ANALYSIS OF STRUCTURE

Lectures Notes

Government Polytechnic, Bhubaneswar
Diploma in Architecture Assistantship | IV Semester

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COURSE CONTENTS:

1.Introduction:
Aim, object and scope of study the subject.

2.Solution of determinate beams.
Define a beam
Explain various types of supports.
Explain various types of beams.
State and illustrate the concept of shear force, bending moment, shear force and bending moment diagram in case of cantilever and simply supported beam subjected to concentrated load and U.D.L acting separately.

3.Bending stress in beams.
Show the use of pure bending equation (No derivation) for followings.
Rectangular solid.
Circular, solid.

4.Slope and deflection of beams by double integration method.
State and explain the differential equation of elastic curve (expression only).
State and explain the sign conventions for slopes and deflection.
State and explain the slope and deflection calculation for simply supported beam subject to single concentrated load at mid span and U.D.L over entire span.

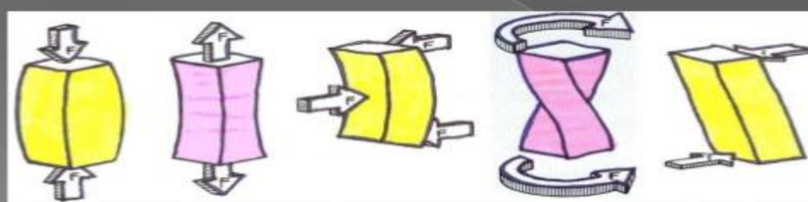
CHAPTER - 2

1.Introduction:

Aim, object and scope of study the subject.

- The strength of a material may be defined as ability, to resist its failure and behaviour, under the action of external forces. It has been observed that, under the action of these forces the material is first deformed and then its failure takes place. A detailed study of forces and their effects , alongwith some suitable protective measures for the safe working condition is known as strength of material.

Behaviour of solid bodies subjected to various types of loading



Compression Tension Bending Torsion (twisted) Shearing

Objectives.

1. This subject is useful for a detailed study of forces and their effects . This knowledge is very essential for an engineer, to enable him, in designing all type of structure and machine.
2. To Provide the basic concepts and principles of strength of materials and to give an ability to analyze a given problem in a simple manner.
3. To give an ability to calculate stresses and deformations of objects under external forces.
4. To give an ability to apply the knowledge of strength of materials on engineering applications and design problems.

CHAPTER - 2

2. Solution of determinate beams

Define a beam:

Beam is the horizontal structural member subjected to a system of external forces at right angles to its axis, bending moment and shear force. The loads are applied over the span as udl or point load.

Explain various types of supports:

Types of supports

1) ***Simple support*** :-

In this type of support,beam is simply supported on the support.

There is no connection between beam and support.

At this type of support,only vertical reaction will be produced.

Simple 

"Support Reactions"

- ***Support*** :- A support prevents translation of a body in a given direction, a force is developed on the body in that direction.
- ***Reactions***:- The forces and moments exerted on an object by its supports are called reactions.

<u>2) Fix end support :-</u>

Beam is completely fixed at end in the wall or support.

Beam cannot rotate at end.

Reactions may be vertical,horizontal,inclined and moment.

Fixed End Support

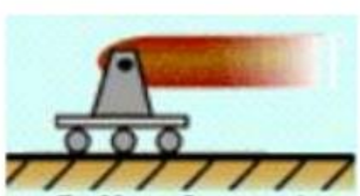
3) <u>Roller support:-</u>

In this type of support,rollers are placed below beam and beam can slide over the rollers.

Reaction will be perpendicular to the surface on which rollers are supported.

This type of support is normally provided at the end of a bridge.

Due to breaking forces of vehicles and temperature forces,bridge slab can slide over the roller support and damage to bridge pier can be avoided.



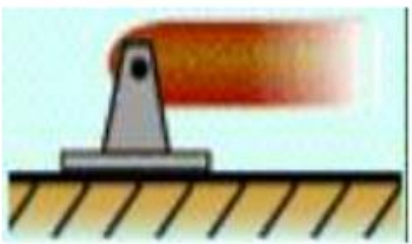
Roller Support

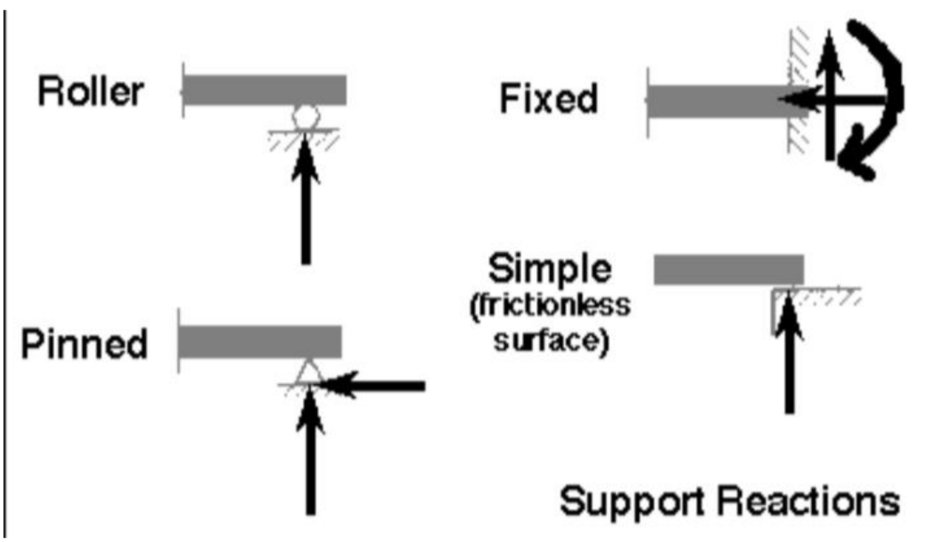
4) hinge support:-

Beam and support are connected by a hinge.

Beam can rotate about the hinge.

Reactions may be vertical, horizontal or inclined.





Explain various types of beams:

In engineering, beams are of several types:

1. ***Simply supported Beam*** –

A beam supported on the ends which are free to rotate and have no moment resistance.

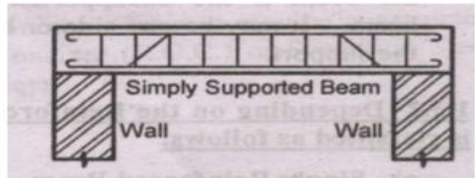


FIG. 1 SIMPLY SUPPORTED BEAM

2.***Fixed beam*** - a beam supported on both ends and restrained from rotation.

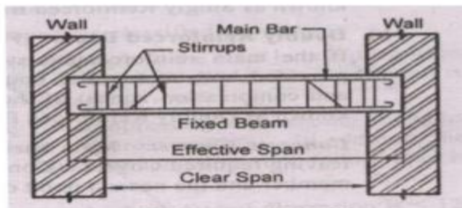
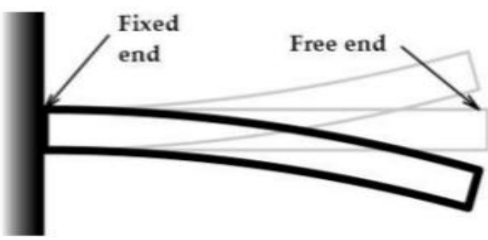


FIG. 2 FIXED BEAM

3) ***Cantilever*** :- it has one end fixed and other end free.



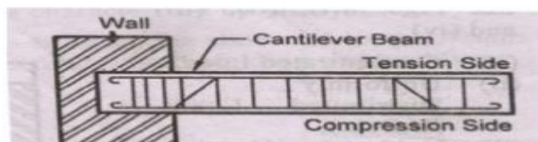
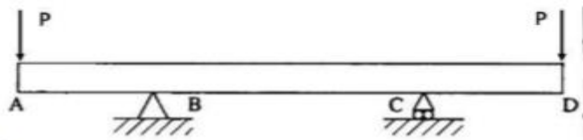
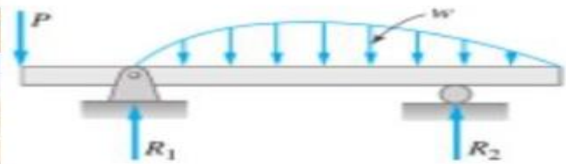


FIG. 3 CANTILEVER BEAM

4) ***Over hanging*** :– A simple beam extending beyond its support on one end.





5) Continuous Beam-

A beam extending over more than two supports.

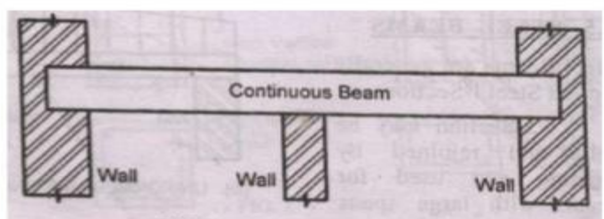


FIG. CONTINUOUS BEAM

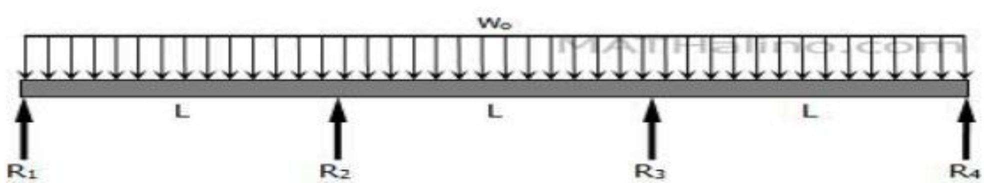


Figure P-829

8) Propped cantilever Beam

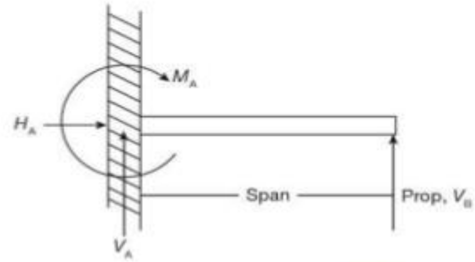


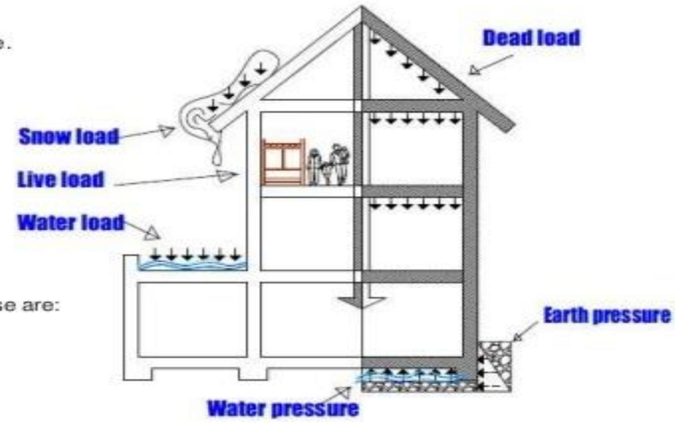
Fig. 4.7 Propped cantilever beam.

Structural load :

Structural loads are forces, deformations, or accelerations applied to a structure or its components.

Types of loads

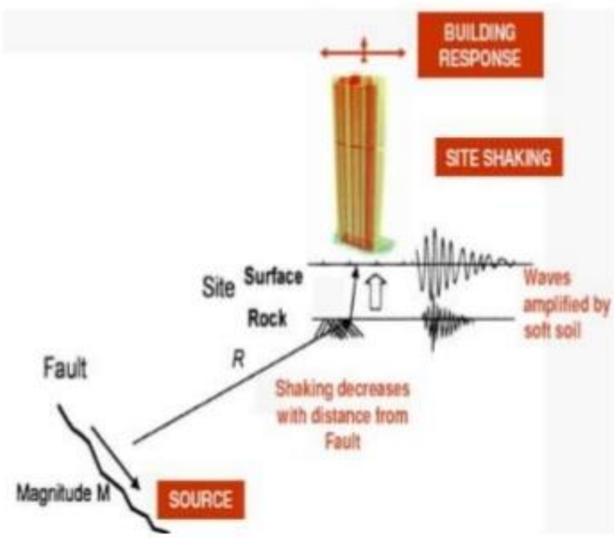
- **Dead load**
  - Loads that are relatively constant over time.
  - Also known as permanent or static loads.
- **Live load**
  - Dynamic or impose or moving loads, temporary of short duration.
  - Considerations: impact, momentum, vibration, slosh dynamic of fluid.



• Environmental loads

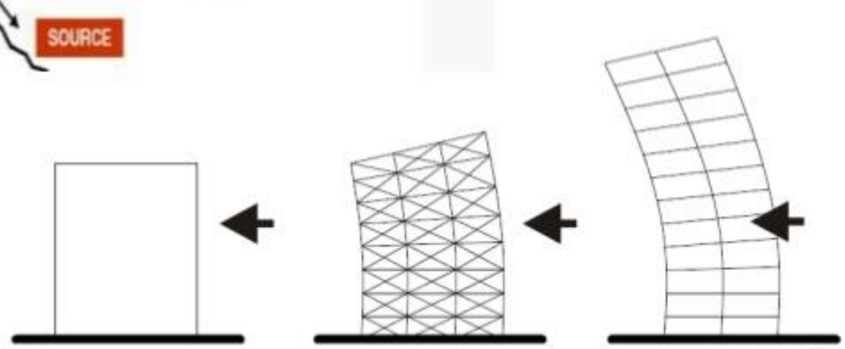
These are loads that act as a result of weather, topography and other natural phenomena. These are:

- **Seismic load**
- Snow, rain and ice load
- **Wind loads**
- Thermal loads (temperature changes leading to thermal expansion)
- Lateral pressure of soil, groundwater or bulk materials



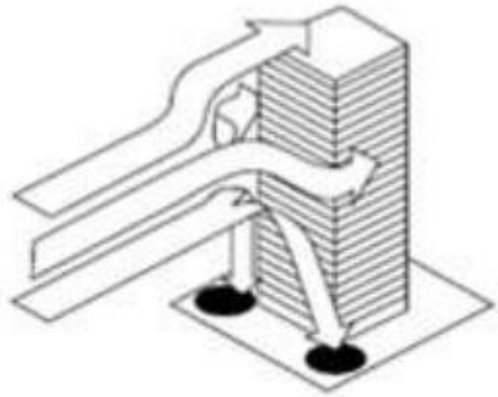
Seismic Load:

- Buildings undergoes dynamic motion during earthquake.
- Building is subjected to inertia forces that act in opposite direction to the acceleration of earthquake excitations.
- These inertia forces, called seismic loads, are usually dealt with by assuming forces external to the building.

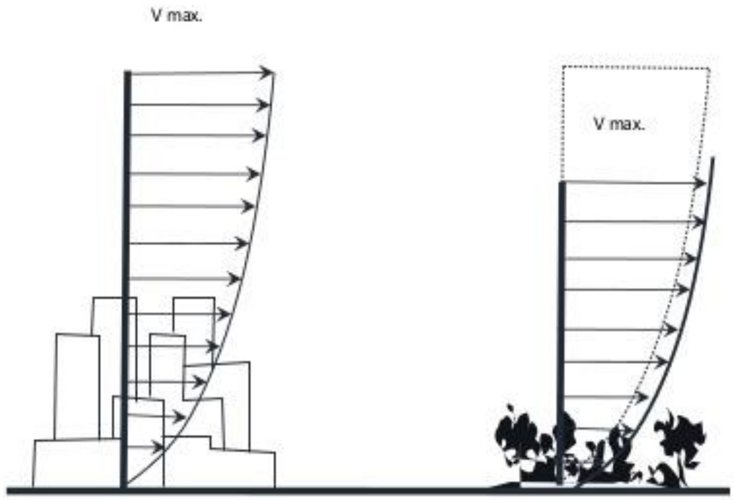


Wind Load:

Wind load has the ability to bring a building to sway.



Wind velocity increases with the increase of height.



Variation of wind velocity with height

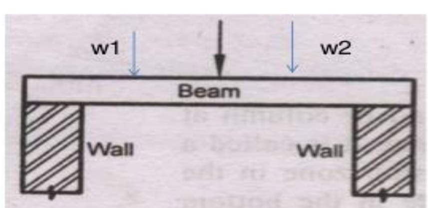
Types of Loads acting on Beams

1) **Point load(concentrated load)-** w1 and w2 are point loads.

➔the load concentrated at one point is called point load.

➔Unit of point load is n or kn.

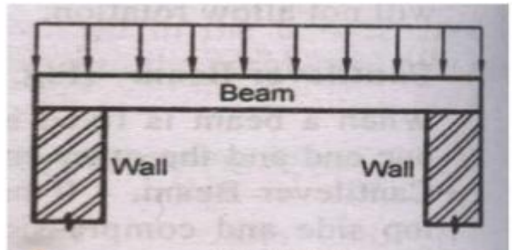
➔eg. 20 kn,100kn,60n,etc



(i) CONCENTRATED LOAD

2) **Uniformly Distributed Load(U.D.L)-** Load uniformly distributed on certain length of beam is called uniformly distributed load.

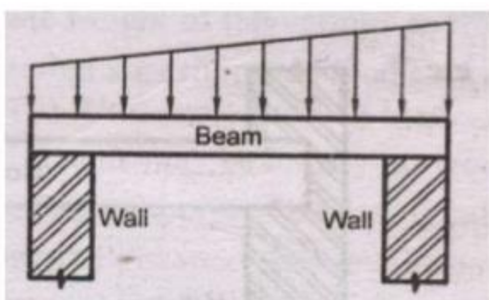
- ➔it is written as u.d.l
- ➔unit of u.d.l is kn/m or n/m.



(ii) U.D.L.

3) Uniformly Varying Load (U.V.L)-

this type of load is gradually increase Or decrease on the length of the beam. it is also called triangular load.



(iii) UNIFORMLY VARYING LOAD

Definition of Shear Force & Bending Moment:

Shear Force:-

It is the algebraic sum of the vertical forces acting to the left or right of a cut section along the span of the beam

- Unit of S.F is N or kN

Bending Moment:-

It is the algebraic sum of the moment of the forces to the left or to the right of the section taken about the section

- Unit of B.M is N.m or kN.m

SF is to be calculated on both (left and right) sides of the point that you are interested in to finally get your shear force diagram .

Left side : force directed up is positive & down is negative

Right side : force directed up is negative and down is positive .

What I prefer for SF is imagining a moment created by the force ... if *clockwise take positive value of force and vice versa*.

BM

Here the value on left as well as right is same ..so depending on saving the complexity of calculations you can prefer any side for BM diagram.

Force going up is positive

Going down is negative .

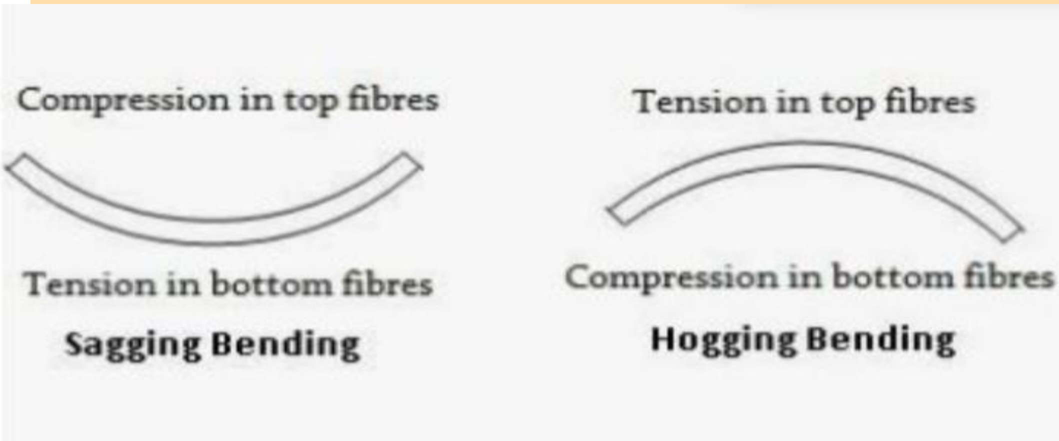
If there is a moment acting ...add according to basic moment conventions. (*CW negative & CCW Positive*)

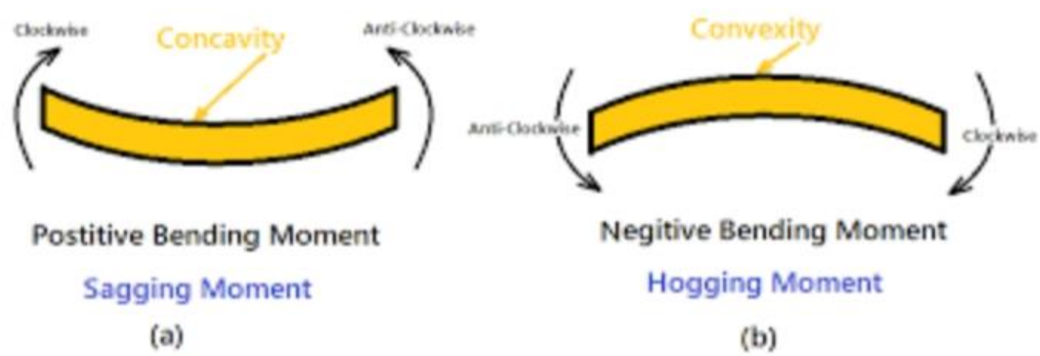
POINT OF CONTRAFLEXURE:

In B.M diagram, The point at which B.M change its sign from positive to negative or negative to positive is called point of contraflexure.

It is a point where the beam tends to bend in opposite direction. It is the point at which curvature of beam changes.

	Effects	Action
Loading	Shear Force	Design Shear reinforcement
Loading	Bending Moment	Design flexure reinforcement





13.8. Cantilever with a Point Load at its Free End

Consider a *cantilever AB of length l and carrying a point load W at its free end B as shown in Fig. 13.2 (*a*). We know that shear force at any section X, at a distance x from the free end, is equal to the total unbalanced vertical force. *i.e.*,

$$F_x = -W \qquad \text{...(Minus sign due to right downward)}$$

* It is a beam fixed at one end and free at the other.

and bending moment at this section,

$$M_x = -W \cdot x \qquad \text{...(Minus sign due to hogging)}$$

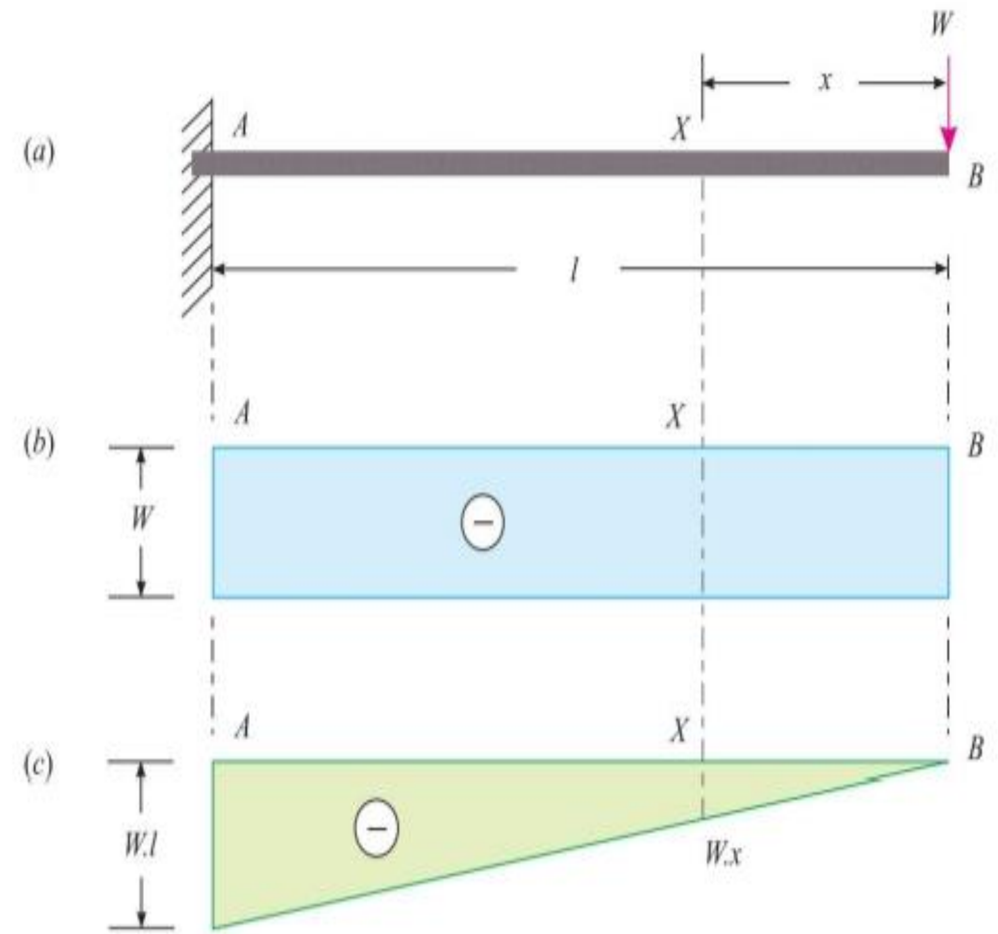


Fig. 13.2. Cantilever with a point load

Thus from the equation of shear force, we see that the shear force is constant and is equal to – *W* at all sections between *B* and *A*. And from the bending moment equation, we see that the bending moment is zero at *B* (where $x = 0$) and increases by a straight line law to – *Wl*; . at (where $x = l$). Now draw the shear force and bending moment diagrams as shown in Fig. 13.2 (*b*) and 13.2 (*c*) respectively.

EXAMPLE 13.1. *Draw shear force and bending moment diagrams for a cantilever beam of span 1.5 m carrying point loads as shown in Fig. 13.3 (a).*

SOLUTION. Given : Span (l) = 1.5 m ; Point load at B (W_1) = 1.5 kN and point load at C (W_2) = 2 kN.

Shear force diagram

The shear force diagram is shown in Fig. 13.3 (*b*) and the values are tabulated here:

$$F_B = -W_1 = -1.5 \text{ kN}$$

$$F_C = -(1.5 + W_2) = -(1.5 + 2) = -3.5 \text{ kN}$$

$$F_A = -3.5 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.3 (*c*) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -[1.5 \times 0.5] = -0.75 \text{ kN-m}$$

$$M_A = -[(1.5 \times 1.5) + (2 \times 1)] = -4.25 \text{ kN-m}$$

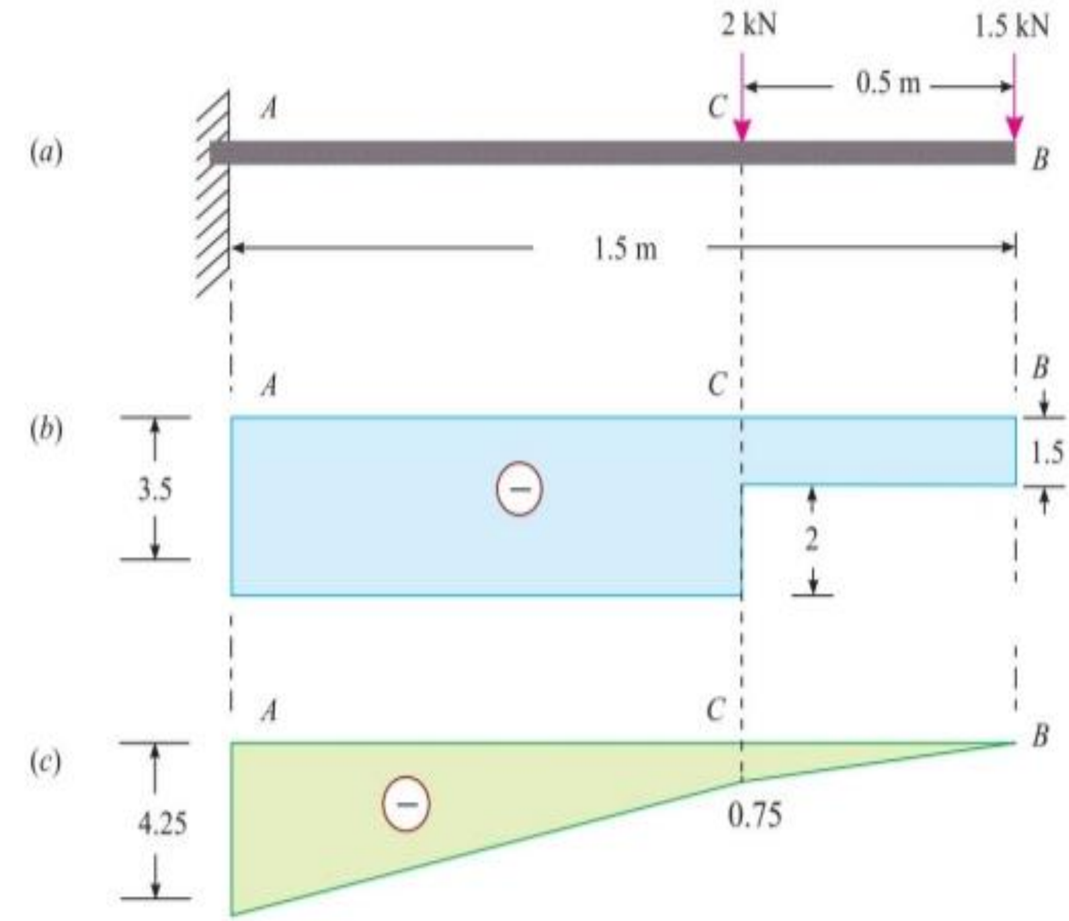


Fig. 13.3

13.9. Cantilever with a Uniformly Distributed Load

Consider a cantilever *AB* of length *l* and carrying a uniformly distributed load of *w* per unit length, over the entire length of the cantilever as shown in Fig. 13.4 (*a*).

We know that shear force at any section *X*, at a distance *x* from *B*,

$$F_x = -w \cdot x \qquad \text{... (Minus sign due to right downwards)}$$

Thus we see that shear force is zero at *B* (where $x = 0$) and increases by a straight line law to $-wl$ at *A* as shown in Fig. 13.4 (*b*).

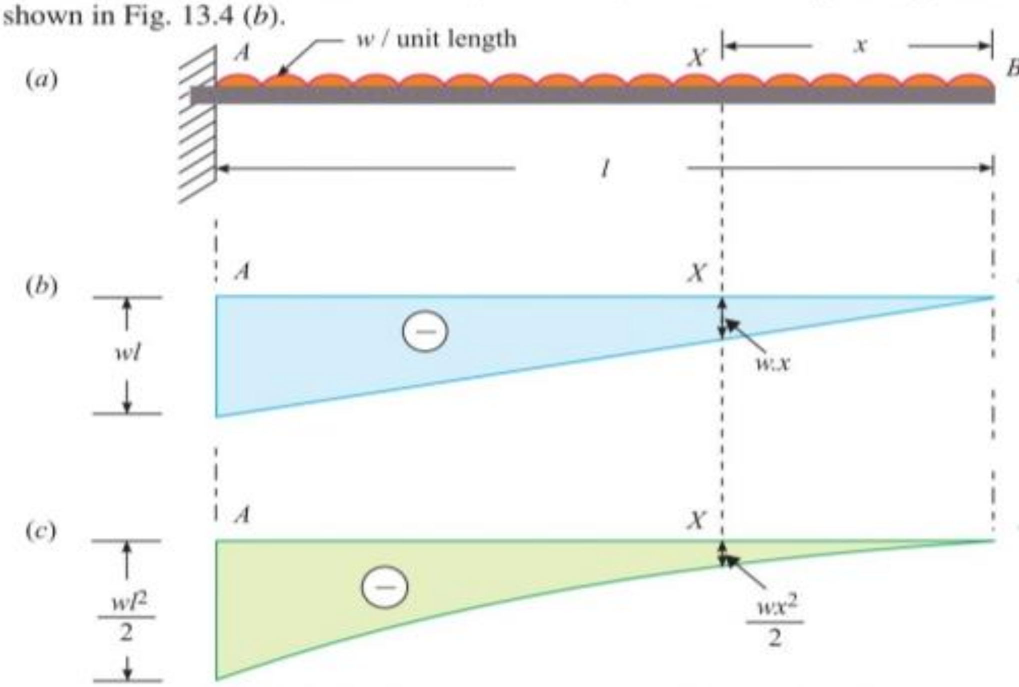


Fig. 13.4. Cantilever with a uniformly distributed load

We also know that bending moment at X,

$$M_x = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2} \qquad \text{...(Minus sign due to hogging)}$$

Thus we also see that the bending moment is zero at B (where $x = 0$) and increases in the form of a parabolic curve to $-\frac{wl^2}{2}$ at B (where $x = 1$) as shown in Fig. 13.4 (c).

EXAMPLE 13.2. *A cantilever beam AB, 2 m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam.*

SOLUTION. Given : span (l) = 2 m ; Uniformly distributed load (w) = 1.5 kN/m and length of the cantilever CB carrying load (a) = 1.6 m.

Shear force diagram

The shear force diagram is shown in Fig. 13.5 (b) and the values are tabulated here:

$$F_B = 0$$

$$F_C = -w \cdot a = -1.5 \times 1.6 = -2.4 \text{ kN}$$

$$F_A = -2.4 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.5 (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -\frac{wa^2}{2} = \frac{1.5 \times (1.6)^2}{2} = -1.92 \text{ kN-m}$$

$$M_A = -\left[(1.5 \times 1.6)\left(0.4 + \frac{1.6}{2}\right)\right] = -2.88 \text{ kN-m}$$

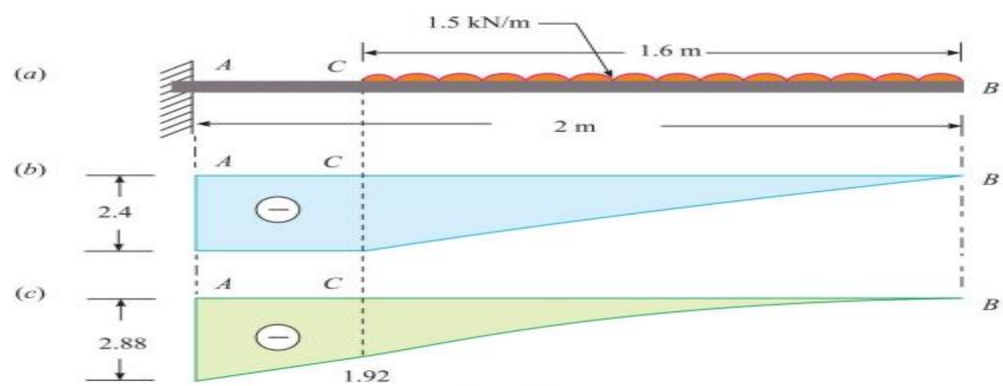


Fig. 13.5

NOTE. The bending moment at A is the moment of the load between C and B (equal to $1.5 \times 1.6 = 2.4$ kN) about A. The distance between the centre of the load and A is $0.4 + \frac{1.6}{2} = 1.2$ m.

EXAMPLE 13.3. *A cantilever beam of 1.5 m span is loaded as shown in Fig. 13.6 (a). Draw the shear force and bending moment diagrams.*

SOLUTION. Given : Span (l) = 1.5 m ; Point load at B (W) = 2 kN ; Uniformly distributed load (w) = 1 kN/m and length of the cantilever AC carrying the load (a) = 1 m.

Shear force diagram

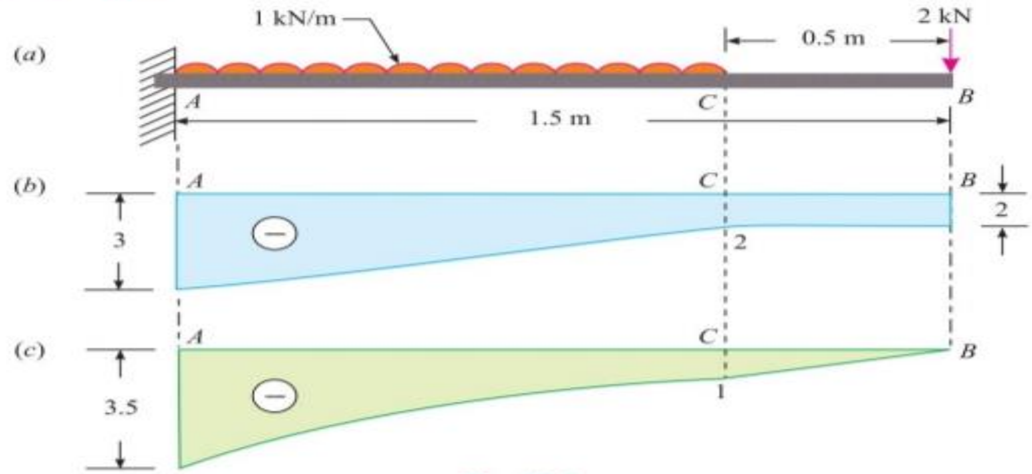


Fig. 13.6

The shear force diagram is shown in Fig. 13.6 (b) and the values are tabulated here:

$$F_B = -W = -2 \text{ kN}$$
$$F_C = -2 \text{ kN}$$
$$F_A = -[2 + (1 \times 1)] = -3 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.6 (c) and the values are tabulated here:

$$M_B = 0$$
$$M_C = -[2 \times 0.5] = -1 \text{ kN-m}$$
$$M_A = -\left[(2 \times 1.5) + (1 \times 1) \times \frac{1}{2}\right] = -3.5 \text{ kN-m}$$

1. A cantilever beam 2 m long carries a point load of 1.8 kN at its free end. Draw shear force and bending moment diagrams for the cantilever. [**Ans.** $F_{max} = -1.8$ kN ; $M_{max} = -3.6$ kN-m]
2. A cantilever beam 1.5 m long carries point loads of 1 kN, 2 kN and 3 kN at 0.5 m, 1.0 m and 1.5 m from the fixed end respectively. Draw the shear force and bending moment diagrams for the beam. [**Ans.** $F_{max} = -6$ kN ; $M_{max} = -7$ kN-m]
3. A cantilever beam of 1.4 m length carries a uniformly distributed load of 1.5 kN/m over its entire length. Draw S.F. and B.M. diagrams for the cantilever. [**Ans.** $F_{max} = -2.1$ kN ; $M_{max} = -1.47$ kN-m]
4. A cantilever *AB* 1.8 m long carries a point load of 2.5 kN at its free end and a uniformly distributed load of 1 kN/m from *A* to *B*. Draw the shear force the bending moment diagrams for the beam. [**Ans.** $F_{max} = -4.3$ kN ; $M_{max} = -6.12$ kN-m]
5. A cantilever 1.5 m long is loaded with a uniformly distributed load of 2 kN/m and a point load of 3 kN as shown in Fig. 13.11

13.11. Simply Supported Beam with a Point Load at its Mid-point

Consider a *simply supported beam *AB* of span *l* and carrying a point load *W* at its mid-point *C* as shown in Fig. 13.12 (*a*). Since the load is at the mid-point of the beam, therefore the reaction at the support *A*,

$$R_A = R_B = 0.5 \text{ W}$$

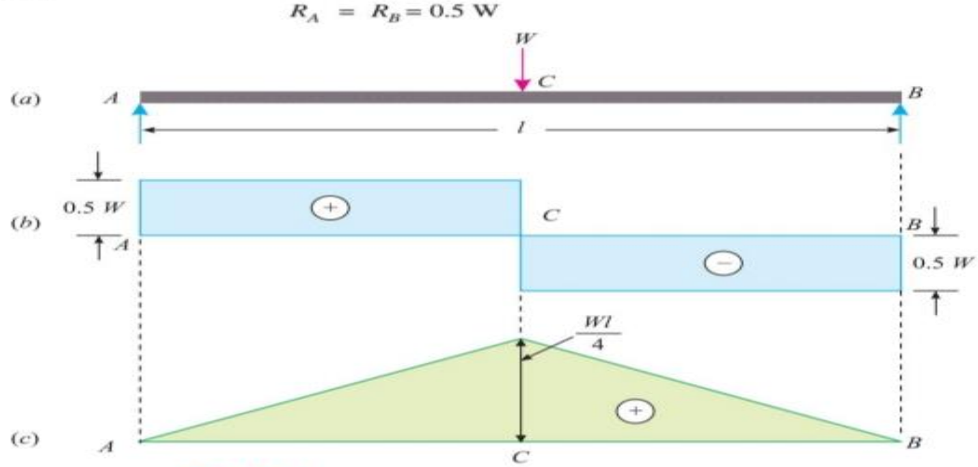


Fig. 13.12. Simply supported beam with a point load

Thus we see that the shear force at any section between *A* and *C* (*i.e.*, up to the point just before the load *W*) is constant and is equal to the unbalanced vertical force, *i.e.*, + 0.5 *W*. Shear force at any section between *C* and *B* (*i.e.*, just after the load *W*) is also constant and is equal to the unbalanced vertical force, *i.e.*, − 0.5 *W* as shown in Fig. 13.12 (*b*).

We also see that the bending moment at *A* and *B* is zero. It increases by a straight line law and is maximum at centre of beam, where shear force changes sign as shown in Fig. 13.12 (*c*).

* It is beam supported or resting freely on the walls or columns on both ends.

Therefore bending moment at *C*,

$$M_C = \frac{W}{2} \times \frac{1}{2} = \frac{Wl}{4} \quad \text{...(Plus sign due to sagging)}$$

Note. If the point load does not act at the mid-point of the beam, then the two reactions are obtained and the diagrams are drawn as usual.

EXAMPLE 13.6. *A simply supported beam AB of span 2.5 m is carrying two point loads as shown in Fig. 13.13.*

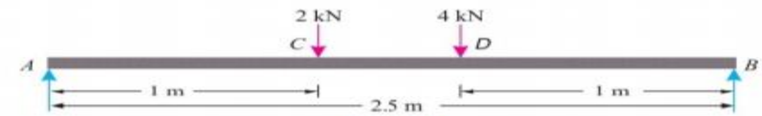


Fig. 13.13

Draw the shear force and bending moment diagrams for the beam.

SOLUTION. Given : Span (l) = 2.5 m ; Point load at C (W_1) = 2 kN and point load at B (W_2) = 4 kN.

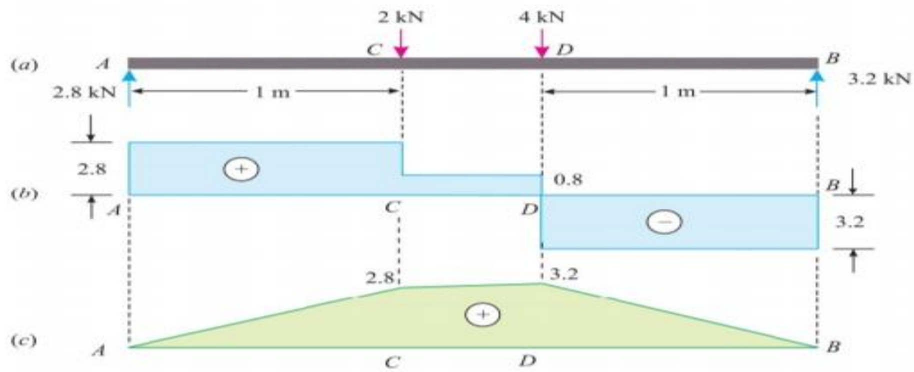


Fig. 13.14

First of all let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 2.5 = (2 \times 1) + (4 \times 1.5) = 8$$

$$R_B = 8/2.5 = 3.2 \text{ kN}$$

and

$$R_A = (2 + 4) - 3.2 = 2.8 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Fig. 13.14 (b) and the values are tabulated here:

$$F_A = +R_A = 2.8 \text{ kN}$$

$$F_C = +2.8 - 2 = 0.8 \text{ kN}$$

$$F_D = 0.8 - 4 = -3.2 \text{ kN}$$

$$F_B = -3.2 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.14 (c) and the values are tabulated here:

$$M_A = 0$$

$$M_C = 2.8 \times 1 = 2.8 \text{ kN-m}$$

$$M_D = 3.2 \times 1 = 3.2 \text{ kN-m}$$

$$M_B = 0$$

NOTE. The value of M_D may also be found and from the reaction R_A. *i.e.*,

$$M_D = (2.8 \times 1.5) - (2 \times 0.5) = 4.2 - 1.0 = 3.2 \text{ kN-m}$$

13.12. Simply Supported Beam with a Uniformly Distributed Load

Consider a simply supported beam *AB* of length *l* and carrying a uniformly distributed load of *w* per unit length as shown in Fig. 13.15. Since the load is uniformly distributed over the entire length of the beam, therefore the reactions at the supports *A*,

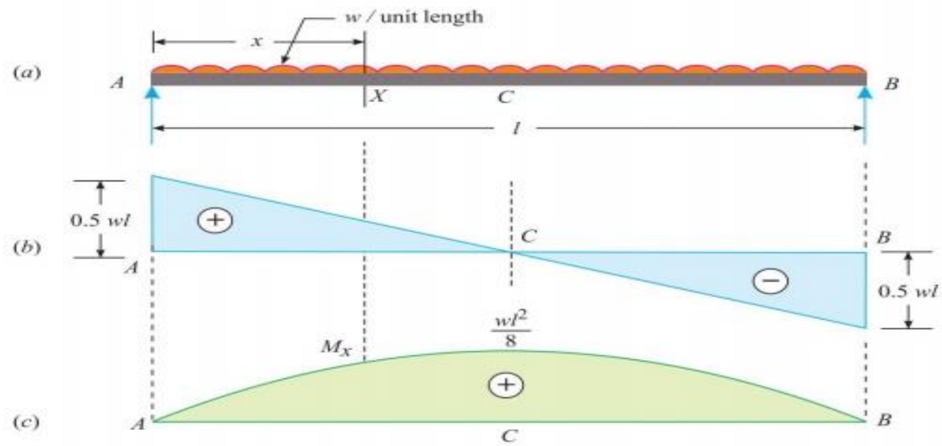


Fig. 13.15. Simply supported beam with a uniformly distributed load

$$R_A = R_B = \frac{wl}{2} = 0.5\, wl$$

We know that shear force at any section *X* at a distance *x* from *A*,

$$F_x = R_A - wx = 0.5\, wl - wx$$

We see that the shear force at *A* is equal to $R_A = 0.5\, wl$, where $x = 0$ and decreases uniformly by a straight line law, to zero at the mid-point of the beam ; beyond which it continues to decrease uniformly to $-0.5\, wl$ at *B* *i.e.*, R_B as shown in Fig. 13.15 (*b*). We also know that bending moment at any section at a distance *x* from *A*,

$$M_x = R_A \cdot x - \frac{wx^2}{2} = \frac{wl}{2}x - \frac{wx^2}{2}$$

We also see that the bending moment is zero at A and B (where $x = 0$ and $x = l$) and increases in the form of a parabolic curve at C, *i.e.*, mid-point of the beam where shear force changes sign as shown in Fig. 13.15 (*c*). Thus bending moment at C,

$$M_C = \frac{wl}{2}\left(\frac{l}{2}\right) - \frac{w}{2}\left(\frac{l}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}$$

EXAMPLE 13.7. *A simply supported beam 6 m long is carrying a uniformly distributed load of 5 kN/m over a length of 3 m from the right end. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.*

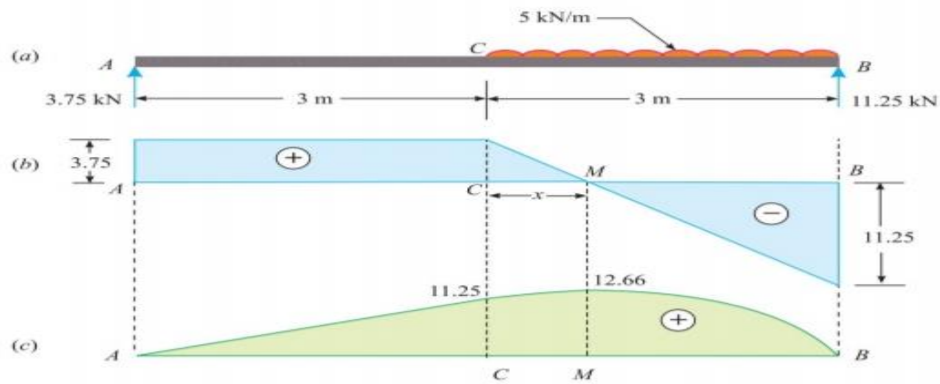


Fig. 13.16

SOLUTION. Given : Span (l) = 6 m ; Uniformly distributed load (w) = 5 kN/m and length of the beam CB carrying load (a) = 3 m.

First of all, let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 6 = (5 \times 3) \times 4.5 = 67.5$$

$\therefore$ $$R_B = \frac{67.5}{6} = 11.25 \text{ kN}$$

and $$R_A = (5 \times 3) - 11.25 = 3.75 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Fig. 13.16 (*b*) and the values are tabulated here:

$$F_A = + R_A = + 3.75 \text{ kN}$$
$$F_C = + 3.75 \text{ kN}$$
$$F_B = + 3.75 - (5 \times 3) = - 11.25 \text{ kN}$$

Bending moment diagram

The bending moment is shown in Fig. 13.16 (*c*) and the values are tabulated here:

$$M_A = 0$$
$$M_C = 3.75 \times 3 = 11.25 \text{ kN}$$
$$M_B = 0$$

We know that the maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between C and M. From the geometry of the figure between C and B, we find that

$$\frac{x}{3.75} = \frac{3-x}{11.25} \quad \text{or} \quad 11.25\,x = 11.25 - 3.75\,x$$

$$15\,x = 11.25 \quad \text{or} \quad x = 11.25/15 = 0.75 \text{ m}$$

$\therefore$ $$M_M = 3.75 \times (3 + 0.75) - 5 \times \frac{0.75}{2} = 12.66 \text{ kN-m}$$

EXAMPLE 13.8. *A simply supported beam 5 m long is loaded with a uniformly distributed load of 10 kN/m over a length of 2 m as shown in Fig. 13.17.*

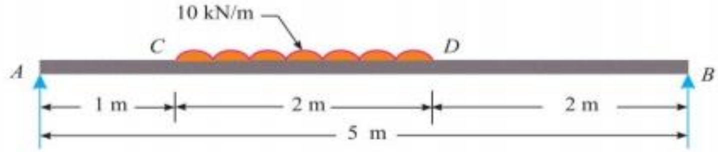


Fig. 13.17

Draw shear force and bending moment diagrams for the beam indicating the value of maximum bending moment.

SOLUTION. Given : Span (l) = 5 m ; Uniformly distributed load (w) = 10 kN/m and length of the beam CD carrying load (a) = 2 m.

First of all, let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 5 = (10 \times 2) \times 2 = 40$$

$\therefore$ $$R_B = 40/5 = 8 \text{ kN}$$

and $$R_A = (10 \times 2) - 8 = 12 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Fig. 13.18 (b) and the values are tabulated here:

$$F_A = +R_A = +12 \text{ kN}$$
$$F_C = +12 \text{ kN}$$
$$F_D = +12 - (10 \times 2) = -8 \text{ kN}$$
$$F_B = -8 \text{ kN}$$

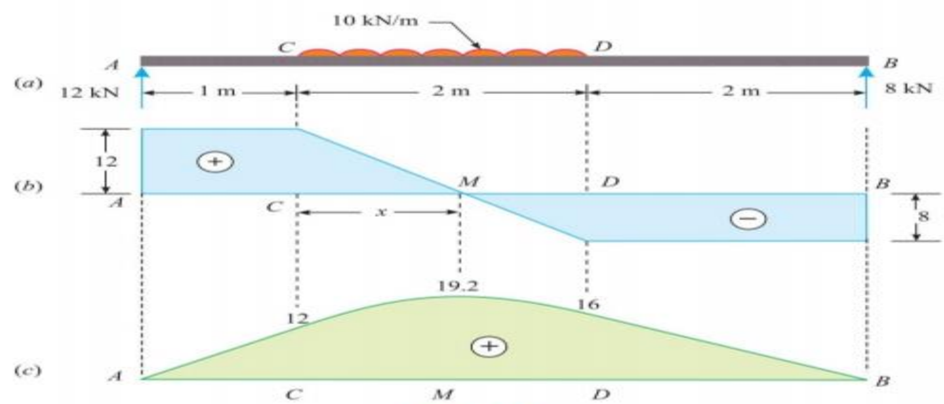


Fig. 13.18

Bending moment diagram

The bending moment diagram is shown in Fig. 13.18 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = 12 \times 1 = 12 \text{ kN-m}$$
$$M_D = 8 \times 2 = 16 \text{ kN-m}$$

We know that maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between C and M. From the geometry of the figure between C and D, we find that

$$\frac{x}{12} = \frac{2-x}{8} \quad \text{or} \quad 8x = 24 - 12x$$

$$20x = 24 \quad \text{or} \quad x = 24/20 = 1.2 \text{ m}$$

$$M_M = 12(1 + 1.2) - 10 \times 1.2 \times \frac{1.2}{2} = 19.2 \text{ kN-m}$$

EXAMPLE 13.9. *A simply supported beam of 4 m span is carrying loads as shown in Fig. 13.19.*

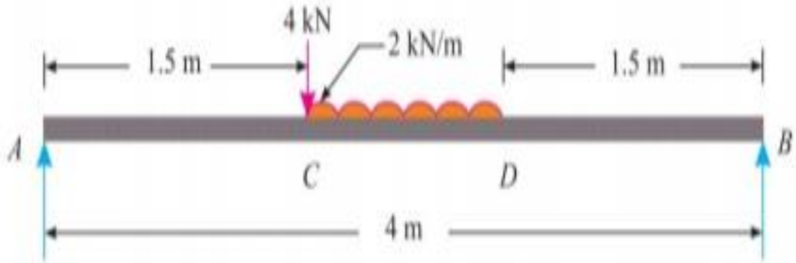


Fig. 13.19

Draw shear force and bending moment diagrams for the beam.

Solution. Given : Span (l) = 4 m ; Point load at C (W) = 4 kN and uniformly distributed load between C and D (w) = 2 kN/m.

First of all, let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 4 = (4 \times 1.5) + (2 \times 1) \times 2 = 10$$

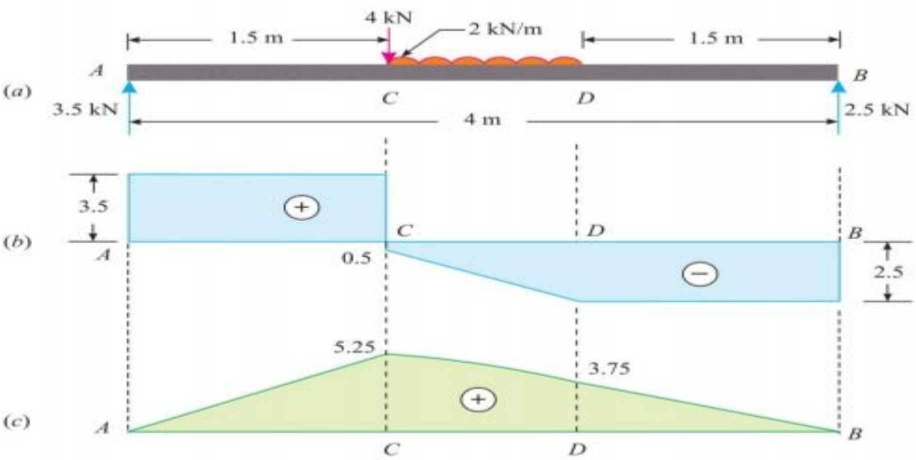


Fig. 13.20

$$R_B = 10/4 = 2.5 \text{ kN}$$

and $$R_A = 4 + (2 \times 1) - 2.5 = 3.5 \text{ kN}$$

Shear force diagram

The shear force diagram is shown in Fig. 13.20 (b) and the values are tabulated here:

$$F_A = +R_A = +3.5 \text{ kN}$$
$$F_C = +3.5 - 4 = -0.5 \text{ kN}$$
$$F_D = -0.5 - (2 \times 1) = -2.5 \text{ kN}$$
$$F_B = -2.5 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.20 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = 3.5 \times 1.5 = 5.25 \text{ kN-m}$$
$$M_D = 2.5 \times 1.5 = 3.75 \text{ kN-m}$$
$$M_B = 0$$

We know that the maximum bending moment will occur at C, where the shear force changes sign, *i.e.*, at C as shown in the figure.

EXAMPLE 13.10. *A simply supported beam AB, 6 m long is loaded as shown in Fig. 13.21.*

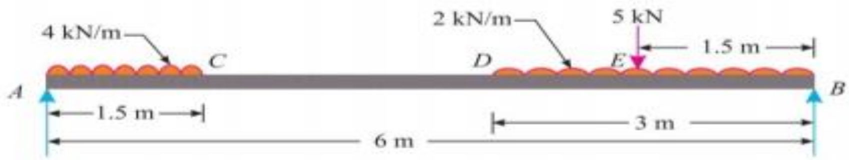


Fig. 13.21

Construct the shear force and bending moment diagrams for the beam and find the position and value of maximum bending moment.

SOLUTION. Given : Span (l) = 6 m ; Point load at E (W) = 5 kN ; Uniformly distributed load between A and C (w_1) = 4 kN/m and uniformly distributed load between D and B = 2 kN/m.

First of all, let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 6 = (4 \times 1.5 \times 0.75) + (2 \times 3 \times 4.5) + (5 \times 4.5) = 54$$

$$R_B = 54/6 = 9 \text{ kN}$$

and

$$R_A = (4 \times 1.5) + (2 \times 3) + 5 - 9 = 8 \text{ kN}$$

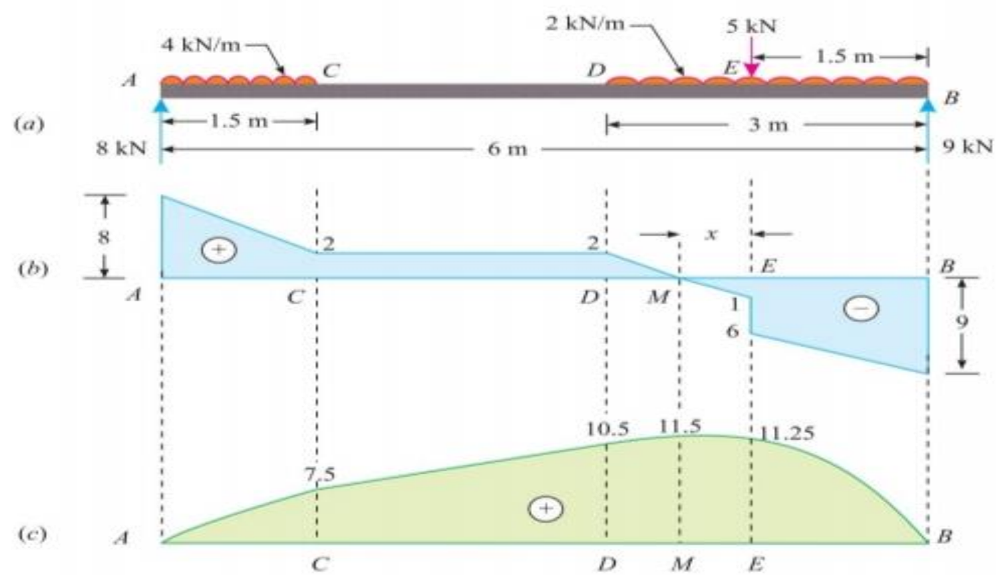


Fig. 13.22

Shear force diagram

The shear force diagram is shown in Fig. 13.22 (*b*) and the values are tabulated here:

$$F_A = +R_A = +8 \text{ kN}$$
$$F_C = 8 - (4 \times 1.5) = 2 \text{ kN}$$
$$F_D = 2 \text{ kN}$$
$$F_E = 2 - (2 \times 1.5) - 5 = -6 \text{ kN}$$
$$F_B = -6 - (2 \times 1.5) = -9 \text{ kN}$$

Bending moment diagram

The bending moment diagram is shown in Fig. 13.22 (c) and the values are tabulated here:

$$M_A = 0$$
$$M_C = (8 \times 1.5) - (4 \times 1.5 \times 0.75) = 7.5 \text{ kN-m}$$
$$M_D = (8 \times 3) - (4 \times 1.5 \times 2.25) = 10.5 \text{ kN-m}$$
$$M_E = (9 \times 1.5) - (2 \times 1.5 \times 0.75) = 11.25 \text{ kN-m}$$
$$M_B = 0$$

We know that maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between E and M. From the geometry of the figure between D and E, we find that

$$\frac{x}{1} = \frac{1.5 - x}{2} \quad \text{or} \quad 2x = 1.5 - x$$
$$3x = 1.5 \quad \text{or} \quad x = 1.5/3 = 0.5 \text{ m}$$
$$\therefore \quad M_M = 9(1.5 + 0.5) - (2 \times 2 \times 1) - (5 \times 0.5) = 11.5 \text{ kN-m}$$

1. A simply supported beam of 3 m span carries two loads of 5 kN each at 1 m and 2 m from the left hand support. Draw the shear force and bending moment diagrams for the beam.

[**Ans.** M_{max} = 5 kN-m]

2. A simply supported beam of span 4.5 m carries a uniformly distributed load of 3.6 kN/m over a length of 2 m from the left end *A*. Draw the shear force and bending moment diagrams for the beam. [**Ans.** M_{max} = 4.36 kN-m at 1.56 m from *A*]

3. A simply supported beam *ABCD* is of 5 m span, such that *AB* = 2 m, *BC* = 1 m and *CD* = 2 m. It is loaded with 5 kN/m over *AB* and 2 kN/m over *CD*. Draw shear force and bending moment diagrams for the beam. [**Ans.** M_{max} = 7.74 kN-m at 1.76 m from *A*]

4. Draw shear force and bending moment diagrams for a simply supported beam, loaded as shown in Fig. 13.28.

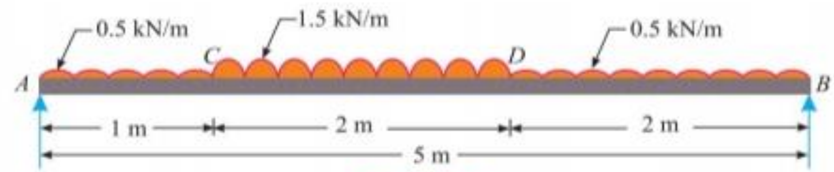


Fig. 13.28

Find the position and value of the maximum bending moment that will occur in the beam.

[**Ans.** 3.47 kN-m at 1.3 m from *C*]

5. A simply supported beam *AB*, 6 m long is loaded as shown in Fig. 13.29.

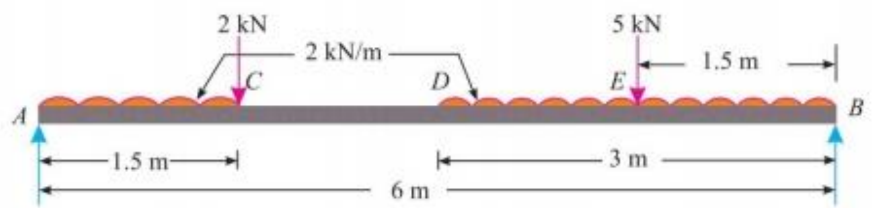


Fig. 13.29

Draw the shear force and bending moment diagrams for the beam.

[*Ans.* M_{max} = 11.75 kN-m at 0.56 m from *E*]

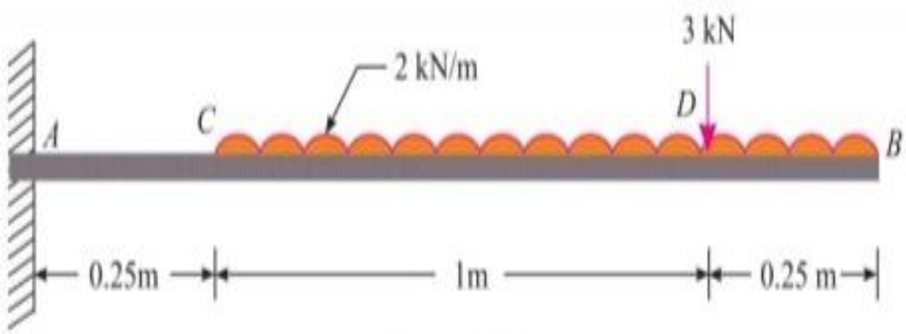


Fig. 13.11

Draw the shear force and bending moment diagrams for the cantilever.

[**Ans.** F_{max} = – 5.5 kN ; M_{max} = – 5.94 kN-m]

CHAPTER - 3

3.Bending stress in beams.

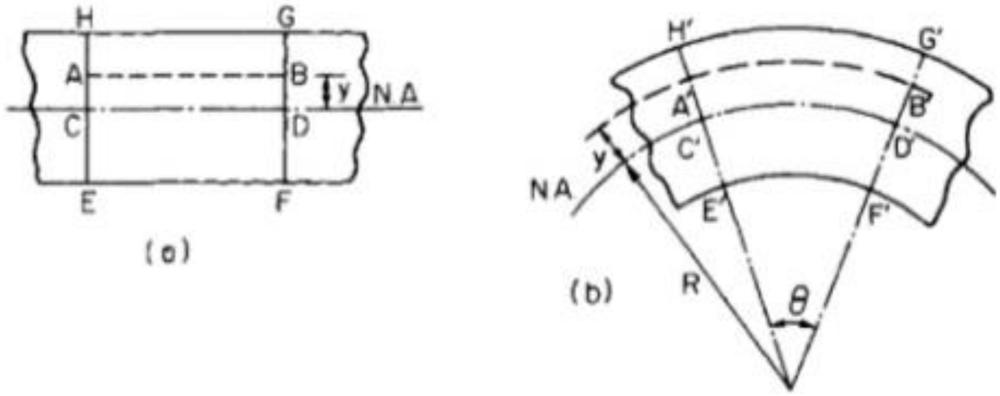
. INTRODUCTION

When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses with certain assumptions can be calculated. The stresses introduced by bending moment are known as *bending stresses.* In this chapter, the theory of pure bending, expression for bending stresses, bending stress in symmetrical and unsymmetrical sections, strength of a beam and composite beams will be discussed.

E.g., Consider a piece of rubber, most conveniently of rectangular cross-section, is bent between one's fingers it is readily apparent that one surface of the rubber is stretched, i.e. put into tension, and the opposite surface is compressed.

SIMPLE BENDING

A theory which deals with finding stresses at a section due to pure moment is called bending theory. If we now consider a beam initially unstressed and subjected to a constant B.M. along its length, it will bend to a radius R as shown in Fig. b. As a result of this bending the top fibres of the beam will be subjected to tension and the bottom to compression. Somewhere between the two surfaces, there are points at which the stress is zero. The locus of all such points is termed the neutral axis (N.A). The radius of curvature R is then measured to this axis. For symmetrical sections the N.A. is the axis of symmetry, but whatever the section the N.A. will always pass through the centre of area or centroid.



Beam subjected to pure bending (a) before, and (b) after, the moment *M* has been applied.

In simple bending the plane of transverse loads and the centroidal plane coincide. The theory of simple bending was developed by Galelio, Bernoulli and St. Venant. Sometimes this theory is called Bernoulli's theory of simple bending.

· ASSUMPTIONS IN SIMPLE BENDING

The following assumptions are made in the theory of simple bending:

1 The beam is initially straight and unstressed.

2 The material of the beam is perfectly homogeneous and isotropic, i.e. of the same density and elastic properties throughout.

3 The elastic limit is nowhere exceeded.

4 Young's modulus for the material is the same in tension and compression.

5 Plane cross-sections remain plane before and after bending.

6 Every cross-section of the beam is symmetrical about the plane of bending, i.e. about an axis perpendicular to the N.A.

7 There is no resultant force perpendicular to any cross-section.

8 The radius of curvature is large compared to depth of beam.

.. DERIVATION OF BENDING EQUATION

Consider a length of beam under the action of a bending moment M as shown in Fig. 6.2a. N-N is the original length considered of the beam. The neutral surface is a plane through X-X. In the side view NA indicates the neutral axis. O is the centre of curvature on bending (Fig. 6.2b).

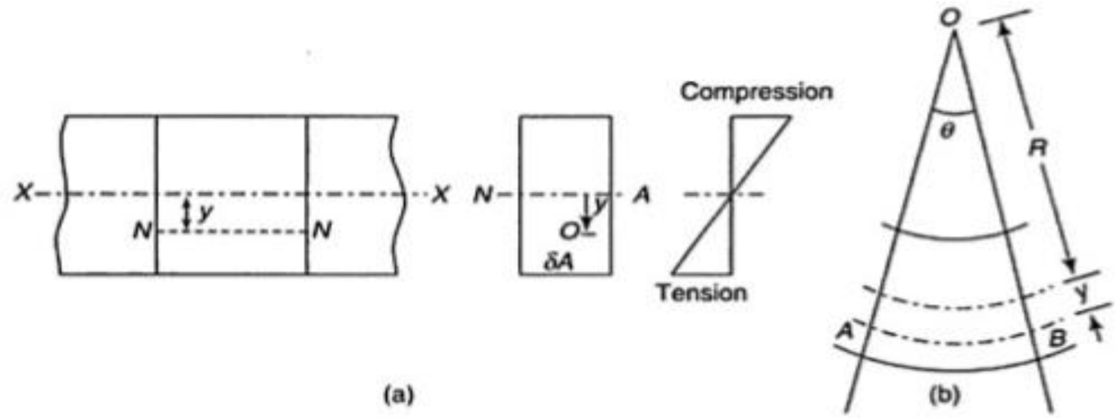


Fig. 6.2

Let R = radius of curvature of the neutral surface

θ = angle subtended by the beam length at centre O

σ = longitudinal stress

A filament of original length NN at a distance v from the neutral axis will be elongated to a length AB

$$\text{The strain in } AB = \frac{AB - NN}{NN}$$

$$\frac{\sigma}{E} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = y\frac{E}{R} \propto y \quad ..(i)$$

Thus stress is proportional to the distance from the neutral axis NA. This suggests that for the sake of weight reduction and economy, it is always advisable to make the cross-section of beams such that most of the material is concentrated at the greatest distance from the neutral axis. Thus there is universal adoption of the I-section for steel beams. Now let δA be an element of cross-sectional area of a transverse plane at a distance v from the neutral axis NA (Fig. 6.2).

For pure bending, Net normal force on the cross-section = 0

$$\int \sigma \cdot dA = 0$$

$$\int \frac{E}{R} y \cdot dA = 0 \text{ or } \frac{E}{R}\int y \cdot dA = 0$$

$$\int y \cdot dA = 0$$

This indicates the condition that the neutral axis passes through the centroid of the section. Also, bending moment = moment of the normal forces about neutral axis

$$M = \int (\sigma \cdot dA) y = \int \frac{E}{R} y \cdot dA \cdot y = \frac{E}{R} \int y^2 \cdot dA$$

$$= \frac{EI}{R}.$$

Or

$$\frac{M}{I} = \frac{E}{R} \qquad \text{(ii)}$$

Where $I = \int y^2 dA$ and is known as the *moment of inertia* or *second moment of area* of the section. From (i) and (ii),

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

pure bending equation

Where,

M = Bending Moment at a section (N-mm).

I = Moment of Inertia of the cross section of the beam about Neutral axis (mm^4).

σ = Bending stress in a fibre located at distance y from neutral axis (N/mm^2). This stress could be compressive stress or tensile stress depending on the location of the fibre.

y = Distance of the fibre under consideration from neutral axis (mm).

E = Young's Modulus of the material of the beam (N/mm^2).

R = Radius of curvature of the bent beam (mm).

PROBLEMS:

EXAMPLE 14.1. *A steel wire of 5 mm diameter is bent into a circular shape of 5 m radius. Determine the maximum stress induced in the wire. Take E = 200 GPa.*

SOLUTION. Given : Diameter of steel wire $(d) = 5$ mm ; Radius of circular shape $(R) = 5 \text{ m} = 5 \times 10^3$ mm and modulus of elasticity $(E) = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$.

We know that distance between the neutral axis of the wire and its extreme fibre,

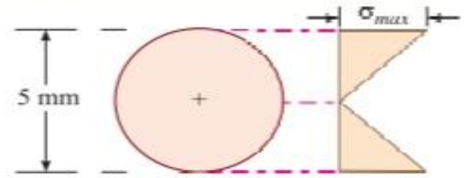


Fig. 14.3

$$y = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}$$

and maximum bending stress induced in the wire,

$$\sigma_{b\,(max)} = \frac{E}{R} \times y = \frac{200 \times 10^3}{5 \times 10^3} \times 2.5 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \qquad \textbf{Ans.}$$

EXAMPLE 14.2. *A copper wire of 2 mm diameter is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80 MPa. Take modulus of elasticity for the copper as 100 GPa.*

SOLUTION. Given : Diameter of wire $(d) = 2$ mm ; Maximum bending stress $\sigma_{b\,(max)} = 80 \text{ MPa} = 80 \text{ N/mm}^2$ and modulus of elasticity $(E) = 100 \text{ GPa} = 100 \times 10^3 \text{ N/mm}^2$.

We know that distance between the neutral axis of the wire and its extreme fibre

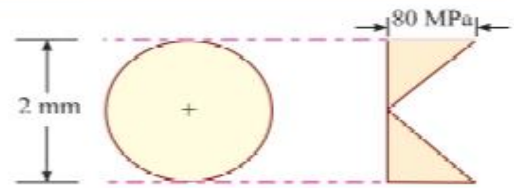


Fig. 14.4

$$y = \frac{2}{2} = 1 \text{ mm}$$

∴ Minimum radius of the drum

$$R = \frac{y}{\sigma_{b(\max)}} \times E = \frac{1}{80} \times 100 \times 10^3 \qquad \ldots\left(\because \frac{\sigma_b}{y} = \frac{E}{R}\right)$$

$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m} \qquad \textbf{Ans.}$$

EXAMPLE 14.3. *A metallic rod of 10 mm diameter is bent into a circular form of radius 6 m. If the maximum bending stress developed in the rod is 125 MPa, find the value of Young's modulus for the rod material.*

SOLUTION. Given : Diameter of rod $(d) = 10$ mm ; Radius $(R) = 6 \text{ m} = 6 \times 10^3$ mm and maximum bending stress $\sigma_{b\,(max)} = 125 \text{ MPa} = 125 \text{ N/mm}^2$.

We know that distance between the neutral axis of the rod and its extreme fibre,

$$y = \frac{10}{2} = 5$$

∴ Value of Young's modulus for the rod material,

$$E = \frac{\sigma_{b(\max)}}{y} \times R = \frac{125}{5} \times (6 \times 10^3) \text{ N/mm}^2 \qquad \ldots\left(\because \frac{\sigma_b}{y} = \frac{E}{R}\right)$$

$$= 150 \times 10^3 \text{ N/mm}^2 = 150 \text{ GPa} \qquad \textbf{Ans.}$$

CHAPTER - 4

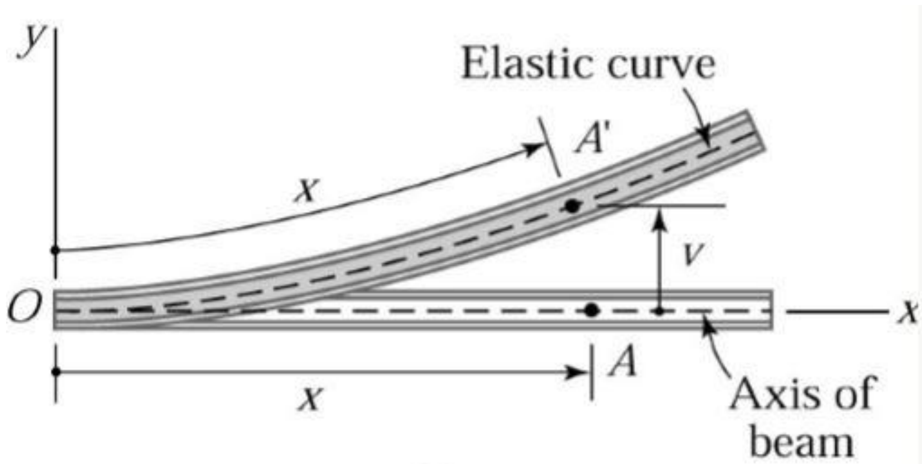
4.Slope and deflection of beams by double integration method

Slope of a Beam: Slope of a beam is the angle between deflected beam to the actual beam at the same point.

Deflection of Beam: Deflection is defined as the vertical displacement of a point on a loaded beam. There are many methods to find out the slope and deflection at a section in a loaded beam.

The maximum deflection occurs where slope is zero. The position of the maximum deflection is found out by equating the slope equation zero. Then the value of x is substituted in the deflection equation to calculate the maximum deflection

Double Integration Method: This is most suitable when concentrated or udl over entire length is acting on the beam.The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.



$$EI\ \frac{d^2y}{dx^2}=-M$$

Integrating one time :

$$EI\ \frac{dy}{dx}=-\int M$$

Integrating again :

$$EI\ y=-\int\int M$$

Where,

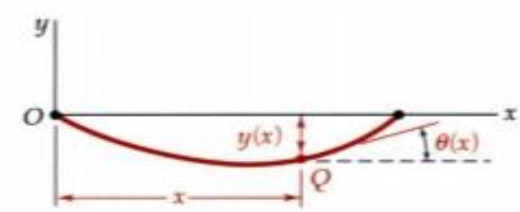
M = Bending moment

I = Moment of inertia of the beam section

y/v = Deflection of the beam

E = Modulus of elasticity of beam material.

Equation of the Elastic Curve



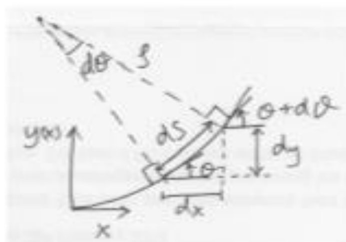
$$\frac{dy}{dx} = \tan\theta$$

$$\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}}$$

$$\frac{d(\tan\theta)}{dx} = \frac{d^2y}{dx^2} = \frac{d(\tan\theta)}{d\theta}\frac{d\theta}{dx} = \left(1+\tan^2\theta\right)\frac{d\theta}{dx} = \left(1+\left(\frac{dy}{dx}\right)^2\right)\frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = \frac{\frac{d^2y}{dx^2}}{1+\left(\frac{dy}{dx}\right)^2}$$

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}$$



- Thus,

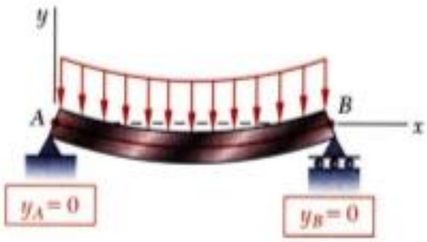
$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}} \approx \frac{d^2y}{dx^2}$$

- Substituting and integrating,

$$EI\frac{1}{\rho} = EI\frac{d^2y}{dx^2} = M(x)$$

$$EI\,\theta \approx EI\frac{dy}{dx} = \int M(x)dx + C_1$$

$$EI\,y = \int dx \int M(x)dx + C_1x + C_2$$



- Constants are determined from boundary conditions

$$EI\ y = \int dx \int M(x)dx + C_1 x + C_2$$

- Three cases for statically determinate beams,
  - Simply supported beam
    $y_A = 0, \quad y_B = 0$
  - Overhanging beam
    $y_A = 0, \quad y_B = 0$
  - Cantilever beam
    $y_A = 0, \quad \theta_A = 0$
- More complicated loadings require multiple integrals and application of requirement for continuity of displacement and slope.

The following relationships exist between loading, shearing force (S.F.), bending moment (B.M.), slope and deflection of a beam:

$$\text{deflection} = y \quad (\text{or } \delta)$$

$$\text{slope} = i \text{ or } \theta = \frac{dy}{dx}$$

$$\text{bending moment} = M = EI\frac{d^2y}{dx^2}$$

$$\text{shearing force} = Q = EI\frac{d^3y}{dx^3}$$

$$\text{loading} = w = EI\frac{d^4y}{dx^4}$$

In order that the above results should agree mathematically the sign convention illustrated in Fig. 5.4 must be adopted.

Using the above formulae the following standard values for *maximum slopes* and *deflections* of simply supported beams are obtained. (These assume that the beam is uniform, i.e. EI is constant throughout the beam.)

MAXIMUM SLOPE AND DEFLECTION OF SIMPLY SUPPORTED BEAMS

Loading condition	Maximum slope	Deflection (y)	Max. deflection (y_{max})
Cantilever with concentrated load W at end	$\frac{WL^2}{2EI}$	$\frac{W}{6EI}[2L^3 - 3L^2x + x^3]$	$\frac{WL^3}{3EI}$
Cantilever with u.d.l. across the complete span	$\frac{wL^3}{6EI}$	$\frac{w}{24EI}[3L^4 - 4L^3x + x^4]$	$\frac{wL^4}{8EI}$
Simply supported beam with concentrated load W at the centre	$\frac{WL^2}{16EI}$	$\frac{Wx}{48EI}[3L^2 - 4x^2]$	$\frac{WL^3}{48EI}$
Simply supported beam with u.d.l. across complete span	$\frac{wL^3}{24EI}$	$\frac{wx}{24EI}[L^3 - 2Lx^2 + x^3]$	$\frac{5wL^4}{384EI}$
Simply supported beam with concentrated load W offset from centre (distance a from one end b from the other)	$0.062\frac{WL^2}{EI}$		$\frac{Wa}{3EIL}\left[\frac{L^2 - a^2}{3}\right]^{3/2}$

19.2. Curvature of the Bending Beam

Consider a beam AB subjected to a bending moment. As a result of loading, let the beam deflect from ABC to ADB into a circular arc as shown in Fig. 19.1.

Let l = Length of the beam AB,

M = Bending moment,

R = Radius of curvature of the bent up beam,

I = Moment of inertia of the beam section,

E = Modulus of elasticity of beam material,

y = Deflection of the beam (*i.e.*, CD) and

i = Slope of the beam (*i.e* angle which the tangent at A makes with AB).

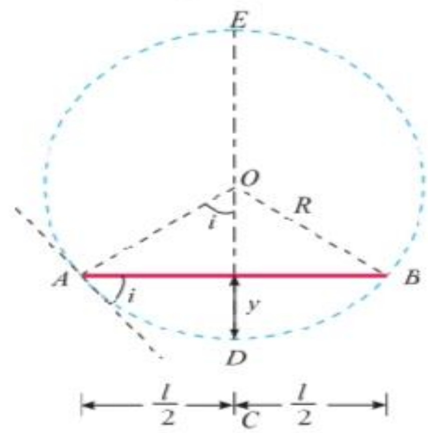


Fig. 19.1. Curvature of the beam.

From the geometry of a circle, we know that

$$AC \times CB = EC \times CD$$

or

$$\frac{l}{2} \times \frac{l}{2} = (2R - y) \times y$$

$$\therefore \quad \frac{l^2}{4} = 2Ry - y^2 = 2Ry \qquad \text{...(Neglecting } y^2\text{)}$$

or

$$y = \frac{l^2}{8R} \qquad ...(i)$$

We have already discussed in Art. 14.6 that for a loaded beam,

$$\frac{M}{I} = \frac{E}{R} \quad \text{or} \quad R = \frac{EI}{M}$$

Now substituting this value of R in equation (*i*),

$$y = \frac{l^2}{8 \times \frac{EI}{M}} = \frac{El^2}{8\,EI}$$

From the geometry of the figure, we find that the slope of the beam i at A or B is also equal to angle AOC.

$$\therefore \quad \sin i = \frac{AC}{OA} = \frac{l}{2R}$$

Since the angle i is very small, therefore, $\sin i$ may be taken equal to i (in radians).

$$\therefore \quad i = \frac{l}{2R} \text{ radians} \qquad ...(ii)$$

Again substituting the value of R in equation (*ii*),

$$i = \frac{l}{2R} = \frac{l}{2 \times \frac{EI}{M}} = \frac{Ml}{2El} \text{ radians} \qquad ...(iii)$$

* As per Indian Standard Specifications, this limit is Span/325.

NOTES: 1. The above equations for deflection (y) and slope (i) have been derived from the bending moment only *i.e.*, the effect of shear force has been neglected. This is due to the reason that the effect of shear force is extremely small as compared to the effect of bending moment.

2. In actual practice the beams bend into an arc of a circle only in a few cases. A little consideration will show that a beam will bend to an arc of a circle only if (i) the beam is of uniform section and (ii) the beam is subjected to a constant moment throughout its length or the beam is of uniform strength.

19.3. Relation between Slope, Deflection and Radius of Curvature

Consider a small portion PQ of a beam, bent into an arc as shown in Fig. 19.2.

Let ds = Length of the beam PQ,

R = Radius of the arc, into which the beam has been bent,

C = Centre of the arc,

Ψ = Angle, which the tangent at P makes with x-x axis and

$\Psi + d\Psi$ = Angle which the tangent at Q makes with x-x axis.

From the geometry of the figure, we find that

$$\angle PCQ = d\Psi$$

and

$$ds = R \,.\, d\Psi$$

$$\therefore \quad R = \frac{ds}{d\Psi} = \frac{dx}{d\Psi} \qquad \text{... (Considering } ds = dx\text{)}$$

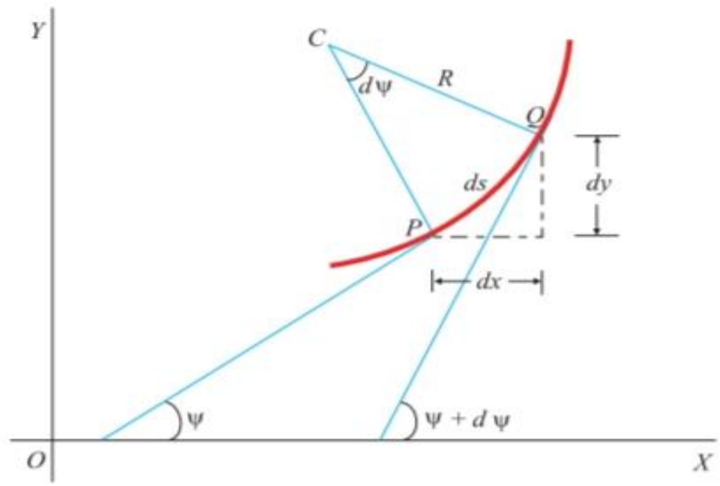


Fig. 19.2. Beam bent into an arc.

or

$$\frac{1}{R} = \frac{d\Psi}{dx} \qquad \text{...}(i)$$

We know that if x and y be the co-ordinates of point P, then

$$\tan \Psi = \frac{dy}{dx}$$

Since Ψ is a very small angle, therefore taking $\tan \Psi = \Psi$,

$$\therefore \quad \frac{d\Psi}{dx} = \frac{d^2y}{dx^2} \qquad \text{...}\left(\because \frac{1}{R} = \frac{d\Psi}{dx}\right)$$

We also know that

$$\frac{M}{I} = \frac{E}{R} \quad \text{or} \quad M = EI \times \frac{I}{R}$$

$$\therefore \quad M = EI \times \frac{d^2 y}{dx^2} \qquad \ldots \left(\text{Substituting value of } \frac{1}{R}\right)$$

NOTE. The above equation is also based only on the bending moment. The effect of shear force, being very small as compared to the bending moment, is neglected.

19.4. Methods for Slope and Deflection at a Section

Though there are many methods to find out the slope and deflection at a section in a loaded beam, yet the following two methods are important from the subject point of view:

1. Double integration method.
2. Macaulay's method.

It will be interesting to know that the first method is suitable for a single load, whereas the second method is suitable for several loads.

19.5. Double Integration Method for Slope and Deflection

We have already discussed in Art. 19.3 that the bending moment at a point,

$$M = EI \frac{d^2 y}{dx^2}$$

Integrating the above equation,

$$EI \frac{dy}{dx} = \int M \qquad \ldots (i)$$

and integrating the above equation once again,

$$EI \,.\, y = \iint M \qquad \ldots (ii)$$

It is thus obvious that after first integration the original differential equation, we get the value of slope at any point. On further integrating, we get the value of deflection at any point.

NOTE. While integrating twice the original differential equation, we will get two constants C_1 and C_2. The values of these constants may be found out by using the end conditions.

Simply Supported beam with a point load problem:

EXAMPLE 19.1. *A simply supported beam of span 3 m is subjected to a central load of 10 kN. Find the maximum slope and deflection of the beam. Take $I = 12 \times 10^6$ mm^4 and E = 200 GPa.*

SOLUTION. Given: Span (l) = 3 m = 3×10^3 mm ; Central load (W) = 10 kN = 10×10^3 N ; Moment of inertia (I) = 12×10^6 mm^4 and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

Maximum slope of the beam

We know that maximum slope of the beam,

$$i_A = \frac{Wl^2}{16EI} = \frac{(10 \times 10^3) \times (3 \times 10^3)^2}{16 \times (200 \times 10^3) \times (12 \times 10^6)} = \mathbf{0.0023\ rad} \qquad \textbf{Ans.}$$

Maximum deflection of the beam

We also know that maximum deflection of the beam,

$$y_C = \frac{Wl^3}{48EI} = \frac{(10 \times 10^3) \times (3 \times 10^3)^3}{48 \times (200 \times 10^3) \times (12 \times 10^6)} = \mathbf{2.3\ mm} \qquad \textbf{Ans.}$$

EXAMPLE 19.2. *A wooden beam 140 mm wide and 240 mm deep has a span of 4 m. Determine the load, that can be placed at its centre to cause the beam a deflection of 10 mm. Take E as 6 GPa.*

SOLUTION. Given: Width (b) = 140 mm ; Depth (d) = 240 mm ; Span (l) = 4 m = 4×10^3 mm ; Central deflection (y_C) = 10 mm and modulus of elasticity (E) = 6 GPa = 6×10^3 N/mm^2.

Let W = Magnitude of the load,

We know that moment of inertia of the beam section,

$$I = \frac{bd^3}{12} = \frac{140 \times (240)^3}{12} = 161.3 \times 10^6 \text{ mm}^4$$

and deflection of the beam at its centre (y_C),

$$10 = \frac{Wl^3}{48EI} = \frac{W \times (4 \times 10^3)^3}{48 \times (6 \times 10^3) \times (161.3 \times 10^6)}$$

$$\therefore \quad W = \frac{10}{1.38 \times 10^{-3}} = 7.25 \times 10^3 \text{ N} = \mathbf{7.25\ kN} \quad \textbf{Ans.}$$

Simply Supported beam with a udl problem:

EXAMPLE 19.6. *A simply supported beam of span 4 m is carrying a uniformly distributed load of 2 kN/m over the entire span. Find the maximum slope and deflection of the beam. Take EI for the beam as 80 × 10^9 N-mm^2.*

SOLUTION. Given: Span (l) = 4 m = 4×10^3 mm ; Uniformly distributed load (w) = 2 kN/m = 2 N/mm and flexural rigidity (E) = 80×10^9 N-mm^2.

Maximum slope of the beam

We know that maximum slope of the beam,

$$i_A = \frac{wl^3}{24EI} = \frac{2 \times (4 \times 10^3)^3}{34 \times (80 \times 10^9)} = \mathbf{0.067\ rad} \qquad \textbf{Ans.}$$

Maximum deflection of the beam

We also know that maximum deflection of the beam,

$$y_C = \frac{5wl^4}{384EI} = \frac{5 \times 2 \times (4 \times 10^3)^4}{384 \times (80 \times 10^9)} = \mathbf{83.3\ mm} \qquad \textbf{Ans.}$$