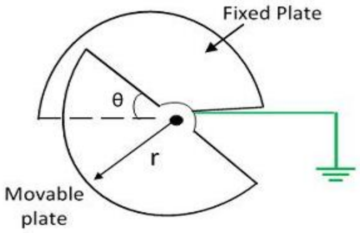
$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon x \omega}{d} F$$

where x – the length of overlapping part of plates
ω – the width of overlapping part of plates.

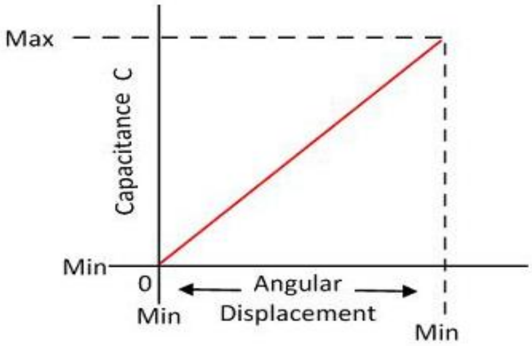
The sensitivity of the displacement is constant, and therefore it gives the linear relation between the capacitance and displacement.

$$S = \frac{\partial C}{\partial x} = \varepsilon \frac{\omega}{d} F/m$$

The capacitive transducer is used for measuring the angular displacement. It is measured by the movable plates shown below. One of the plates of the transducer is fixed, and the other is movable.



The phasor diagram of the transducer is shown in the figure below.

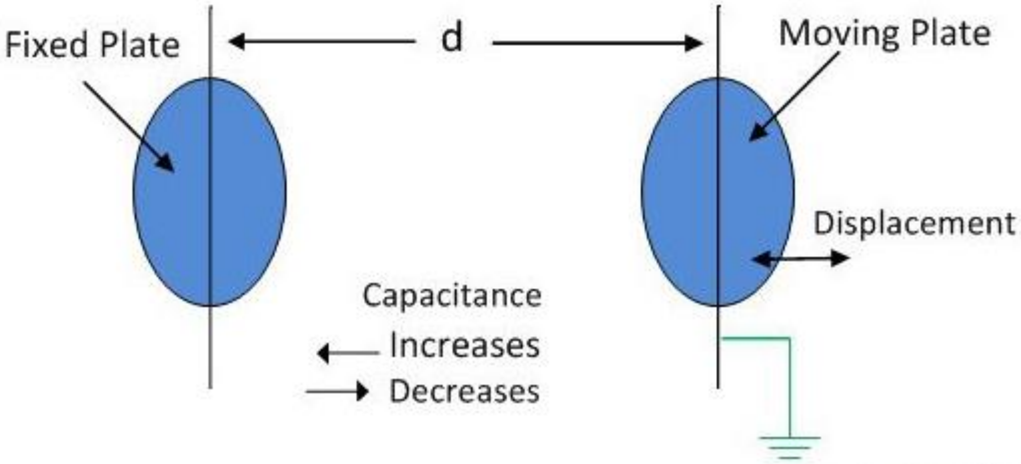


The angular movement changes the capacitance of the transducers. The capacitance between them is maximum when these plates overlap each other. The maximum value of capacitance is expressed as $C_{max} = \frac{\varepsilon A}{d} = \frac{\pi \varepsilon r^2}{2d}$

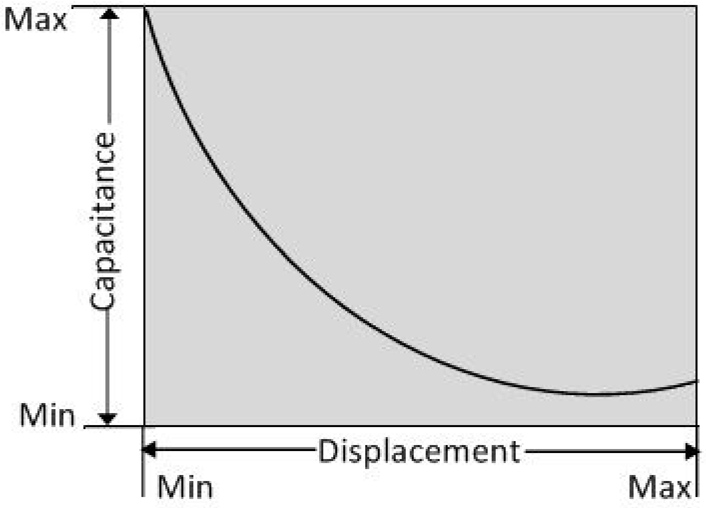
The capacitance at angle θ is given expressed as, $C = \frac{\varepsilon \theta r^2}{2d}$ θ – angular displacement in radian. The sensitivity for the change in capacitance is given as $S = \frac{\partial C}{\partial \theta} = \frac{\varepsilon r^2}{2d}$

The 180° is the maximum value of the angular displacement of the capacitor.

2. The transducer using the change in distance between the plates – The capacitance of the transducer is inversely proportional to the distance between the plates. The one plate of the transducer is fixed, and the other is movable. The displacement which is to be measured links to the movable plates.



The capacitance is inversely proportional to the distance because of which the capacitor shows the nonlinear response. Such type of transducer is used for measuring the small displacement. The phasor diagram of the capacitor is shown in the figure below.



The sensitivity of the transducer is not constant and vary from places to places.

Advantage of Capacitive Transducer

The following are the major advantages of capacitive transducers.

1. It requires an external force for operation and hence very useful for small systems.
2. The capacitive transducer is very sensitive.
3. It gives good frequency response because of which it is used for the dynamic study.
4. The transducer has high input impedance hence they have a small loading effect.
5. It requires small output power for operation.

Disadvantages of capacitive Transducer

The main disadvantages of the transducer are as follows.

1. The metallic parts of the transducers require insulation.
2. The frame of the capacitor requires earthing for reducing the effect of the stray magnetic field.
3. Sometimes the transducer shows the nonlinear behaviours because of the edge effect which is controlled by using the guard ring.
4. The cable connecting across the transducer causes an error.

Uses of Capacitive Transducer

The following are the uses of capacitive transducer.

1. The capacitive transducer uses for measurement of both the linear and angular displacement. It is extremely sensitive and used for the measurement of very small distance.

2. It is used for the measurement of the force and pressures. The force or pressure, which is to be measured is first converted into a displacement, and then the displacement changes the capacitances of the transducer.
3. It is used as a pressure transducer in some cases, where the dielectric constant of the transducer changes with the pressure.
4. The humidity in gases is measured through the capacitive transducer.
5. The transducer uses the mechanical modifier for measuring the volume, density, weight etc.

The accuracy of the transducer depends on the variation of temperature to the high level.

Hall Effect Transducer

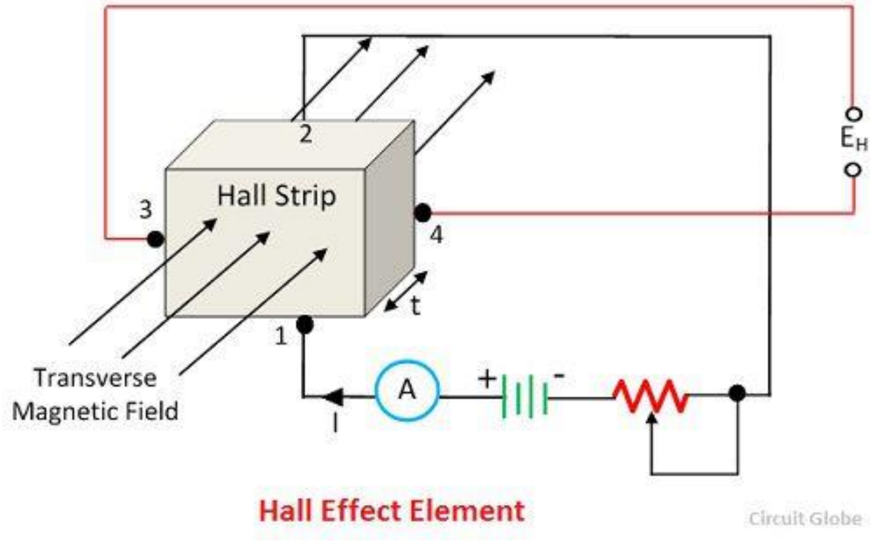
Definition: The hall effect element is a type of transducer used for measuring the magnetic field by converting it into an emf. The direct measurement of the magnetic field is not possible. Thus the Hall Effect Transducer is used. The transducer converts the magnetic field into an electric quantity which is easily measured by the analogue and digital meters.

Principle of Hall Effect Transducer

The principle of hall effect transducer is that if the current carrying strip of the conductor is placed in a transverse magnetic field, then the EMF develops on the edge of the conductor. The magnitude of the develop voltage depends on the density of flux, and this property of a conductor is called the Hall effect. The Hall effect element is mainly used for magnetic measurement and for sensing the current.

The metal and the semiconductor has the property of hall effect which depends on the densities and the mobility of the electrons.

Consider the hall effect element shown in the figure below. The current supply through the lead 1 and 2 and the output is obtained from the strip 3 and 4. The lead 3 and 4 are at same potential when no field is applied across the strip.



Hall Effect Element

When the magnetic field is applied to the strip, the output voltage develops across the output leads 3 and 4. The develops voltage is directly proportional to the strength of the material.

The output voltage is, $E_H = K_H I B / t$

where,

$$K_H - Hall\ effect\ coefficient\ ;\ \frac{V - m}{A - Wbm^{-2}}$$

$$t - thickness\ of\ Strip\ ; m$$

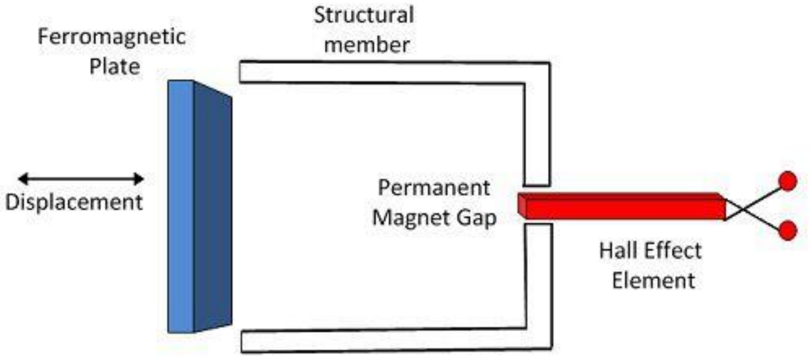
The I is the current in ampere and the B is the flux densities in Wb/m^2

The current and magnetic field strength both can be measured with the help of the output voltages. The hall effect EMF is very small in conductors because of which it is difficult to measure. But semiconductors like germanium produces large EMF which is easily measured by the moving coil instrument.

Applications of Hall Effect Transducer

The following are the application of the Hall effect Transducers.

1. Magnetic to Electric Transducer – The Hall effect element is used for converting the magnetic flux into an electric transducer. The magnetic fields are measured by placing the semiconductor material in the measurand magnetic field. The voltage develops at the end of the semiconductor strips, and this voltage is directly proportional to the magnetic field density.
The Hall Effect transducer requires small space and also gives the continuous signal concerning the magnetic field strength. The only disadvantage of the transducer is that it is highly sensitive to temperature and thus calibration requires in each case.
2. Measurement of Displacement – The Hall effect element measures the displacement of the structural element. For e.g-Consider the ferromagnetic structure which has a permanent magnet.



Measurement of Displacement Using Hall Efect Transducer

Circuit Globe

The hall effect transducer placed between the poles of the permanent magnet. The magnetic field strength across the hall effect element changes by changing the position of the ferromagnetic field.

3. Measurement of Current – The hall effect transducer is also used for measuring the current without any physical connection between the conductor circuit and meter.

The AC or DC is applied across the conductor for developing the magnetic field. The strength of the magnetic field is directly proportional to the applied current. The magnetic field develops the emf across the strips. And this EMF depends on the strength of the conductor.

4. Measurement of Power – The hall effect transducer is used for measuring the power of the conductor. The current is applied across the conductor, which develops the magnetic field. The intensity of the field depends on the current. The magnetic field induces the voltage across the strip. The output voltage of the multiplier is proportional to the power of the transducer.

Cathode Ray Tube (CRT)

A Cathode Ray Tube is the main part of Cathode Ray Oscilloscope (CRO) with additional circuitry to operate the circuit.
Its main parts are:
(1) Electron gun assembly.
(2) Deflection pate assembly.
(3) Fluorescent Screen.
(4) Glass envelope.
(5) Base for making the connection.

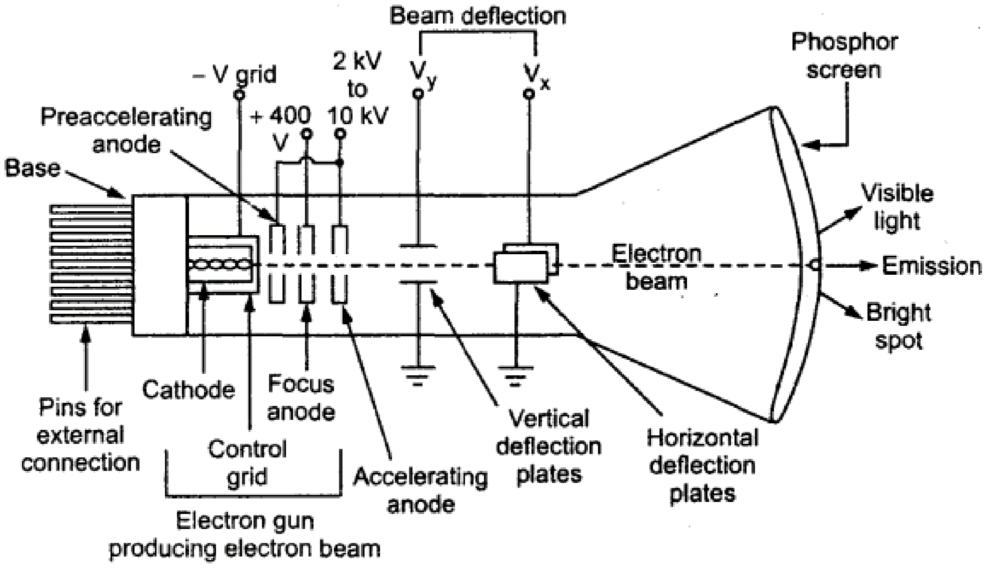


Fig- Cathode ray tube

Cathode ray tube is a vacuum tube and convert an electrical signal into visual one.
Cathode ray tube makes available plenty of electrons. The electrons are accelerated to high velocity and are focussed on a fluorescent screen. Electron beam produces a spot of light wherever it strikes. The electron beam is deflected on its journey in response to the electrical signal under study. Hence the electrical signal is displayed.
Electron gun assembly
The arrangement of electron which produces a focussed beam of electrons is called the electron gun. It consists of:
(1) An indirectly heated cathode.
(2) A control grid surrounding the cathode.
(3) A focusing anode.
(4) An accelerating anode.
Control grid is held at negative potential with respect to cathode. Two anodes focusing anode and accelerating anode are maintained at high positive potential with respect to cathode.
It has the following parts:
· Cathode with the heater.
· Control grid
· Focusing anode

- Accelerating anode.

Cathode: It consists of a nickel cylinder coated with oxide coating of barium and strontium emit plenty of electron. Rate of emission of electrons depend upon magnitude cathode current which can be controlled by the control grid.

Control Grid: It is a nickel cylinder with a centrally located hole. It encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. It is 15mm diameter and 15mm long with a hole of 0.25mm at the centre. Control grid is kept at negative potential with respect to the cathode and its function is to vary the electron emission. The hole in the grid is provided to allow the passage of electrons and concentrate the beam of electron along the axis of the tube.

Focusing anode: The focusing anode focuses the electron beam into sharp pin point by controlling the positive potential on it. Its value is a few hundred volts and more positive than cathode so as to accelerate the electron beam.

Accelerating anode: The positive potential on the accelerating anode is much higher than the focusing anode. It is in the range of 10KV and for this reason the anode accelerate the narrow beam of electrons to high velocity. Therefore the electron gun assembly forms a narrow accelerated beam of electron s which produces a spot of light when strikes the screen.

Deflection Plate assembly

Deflection plate is accomplished by two set of deflecting plates placed within the tube beyond accelerating anode.

(1) One set is vertical deflection plates.

(2) Other set is horizontal defection plates.

Vertical defection plates:

These plates are mounted horizontally and applied a proper potential. These plates move the electron beam in the vertical direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the vertical up and down on the fluorescent screen.

Horizontal defection plates:

These plates are mounted vertically and applied a proper potential. These plates move the electron beam in the horizontal direction when the potential is applied to the plates. These plates are responsible for the movement of electron beams in the horizontalleft and right on the fluorescent screen.

Fluorescent Screen:

The end wall or inside face of tube coated with some fluorescent material called:

- Phosphor
- Zinc oxide.
- Zinc orthosilicate

When high velocity of electron beam strikes the screen a spot of light is produced at the point of impact. It absorbs the kinetic energy of the electron and convert into light. Colour of the light emitted depends on the fluorescent material used.

Glass Envelope

It is a highly evacuated glass housing in which vacuum is maintained inside. Inner wall of the CRT between neck and the screen are usually coated with conducting material called “aquadag”. This coating is electrically connected to the accelerating anode so that electrons which accidentally stroke the wall return to the anode. This prevents the wall from charging to a high negative potential. This aquadag coating prevents the formation of negative charge on the screen.

2.2 Cathode Ray Oscilloscope (CRO)

Cathode ray oscilloscope is the most useful and the most versatile laboratory instrument for studying wave shapes of alternating current, voltage, power, frequency which have amplitude and waveform.

It allows the user to see the amplitude of electrical signal as a function of time on the screen.

With CRO wave shapes of signal can be studied and can be used for measuring

(1) Voltage

(2) Frequency

(3) Phase shift

Application of CRO

In laboratories it is used for

- Tracing actual waveform of current and voltage.
- Determination of amplitude of a variable quantity.
- Comparison of phase and frequency.
- Can be used to check any kind of electronic components in an electronic circuit.
- For finding B-H curve of hysteresis loop.
- For tracing the characteristics of any electronic components like diode, transistors etc.

For commercial purpose:

- In CRT televisions.
- In Radar.

Block diagram of General purpose CRO

It consists of the following components:

- Cathode Ray Tube
- Power supply Block
- Vertical Amplifier
- Horizontal Amplifier.
- Time Base generator.
- Trigger Circuit.

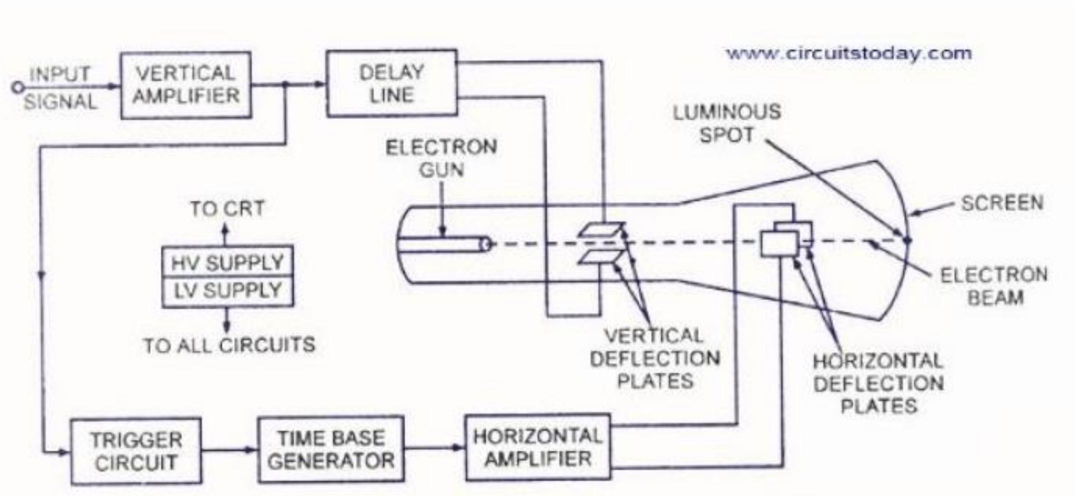
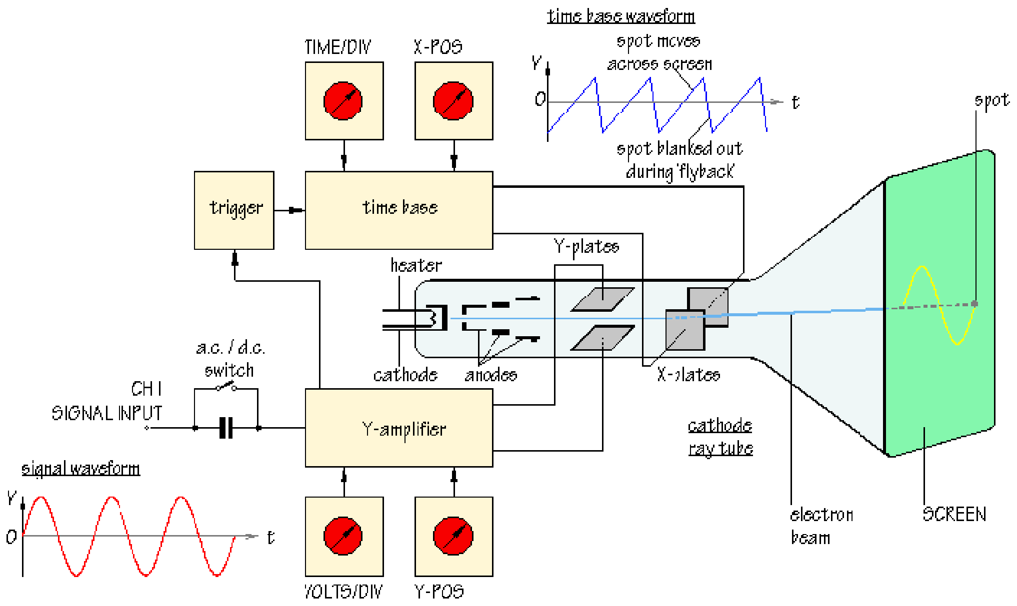


Fig- Block diagram of CRO



Cathode Ray Tube:

It is the heart of the oscilloscope. It generates sharply focused electron beam and accelerate the beam to a high velocity and deflect the beam to create image. It contains the phosphor screen where the electron beam became visible. While travelling form the electron gun to screen the electron beam passes between set of

(1) Vertical Deflection Plate.
(2) Horizontal Deflection Plate.

Voltage applied to the vertical deflection plate to move the beam in the vertical plane and the CRT spot moves up and down.

Voltage applied to the horizontal deflection plate moves the beam in the horizontal plane.

Power supply block

It provides the voltage required by the CRT to generate and accelerate the electron beam as well as to supply the required operating voltages for the other circuit of the oscilloscope. Voltage in the range of few Kilo Volts are required by CRT for acceleration. Low voltage for heater of electron gun of CRT which emit the electron.

Supply voltage for the other circuit is not more than few hundred volt.

Vertical Amplifier

It amplifies the signal waveform to be viewed. This is a wide bad amplifier used to amplify signal in the vertical section.

Horizontal Amplifier

It is fed with saw-tooth voltage. It amplifies saw-tooth voltage before it is applied to horizontal deflection plates.

Time base generator:

It develops saw-tooth voltage waveform required to deflect the beam in horizontal section.

Trigger Circuit

The trigger circuit is used to convert the incoming signal into trigger pulses so that input frequency can be synchronised.

2.3. CRO Measurement

Various parameters which can be measured by CRO are

1. Voltage; 2. Current; 3. Time period; 4. Frequency; 5.Phase angle

6. Amplitude, 7. Peak to peak value

Voltage measurement

Voltage to be measured is applied to the V deflection plates through vertical amplifier.

The ‘X’ deflection plate is excited by time base generator.

Þ After noting down the selection in vect/div from front panel the peak to peak value amplitude and r m s value of sinusoidal voltage can be obtained.

I. Peak to peak value $V_{P-P} = \frac{Volt}{Division} \times no. of\ division$

II. Amplitude$V_m = \frac{V_{P-P}}{2}$

III. R. M. S value$V_{RMS} = \frac{2V_{P-P}}{\sqrt{2}} = \frac{V_P}{\sqrt{2}} + \frac{V_P}{\sqrt{2}}$

Voltage/division = deflection Sensitivity

Deflection factor $= \frac{1}{Deflection\ Sensitivity}$

Current Measurement

A CRO has a very high input impedance and cannot be used for direct measurement. However the current can be measured in terms of voltage drop across a standard resistance I=V/R

Time period measurement:

For the measurement of time period ‘T’ of the waveform. It is displayed on the screen such that one complete cycle is visible on the screen.

After noting the time/division selected on front panel the time period of the wave form can be obtained as Time period T = time/division X number of division occupied by one cycle.

Frequency = f = 1/T

Frequency measurement by (lisajous Method Pattern)

The unknown frequency can be accurately determined with the help of CRO

Step – 1

Known frequency is applied to horizontal input (2000 hz) Unknown frequency to vertical input

The number of loops cut by the horizontal line gives frequency on the vertical plates fy

The number of loops cut by vertical lines gives the frequency on the horizontal plates FH

$$\frac{f_v}{f_H} = \frac{\text{No of loops cut by horizontal line}}{\text{No of loops cut by vertical line}}$$

$$\frac{f_v}{2000} = \frac{1}{2}$$

$$f_v = 2000 \times \frac{1}{2}$$

Unknown frequency = 1000hz

It is a method to determine the unknown frequency by comparing it with known frequency.

This fog = Lissajous fog – Named after French.

Example 1:

$$\frac{f_v}{f_H} = \frac{1}{1}$$

$f_H = 1000$

$f_v = f_H = 1000$hz

No of loops cut by horizontal and vertical line – 1

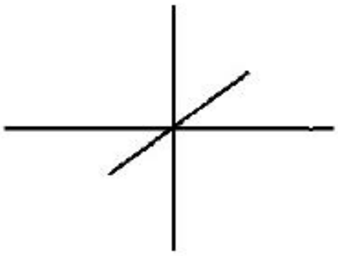
Phase and frequency measurement

An as well as cope can be used to find the phase angle between two sinusoidal quantities of the same frequency.

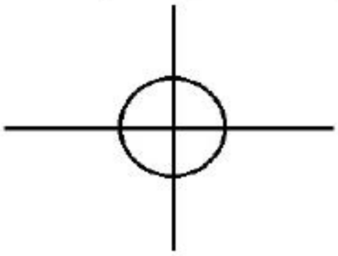
I. One of the signals is applied to Y plates time base generator is switched out and second signal is fed to X plates.

It is necessary that X2 Y are equal magnitude.

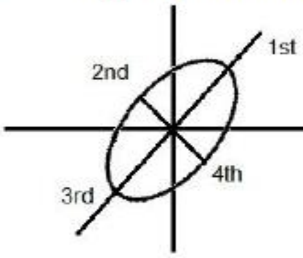
- If the two signals are in phase the display would be a straight line at 45˚ to the horizontal.



- If the phase angle is 90° the display would be a <u>**circle.**</u>

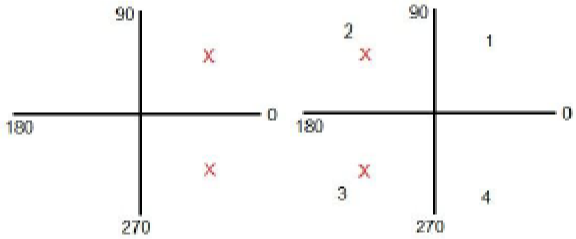


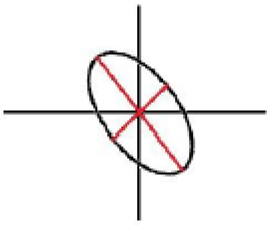
- For any other phase difference the display would be an ellipse.



If the phase angle between (0 - 90°) = (270° - 360°)

The ellipse has its major axis in the 1st quadrant and 3rd quadrant.



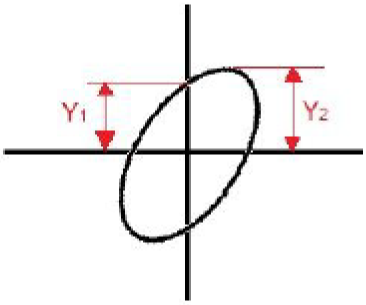


If the phase difference is between 90° and 180° the ellipse has its major axis in the 2nd and 4th quadrant.

Value of phase angle ø is given by

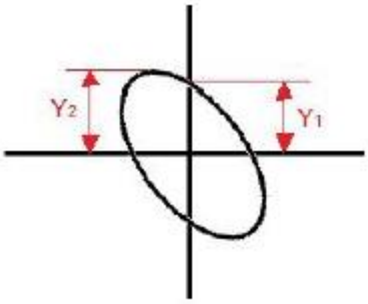
Sinø = Y_1/Y_2 are intercept.

I When the phase angle is between <u>**0 to 90°**</u>
& 270 to 360° major axes lines in 1st quadrant = 3rd quadrant



Sin ø = Y_1/Y_2 = ½ = 0.5
Ø= sin -1 0.5 = 30° or +**5**

II. When phase angle is between 90° to 180° and 180° to 270° major axis would lie in 2nd and 4th quadrant.



Sin ø= Y_1/Y_2 = 0.5
Ø = 150° or 210°